
1 Challenge Problems

Challenge Problems for Chapter 6

Problem 1 Argue geometrically to prove that the following transformations are linear:

1. Rotation of the plane about the origin through angle θ .
2. Reflection of the plane about the line $y = mx$.

Problem 2 Find the matrix of the linear transformation which rotates every vector in $\mathbb{R}^3$ counter clockwise about the z axis when viewed from the positive z axis through an angle of 30° and then reflects through the xy plane.

Click on the arrow to see answer.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \cos\left(\frac{\pi}{6}\right) & -\sin\left(\frac{\pi}{6}\right) & 0 \\ \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2}\sqrt{3} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Problem 3 Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation such that $\ker(T) = \mathbb{R}^n$. Find $\text{im}(T)$.

Problem 4 Let $\mathbf{v}$ be a non-zero vector in $\mathbb{R}^n$. Given any vector $\mathbf{w}$ in $\mathbb{R}^m$, show there exists a linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ with $T(\mathbf{v}) = \mathbf{w}$.

Problem 5 Given $\mathbf{y}$ in $\mathbb{R}^n$, define $S_{\mathbf{y}} : \mathbb{R}^n \rightarrow \mathbb{R}$ by $S_{\mathbf{y}}(\mathbf{x}) = \mathbf{x} \cdot \mathbf{y}$ for all $\mathbf{x}$ in $\mathbb{R}^n$.

1. Show that $S_{\mathbf{y}}$ is a linear transformation.
2. Show that every linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}$ arises in this way (i.e. $T = S_{\mathbf{y}}$ for some $\mathbf{y}$ in $\mathbb{R}^n$.)

Bibliography

Some of these problems come from Chapter 5 of Ken Kuttler's [A First Course in Linear Algebra](#). (CC-BY)

Ken Kuttler, *A First Course in Linear Algebra*, Lyryx 2017, Open Edition, pp. 272–315.

Some of these problems come from the end of Chapter 7 of Keith Nicholson's [Linear Algebra with Applications](#). (CC-BY-NC-SA)

W. Keith Nicholson, *Linear Algebra with Applications*, Lyryx 2018, Open Edition, pp. 376–386.