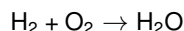
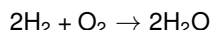

1 Application to Chemical Equations

When a chemical reaction takes place a number of molecules combine to produce new molecules. Hence, when hydrogen H_2 and oxygen O_2 molecules combine, the result is water H_2O . We express this as

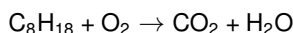


Individual atoms are neither created nor destroyed, so the number of hydrogen and oxygen atoms going into the reaction must equal the number coming out (in the form of water). In this case the reaction is said to be *balanced*. Note that each hydrogen molecule H_2 consists of two atoms as does each oxygen molecule O_2 , while a water molecule H_2O consists of two hydrogen atoms and one oxygen atom. In the above reaction, this requires that twice as many hydrogen molecules enter the reaction; we express this as follows:

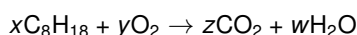


This is now balanced because there are 4 hydrogen atoms and 2 oxygen atoms on each side of the reaction.

Example 1. Balance the following reaction for burning octane C_8H_{18} in oxygen O_2 :



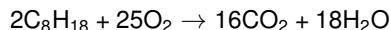
where CO_2 represents carbon dioxide. We must find positive integers x , y , z , and w such that



Equating the number of carbon, hydrogen, and oxygen atoms on each side gives $8x = z$, $18x = 2w$ and $2y = 2z + w$, respectively. These can be written as a homogeneous linear system

$$\begin{array}{ccccccc} 8x & & & - & z & & = 0 \\ 18x & & & & & - & 2w = 0 \\ & 2y & - & 2z & - & w & = 0 \end{array}$$

which can be solved by gaussian elimination. In larger systems this is necessary but, in such a simple situation, it is easier to solve directly. Set $w = t$, so that $x = \frac{1}{9}t$, $z = \frac{8}{9}t$, $2y = \frac{16}{9}t + t = \frac{25}{9}t$. But x , y , z , and w must be positive integers, so the smallest value of t that eliminates fractions is 18. Hence, $x = 2$, $y = 25$, $z = 16$, and $w = 18$, and the balanced reaction is



The reader can verify that this is indeed balanced.

It is worth noting that this problem introduces a new element into the theory of linear equations: the insistence that the solution must consist of positive integers.

Practice Problems

Problems ??-??

Balance the chemical reaction.

Problem 1 $CH_4 + O_2 \rightarrow CO_2 + H_2O$. This is the burning of methane CH_4 .

Problem 2 $\dots NH_3 + \dots CuO \rightarrow N_2 + \dots Cu + \dots H_2O$. Here NH_3 is ammonia, CuO is copper oxide, Cu is copper, and N_2 is nitrogen.

Problem 3 $CO_2 + H_2O \rightarrow C_6H_{12}O_6 + O_2$. This is called the photosynthesis reaction— $C_6H_{12}O_6$ is glucose.

Problem 4 $\dots Pb(N_3)_2 + \dots Cr(MnO_4)_2 \rightarrow \dots Cr_2O_3 + \dots MnO_2 + \dots Pb_3O_4 + \dots NO$.

Problem 5 $KNO_3 + H_2CO_3 \rightarrow K_2CO_3 + HNO_3$

Problem 6 $AgI + Na_2S \rightarrow Ag_2S + NaI$

Problem 7 $Ba_3N_2 + H_2O \rightarrow Ba(OH)_2 + NH_3$

Problem 8 $CaCl_2 + Na_3PO_4 \rightarrow Ca_3(PO_4)_2 + NaCl$

Text Source

This application was adapted from Section 1.4 of Keith Nicholson's *Linear Algebra with Applications*. (CC-BY-NC-SA)

W. Keith Nicholson, *Linear Algebra with Applications*, Lyryx 2018, Open Edition, p. 32 Practice Problems ??, ??, ??, and ?? are Exercises 1.2.60 from Keith Nicholson's *Linear Algebra with Applications*. (CC-BY-NC-SA)

Ken Kuttler, *A First Course in Linear Algebra*, Lyryx 2017, Open Edition, p. 49.