
1 The Characteristic Equation

Let A be an $n \times n$ matrix. In the previous section we learned that the eigenvectors and eigenvalues of A are vectors $\mathbf{x}$ and scalars λ that satisfy the equation

$$A\mathbf{x} = \lambda\mathbf{x} \quad (1)$$

We listed a few reasons why we are interested in finding eigenvalues and eigenvectors, but we did not give any process for finding them. In this section we will focus on a process which can be used for small matrices. For larger matrices, the best methods we have are iterative methods, and we will explore some of these in [The Power Method](#) and [the Dominant Eigenvalue](#).

For an $n \times n$ matrix, we will see that the eigenvalues are the roots of a polynomial called the *characteristic polynomial*. So finding eigenvalues is equivalent to solving a polynomial equation of degree n . Finding the corresponding eigenvectors turns out to be a matter of computing the null space of a matrix, as the following exploration demonstrates.

Exploration 1 If a vector $\mathbf{x}$ is an eigenvector satisfying Equation (1), then clearly it also satisfies $A\mathbf{x} - \lambda\mathbf{x} = (0/0)$.

It seems natural at this point to try to factor. We would love to “factor out” $\mathbf{x}$. Here is the procedure:

$$\begin{aligned} A\mathbf{x} - \lambda\mathbf{x} &= \mathbf{0} \\ A\mathbf{x} - \lambda I\mathbf{x} &= \mathbf{0} \\ (A - \lambda I)\mathbf{x} &= \mathbf{0} \end{aligned}$$

This shows that any eigenvector $\mathbf{x}$ of A is in the (row space/ column space/ null space) of the related matrix, $A - \lambda I$.

Since eigenvectors are non-zero vectors, this means that A will have eigenvectors if and only if the null space of $A - \lambda I$ is nontrivial. The only way that $\text{null}(A - \lambda I)$ can be nontrivial is if $\text{rank}(A - \lambda I) (= / < / >) n$.

If the rank of an $n \times n$ matrix is less than n , then the matrix is singular. Since $(A - \lambda I)$ must be singular for any eigenvalue λ , we see that λ is an eigenvalue of A if and only if

$$\det(A - \lambda I) = \dots \dots \quad (2)$$

In theory, Exploration ?? offers us a way to find eigenvalues. To find the eigenvalues of A , one can solve Equation (2) for λ .

Eigenvalues

Definition 1. The equation

$$\det(A - \lambda I) = 0$$

is called the *characteristic equation* of A .

Example 1. Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Compute the eigenvalues of this matrix using the characteristic equation.

Explanation.

$$\begin{aligned}\det(A - \lambda I) &= \begin{vmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{vmatrix} = (2 - \lambda)^2 - 1 \\ &= \lambda^2 - 4\lambda + 3 \\ &= (\lambda - 1)(\lambda - 3)\end{aligned}$$

The characteristic equation $(\lambda - 1)(\lambda - 3) = 0$ has solutions $\lambda_1 = 1$ and $\lambda_2 = 3$. These are the eigenvalues of A .

Example 2. Let $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$. Compute the eigenvalues of B using the characteristic equation. (List your answers in an increasing order.)

$\lambda_1 = \dots$ and $\lambda_2 = \dots$

Example 3. Let $C = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$. Compute the eigenvalues of C using the characteristic equation.

Explanation.

$$\begin{aligned}\det(C - \lambda I) &= \begin{vmatrix} 2 - \lambda & 1 & 1 \\ 1 & 2 - \lambda & 1 \\ 1 & 1 & 2 - \lambda \end{vmatrix} \\ &= (2 - \lambda) \begin{vmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & 2 - \lambda \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 - \lambda \\ 1 & 1 \end{vmatrix} \\ &= (2 - \lambda)^3 - (2 - \lambda) - ((2 - \lambda) - 1) + 1 - (2 - \lambda) \\ &= 8 - 12\lambda + 6\lambda^2 - \lambda^3 - 2 + \lambda - 2 + \lambda + 1 + 1 - 2 + \lambda \\ &= 4 - 9\lambda + 6\lambda^2 - \lambda^3 \\ &= -(\lambda - 4)(\lambda - 1)^2\end{aligned}$$

Matrix C has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 4$.

