
1 Challenge Problems

Challenge Problems for Chapter 3

Problem 1 Suppose $\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$ is a linearly independent set in $\mathbb{R}^n$, and that $\mathbf{w}$ is not in $\text{span}(\mathbf{v}_1, \dots, \mathbf{v}_m)$.

1. Is $\mathbf{w}$ in $\text{span}(\mathbf{v}_1 + \mathbf{w}, \dots, \mathbf{v}_m + \mathbf{w})$ (YES, / NO)
2. Is $\{\mathbf{v}_1 + \mathbf{w}, \dots, \mathbf{v}_m + \mathbf{w}\}$ linearly independent?
(YES, / NO)

Problem 2 Suppose $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ are the rows of the 3×3 matrix A . Then we can interpret the solution to the system of equations $[A|\mathbf{0}]$ as the intersection of three planes containing the origin. Discuss what this intersection would look like geometrically if the reduced row echelon form of $[A|\mathbf{0}]$ is of the form:

1.

$$\left[\begin{array}{ccc|c} 1 & 0 & * & 0 \\ 0 & 1 & * & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

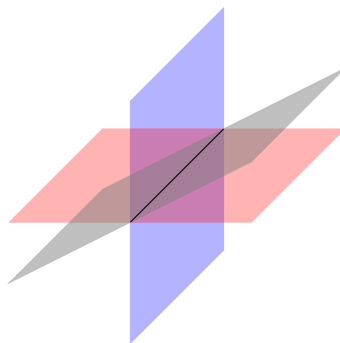
2.

$$\left[\begin{array}{ccc|c} 1 & * & * & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

3. Are there any other possibilities?

Problem 3 Show that for any matrix A , $\text{rref}(A)$ has a row of zeros if and only if one of the rows of A can be expressed as a linear combination of the others.

Problem 4 Recall that a plane in $\mathbb{R}^3$ has an equation of the form $ax+by+cz = d$, where a, b, c are the components of a normal vector to the plane. Suppose three planes intersect in a line, as shown below.



We can find the line of intersection by solving a system of equations. Suppose the augmented matrix $[A|\mathbf{d}]$ corresponds to this system.

1. Explain why $\text{rref}[A|\mathbf{d}]$ will have a row of zeros.
2. Consider the normal vectors to the three planes. What geometric property of these particular three normal vectors explains the row of zeros in the reduced row-echelon form?

Bibliography

The first problem came from the end of Chapter 1 of Keith Nicholson's *Linear Algebra with Applications*. (CC-BY-NC-SA)

W. Keith Nicholson, *Linear Algebra with Applications*, Lyryx 2018, Open Edition, pp. 33–34.