
1 Challenge Problems

Challenge Problems for Chapter 7

Problem 1 Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos t & -\sin t \\ 0 & \sin t & \cos t \end{bmatrix}$$

Does there exist a value of t for which this matrix fails to have an inverse? Explain.

Click the arrow to see answer. No. It has a nonzero determinant for all t .

Problem 2 If A, B , and C are each $n \times n$ matrices and ABC is invertible, show why each of A, B , and C are invertible.

Click the arrow to see answer.

This follows because $\det(ABC) = \det(A)\det(B)\det(C)$ and if this product is nonzero, then each determinant in the product is nonzero and so each of these matrices is invertible.

Problem 3 Suppose A, B are $n \times n$ matrices and that $AB = I$. Show that then $BA = I$.

Problem 4 Suppose A is an upper triangular matrix. Show that A^{-1} exists if and only if all elements of the main diagonal are non zero. Is it true that A^{-1} will also be upper triangular? Explain. Could the same be concluded for lower triangular matrices?

Problem 5 Let A, B , and C denote $n \times n$ matrices. Assume that $\det A = -1$, $\det B = 2$, and $\det C = 3$. Evaluate

1. $\det(A^3 BC^T B^{-1})$
2. $\det(B^2 C^{-1} AB^{-1} C^T)$
3. $\det(A^{-1} B^{-1} AB)$

Problem 6 If A and B are $n \times n$ matrices such that $AB = -BA$, and if n is odd, show that either A or B has no inverse.

Problem 7 Show that no 3×3 matrix A exists such that $A^2 + I = O$. Find a 2×2 matrix A with this property.

Problem 8 Show that $\det(A + B^T) = \det(A^T + B)$ for any $n \times n$ matrices A and B .

Bibliography

These problems come from Chapter 3 of Ken Kuttler's *A First Course in Linear Algebra*. (CC-BY)

Ken Kuttler, *A First Course in Linear Algebra*, Lyryx 2017, Open Edition, pp. 272–315.

Several problems came from the end of Chapter 3 of Keith Nicholson's *Linear Algebra with Applications*. (CC-BY-NC-SA)

W. Keith Nicholson, *Linear Algebra with Applications*, Lyryx 2018, Open Edition, pp. 167.