Remark 1. The factor $(\lambda - 1)$ appears twice. This repeated factor gives rise to the eigenvalue $\lambda_1 = 1$. We say that the eigenvalue $\lambda_1 = 1$ has *algebraic multiplicity* 2.

Definition 2. *Algebraic multiplicity* of an eigenvalue is its multiplicity as a root of the characteristic equation.

It is not always possible to completely factor the characteristic polynomial using only real numbers. However, a fundamental fact from algebra is that every degree n polynomial has n roots (counting multiplicity) provided that we allow complex numbers. This is why sometimes eigenvalues and their corresponding eigenvectors involve complex numbers. The next example illustrates this point.

Example 4. Let $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$. Compute the eigenvalues of this matrix.

Explanation.

$$\begin{aligned}\det(D - \lambda I) &= \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & 1 - \lambda & 1 \\ 0 & -1 & 1 - \lambda \end{vmatrix} \\ &= -\lambda \begin{vmatrix} 1 - \lambda & 1 \\ -1 & 1 - \lambda \end{vmatrix} \\ &= -\lambda((1 - \lambda)^2 + 1) \\ &= -\lambda(\lambda^2 - 2\lambda + 2)\end{aligned}$$

So one of the eigenvalues of D is $\lambda = 0$. To get the other eigenvalues we must solve $\lambda^2 - 2\lambda + 2 = 0$. Using the quadratic formula, we compute that $\lambda_1 = 1 + i$ and $\lambda_2 = 1 - i$ are also eigenvalues of D .

Exploration 2 Let $T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$. Compute the eigenvalues of this matrix. (List your answers in an increasing order.)

$$\lambda_1 = \dots, \quad \lambda_2 = \dots, \quad \text{and} \quad \lambda_3 = \dots$$

What do you observe about the eigenvalues?

What property of the matrix makes this “coincidence” possible?

The matrix in Exploration Problem ?? is a triangular matrix, and the property you observed holds in general.

Theorem 1. Let T be a triangular matrix. Then the eigenvalues of T are the entries on the main diagonal.

Proof See Practice Problem ??. ■

Corollary 1. Let D be a diagonal matrix. Then the eigenvalues of D are the entries on the main diagonal.

One final note about eigenvalues. We began this section with the sentence, “In theory, then, to find the eigenvalues of A , one can solve Equation (??) for λ .” In general, one does not attempt to compute eigenvalues by solving the characteristic equation of a matrix, as there is no simple way to solve this polynomial equation for $n > 4$. Instead, one can often approximate the eigenvalues using *iterative methods*. We will explore some of these techniques in [The Power Method and the Dominant Eigenvalue](#).

Eigenvectors

Once we have computed an eigenvalue λ of an $n \times n$ matrix A , the next step is to compute the associated eigenvectors. In other words, we seek vectors $\mathbf{x}$ such that $A\mathbf{x} = \lambda\mathbf{x}$, or equivalently,

$$(A - \lambda I)\mathbf{x} = \mathbf{0} \tag{3}$$

As we have seen in the previous section, a (non-zero) scalar multiple of an eigenvector is an eigenvector. So for any given eigenvalue λ there are infinitely many eigenvectors associated with it. In fact, the eigenvectors associated with λ , together with the zero vector, form a subspace of $\mathbb{R}^n$.

Theorem 2. Let A be an $n \times n$ matrix and let λ be an eigenvalue of A . Then the set of all eigenvectors associated with λ , together with the zero vector, is a subspace of $\mathbb{R}^n$.

Proof See Practice Problems ?? and ??. ■

This motivates the following definition.

Definition 3. The set of all eigenvectors associated with a given eigenvalue of a matrix, together with the zero vector, is known as the *eigenspace* associated with that eigenvalue.

Given an eigenvalue λ , there is an associated eigenspace S_λ . Our goal is to find a basis of S_λ . Then any eigenvector $\mathbf{x}$ associated with λ will be a linear combination of the vectors in that basis. In seeking a basis for S_λ , we are trying to find a basis for the set of vectors that satisfy $(A - \lambda I)\mathbf{x} = \mathbf{0}$, which means we seek a basis for $\text{null}(A - \lambda I)$.

