
1 Octave for Chapters 5-6

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The templates in this section provide sample Octave code for finding subspaces associated with matrices as well as the kernel and image of a linear transformation. You can access our code through the link at the bottom of each template. Feel free to modify the code and experiment to learn more!

You can write your own code using Octave software or online Octave cells. To access Octave cells online, go to the [Sage Math Cell Webpage](#), select OCTAVE as the language, enter your code, and press EVALUATE.

To "save" or share your online code, click on the *Share* button, select *Permalink*, then copy the address directly from the browser window. You can store this link to access your work later or share this link with others. You will need to get a new Permalink every time you modify the code.

Octave Tutorial

Column Space of a Matrix (Image of a Linear Transformation)

Recall that the column space of a matrix is the span of the columns of the matrix. In [Subspaces of \$\mathbb{R}^n\$ Associated with Matrices](#) we introduced the following procedure for finding the column space.

Procedure 1 (??). Given a matrix B , a basis for $\text{col}(B)$ can be found as follows:

1. Find $\text{rref}(B)$ (or any row-echelon form B' of B .)
2. Identify the pivot columns of $\text{rref}(B)$ (or B').
3. The columns of B corresponding to the pivot columns of $\text{rref}(B)$ (or B') form a basis for $\text{col}(B)$.

We will implement this procedure in Octave.

Template 1. Let

$$A = \begin{bmatrix} 2 & -1 & 1 & -4 & 1 \\ 1 & 0 & 3 & 3 & 0 \\ -2 & 1 & -1 & 5 & 2 \\ 4 & -1 & 7 & 2 & 1 \end{bmatrix}$$

In Example ?? we found that columns 1, 2 and 4 of A constitute a basis of $\text{col}(A)$. We will now verify this using Octave.

We can find the reduced row-echelon form of A using the *rref* function, and identify the pivot columns visually. We can also take advantage of the expanded version of the *rref* function to identify the pivot columns automatically, as shown below.

```

% Define A
A = [2 -1 1 -4 1;
     1 0 3 3 0;
     -2 1 -1 5 2;
     4 -1 7 2 1];

% Compute rref of A
[rref_A, pivot_cols] = rref(A);

% the pivot column indices are...
pivot_cols

% Extract the pivot columns from A, using pivot column indices
colSpace_basis = A(:, pivot_cols)

```

[Link to code.](#)

When you study linear transformations, you will learn that the column space of a matrix is the image of the linear transformation induced by the matrix. So, Template ?? can be used to find the image of a linear transformation.

Null Space of a Matrix (Kernel of a Linear Transformation)

Recall the following definition of the null space of a matrix from [Subspaces of \$\mathbb{R}^n\$ Associated with Matrices](#).

Definition 1 (??). Let A be an $m \times n$ matrix. The *null space* of A , denoted by $\text{null}(A)$, is the set of all vectors $\mathbf{x}$ in $\mathbb{R}^n$ such that $A\mathbf{x} = \mathbf{0}$.

We can use the reduced row-echelon form (*rref* function) to find the null space, as shown in Example ?? of [Subspaces of \$\mathbb{R}^n\$ Associated with Matrices](#). We will omit the details here.

We can also use the *null* function to find a basis for the null space of a matrix. This function finds an *orthonormal* basis for the null space. An orthonormal set of vectors consists of unit vectors that are pairwise orthogonal. We demonstrate the *null* function below.

Template 2. We will use the matrix from Example ??. Let

$$A = \begin{bmatrix} 2 & -1 & 1 & -4 & 1 \\ 1 & 0 & 3 & 3 & 0 \\ -2 & 1 & -1 & 5 & 2 \\ 4 & -1 & 7 & 2 & 1 \end{bmatrix}$$

Find $\text{null}(A)$.

```

% Define A
A = [2 -1 1 -4 1;
     1 0 3 3 0;
     -2 1 -1 5 2;
     4 -1 7 2 1];

```

```
% Basis for null(A)
% Octave constructs an orthonormal basis, meaning that the basis elements are pairwise orthogonal unit vectors
B=null(A)

% To find the dimension of the null space, find the number of columns in B
[rows, cols]=size(B)
```

[Link to code.](#)

The basis for the null space we found in Template ?? looks different from what you would have found by hand using the *rref* function. In Problem ?? you will verify that *B* in Template ?? spans the null space of *A*.

If you are familiar with linear transformations, you may recall that the *kernel* of a linear transformation induced by a matrix is the null space of the matrix. The *null* function gives us a way to compute the kernel.

Octave Exercises

For Problems ?? and ??, use Octave to find $\text{col}(A)$, $\text{null}(A)$, and their respective dimensions. Verify that the Rank-Nullity Theorem (??) holds for *A*.

Problem 1 Let

$$A = \begin{bmatrix} 2 & -1 & 5 & -1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 3 & -2 \\ 1 & -1 & 3 & -2 \end{bmatrix}$$

Problem 2 Let

$$A = \begin{bmatrix} -2 & -9 & -33 & -22 \\ 0 & 4 & 15 & 10 \\ 1 & 3 & 12 & 10 \\ -3 & -9 & -36 & -24 \end{bmatrix}$$

Problem 3 Let

$$H = \text{span} \left\{ \begin{bmatrix} 2 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 15 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ 15 \\ 6 \\ 3 \end{bmatrix} \right\}$$

Find the dimension of *H* and determine a basis.

Problem 4 Find the rank of the following matrix. Also find a basis for the row and column spaces.

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 0 & 3 \\ 3 & 9 & 1 & -7 & 0 & 8 \\ 1 & 3 & 1 & -3 & 1 & -1 \\ 1 & 3 & -1 & -1 & -2 & 10 \end{bmatrix}$$

Problem 5 Find the rank of the following matrix. Also find a basis for the row and column spaces.

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 7 & 3 \\ 3 & 9 & 1 & -7 & 23 & 8 \\ 1 & 3 & 1 & -3 & 9 & 2 \\ 1 & 3 & -1 & -1 & 5 & 4 \end{bmatrix}$$

Problem 6 Find the rank of the following matrix. Also find a basis for the row and column spaces.

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 7 & 0 \\ 3 & 1 & 10 & 0 & 23 & 0 \\ 1 & 1 & 4 & 1 & 7 & 0 \\ 1 & -1 & 2 & -2 & 9 & 1 \end{bmatrix}$$

Problem 7 In Template ?? we used the `null` function to find a basis B for the null space of matrix

$$A = \begin{bmatrix} 2 & -1 & 1 & -4 & 1 \\ 1 & 0 & 3 & 3 & 0 \\ -2 & 1 & -1 & 5 & 2 \\ 4 & -1 & 7 & 2 & 1 \end{bmatrix}$$

1. Verify that the product of A with of each element of the basis B is $\mathbf{0}$.
2. Use `rref(A)` to find another basis for the null space.
3. The basis we found using the `null` function and the basis you found in part (b) look very different. How do we know that they span the same subspace?

Problem 8 Use some of our earlier matrices, together with their null spaces, for the following experiment.

- Given a matrix, find the product of the matrix with some vector in its null space. What do you observe? Why does this make sense?
- Formulate a conjecture about the relationship between the rows of a matrix and the elements of its null space.
- Prove your conjecture.

Bibliography

Matrices used in Problems ??-?? come from the end of Chapter 4 of Ken Kuttler's *A First Course in Linear Algebra*. (CC-BY)

Ken Kuttler, *A First Course in Linear Algebra*, Lyryx 2017, Open Edition, pp. 227–232.