Let's return to the examples we did in the first part of this section.

Example 5. (Finding eigenvectors for Example ??)

Recall that $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 3$. Compute a basis for the eigenspace associated with each of these eigenvalues.

Explanation. Eigenvectors associated with the eigenvalue $\lambda_1 = 1$ are in the null space of $A - I$. So we seek a basis for $\text{null}(A - I)$. We compute:

$$\text{rref}(A - I) = \text{rref} \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix},$$

From this we see that the eigenspace S_1 associated with $\lambda_1 = 1$ consists of vectors of the form $\begin{bmatrix} -1 \\ 1 \end{bmatrix} t$. This means

that $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$ is one possible basis for S_1 .

In a similar way, we compute a basis for S_3 , the subspace of all eigenvectors associated with the eigenvalue $\lambda_2 = 3$. Now we compute:

$$\text{rref}(A - 3I) = \text{rref} \left(\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix},$$

Vectors in the null space have the form $\begin{bmatrix} 1 \\ 1 \end{bmatrix} t$. This means that $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ is one possible basis for the eigenspace S_3 .

These computations reinforce the geometric insights into this problem that we gained in the previous section.

Example 6. (Finding eigenvectors for Example ??) We know from Example ?? that $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ has eigenvalues $\lambda_1 = 0$ and $\lambda_2 = 4$. Compute a basis for the eigenspace associated with each of these eigenvalues.

Explanation. Let's begin by finding a basis for the eigenspace $\mathcal{S}_0$, which is the subspace of $\mathbb{R}^n$ consisting of eigenvectors corresponding to the eigenvalue $\lambda_1 = 0$. We need to compute a basis for $\text{null}(B - 0I) = \text{null}(B)$. We compute:

$$\text{rref}(B) = \text{rref}\left(\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}\right) = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix},$$

From this we see that an eigenvector in $\mathcal{S}_0$ has the form $\begin{bmatrix} -1/2 \\ 1 \end{bmatrix} t$. This means that $\left\{ \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} \right\}$ is one possible basis for the eigenspace $\mathcal{S}_0$. By letting $t = -2$, we obtain an arguably nicer-looking basis: $\left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\}$.

To compute a basis for $\mathcal{S}_4$, the subspace of all eigenvectors associated to the eigenvalue $\lambda_2 = 4$, we compute:

$$\text{rref}(B - 4I) = \text{rref}\left(\begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix}$$

From this we find that $\left\{ \begin{bmatrix} 1 \\ \dots \end{bmatrix} \right\}$ is one possible basis for the eigenspace $\mathcal{S}_4$.

Example 7. (Finding eigenvectors for Example ??) We know from Example ?? that $C = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 4$. Compute a basis for the eigenspace associated to each of these eigenvalues.

Explanation. We first find a basis for the eigenspace $\mathcal{S}_1$. We need to compute a basis for $\text{null}(C - I)$. We compute:

$$\text{rref}(C - I) = \text{rref}\left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

Notice that there are two free variables. The eigenvectors in $\mathcal{S}_1$ have the form

$$\begin{bmatrix} -s - t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

So one possible basis for the eigenspace $\mathcal{S}_1$ is given by $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

Next we find a basis for the eigenspace $\mathcal{S}_4$. We need to compute a basis for $\text{null}(C - 4I)$. We compute:

$$\text{rref}(C - 4I) = \text{rref}\left(\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

This time there is one free variable. The eigenvectors in $\mathcal{S}_4$ have the form $\begin{bmatrix} t \\ t \\ t \end{bmatrix}$, so a possible basis for the eigenspace

$$\mathcal{S}_4 \text{ is given by } \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Remark 2. Recall that $\lambda_1 = 1$ has algebraic multiplicity 2. Note that $\dim(\mathcal{S}_{\lambda_1}) = 2$. We say that λ_1 has geometric multiplicity 2. We have to be careful, and not draw hasty conclusions. Geometric and algebraic multiplicity are NOT always the same! We will revisit this issue later in the text.

Definition 4. The *geometric multiplicity* of an eigenvalue is the dimension of the eigenspace corresponding to it.

Example 8. (Finding eigenvectors for Example ??) We know from Example ?? that $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$ has eigen-

values $\lambda = 0$, $\lambda_1 = 1 + i$, and $\lambda_2 = 1 - i$. Compute a basis for the eigenspace associated with each eigenvalue.

Explanation. We first find a basis for the eigenspace $\mathcal{S}_0$. We need to compute a basis for $\text{null}(D - 0I) = \text{null}(D)$. We compute:

$$\text{rref}(D) = \text{rref} \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \right) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

From this we see that for any eigenvector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in $\mathcal{S}_0$ we have $x_2 = 0$ and $x_3 = 0$, but x_1 is a free variable. So one possible basis for the eigenspace $\mathcal{S}_0$ is given by

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

Next we find a basis for the eigenspace $\mathcal{S}_{1+i}$. We need to compute a basis for $\text{null}(D - (1 + i)I)$. We compute:

$$\begin{aligned} \text{rref}(D - (1 + i)I) &= \text{rref} \left(\begin{bmatrix} -(1 + i) & 0 & 0 \\ 0 & 1 - (1 + i) & 1 \\ 0 & -1 & 1 - (1 + i) \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & i \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

There is one free variable. Setting $x_3 = t$, we get $x_1 = 0$ and $x_2 = ti$. From this we see that eigenvectors in $\mathcal{S}_{1+i}$ have the form $\begin{bmatrix} 0 \\ -i \\ 1 \end{bmatrix} t$, so a possible basis for the eigenspace $\mathcal{S}_{1+i}$ is given by $\left\{ \begin{bmatrix} 0 \\ -i \\ 1 \end{bmatrix} \right\}$. We ask you in Practice Problem

?? to show that $\left\{ \begin{bmatrix} 0 \\ i \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathcal{S}_{1-i}$.

We conclude this section by establishing the significance of a matrix having an eigenvalue of zero.

Theorem 3. A square matrix has an eigenvalue of zero if and only if it is singular.

Proof A square matrix A is singular if and only if $\det A = 0$. But $\det A = 0$ if and only if $\det A - 0I = 0$, which is true if and only if zero is an eigenvalue of A . ■

Practice Problems

Problem 3 Compute the eigenvalues of A , and find the corresponding eigenspaces.

$$A = \begin{bmatrix} 4 & 1 \\ 8 & -3 \end{bmatrix}$$

Answer: (List the eigenvalues in an increasing order.)

$$\lambda_1 = \dots, \quad \lambda_2 = \dots$$

A basis for S_{λ_1} is $\left\{ \begin{bmatrix} \dots \\ 1 \end{bmatrix} \right\}$. A basis for S_{λ_2} is $\left\{ \begin{bmatrix} \dots \\ 1 \end{bmatrix} \right\}$.

Problem 4 Compute the eigenvalues of A , and find the corresponding eigenspaces.

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

Answer:

$$\lambda_1 = \dots + \dots i, \quad \lambda_2 = \dots - \dots i$$

A basis for S_{λ_1} is $\left\{ \begin{bmatrix} i \\ \dots \end{bmatrix} \right\}$. A basis for S_{λ_2} is $\left\{ \begin{bmatrix} -i \\ \dots \end{bmatrix} \right\}$.

Problem 5 Let $T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$. Compute a basis for each of the eigenspaces of this matrix, S_1 , S_5 , and S_9 .

Problem 6 Let

$$A = \begin{bmatrix} 9 & 2 & 8 \\ 2 & -6 & -2 \\ -8 & 2 & -5 \end{bmatrix}$$

Question 6.1 Compute the eigenvalues of A .

Answer:

(List your answers in an increasing order.)

$$\lambda_1 = \dots, \quad \lambda_2 = \dots, \quad \lambda_3 = \dots$$

Question 6.2 Compute a basis for each of the eigenspaces of A , S_{λ_1} , S_{λ_2} , and S_{λ_3} .

Answer: A basis for S_{λ_1} is $\left\{ \begin{bmatrix} 1 \\ \dots \\ \dots \end{bmatrix} \right\}$, a basis for S_{λ_2} is $\left\{ \begin{bmatrix} -2 \\ \dots \\ \dots \end{bmatrix} \right\}$,
and a basis for S_{λ_3} is $\left\{ \begin{bmatrix} 2 \\ \dots \\ \dots \end{bmatrix} \right\}$.

Problem 7 Complete Example ?? by showing that a basis for S_{1-i} is given by $\left\{ \begin{bmatrix} 0 \\ i \\ 1 \end{bmatrix} \right\}$, where S_{1-i} is the

eigenspace associated with the eigenvalue $\lambda = 1 - i$ of the matrix $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$.

Problem 8 Prove Theorem ??.

Problem 9 Recall that a vertical stretch/compression of the plane is a linear transformation whose standard matrix is

$$M_v = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$$

Find the eigenvalues of M_v . Find a basis for the eigenspace corresponding to each eigenvalue.

Answer: A basis for S_1 is $\left\{ \begin{bmatrix} 1 \\ \dots \end{bmatrix} \right\}$ and a basis for S_k is $\left\{ \begin{bmatrix} \dots \\ 1 \end{bmatrix} \right\}$

Sketch several vectors in each eigenspace and use geometry to explain why the eigenvectors you sketched make sense.

Problem 10 Recall that a horizontal shear of the plane is a linear transformation whose standard matrix is

$$M_{hs} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

Find the eigenvalue of M_{hs} .

Answer: $\lambda = \dots$

Find a basis for the eigenspace corresponding to λ .

Answer: A basis for S is $\left\{ \begin{bmatrix} 1 \\ \dots \end{bmatrix} \right\}$

What is the algebraic multiplicity of λ ? ...

What is the geometric multiplicity of λ ? ...

Problem 11 Recall that a counterclockwise rotation of the plane through angle θ is a linear transformation whose standard matrix is

$$M_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Verify that the eigenvalues of M_θ are

$$\lambda = \cos \theta \pm \sqrt{\cos^2 \theta - 1}$$

Use algebra, then use geometry to explain why λ is a real number if and only if θ is a multiple of π .

Suppose $\theta = \pi$. Then $\lambda = \dots$ has algebraic multiplicity $\dots$. Which of the following describes the corresponding eigenspace?

Multiple Choice:

1. All vectors in $\mathbb{R}^2$.
2. All vectors along the x -axis.
3. All vectors along the y -axis.
4. All vectors along the line $y = x$.

Problem 12 Recall that a reflection of the plane about the line $y = mx$ is a linear transformation whose standard matrix is

$$M_{y=mx} = \frac{1}{1+m^2} \begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$$

Verify that the eigenvalues of $M_{y=mx}$ are 1 and -1 .

Find a basis for eigenspaces S_1 and S_{-1} . (For simplicity, assume that $m \neq 0$.)

Answer: A basis for S_1 is $\left\{ \begin{bmatrix} 1 \\ \dots \end{bmatrix} \right\}$ and a basis for S_{-1} is $\left\{ \begin{bmatrix} m \\ \dots \end{bmatrix} \right\}$

Choose the best description of S_1 .

Multiple Choice:

1. All vectors in $\mathbb{R}^2$.
2. All vectors with "slope" m .
3. All vectors with "slope" $1/m$.
4. All vectors with "slope" $-1/m$.

Choose the best description of S_{-1} .

Multiple Choice:

1. All vectors along the line $y = mx$.

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2. All vectors parallel to the x -axis.
 3. All vectors parallel to the y -axis.
 4. All vectors perpendicular to the line $y = mx$.

Use geometry to explain why the eigenspaces you found make sense.

Problem 13 Prove that the eigenvectors associated with an eigenvalue λ of an $n \times n$ matrix A , together with the zero vector, form a subspace of $\mathbb{R}^n$.

Exercise Source

Practice Problem ?? is adopted from Problem 7.1.11 of Ken Kuttler's [A First Course in Linear Algebra](#). (CC-BY)
Ken Kuttler, *A First Course in Linear Algebra*, Lyryx 2017, Open Edition, p. 361.