
1 Octave for Chapter 2

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The templates in this section provide sample Octave code for solving systems of equations. You can access our code through the link at the bottom of each template. Feel free to modify the code and experiment to learn more!

You can write your own code using Octave software or online Octave cells. To access Octave cells online, go to the [Sage Math Cell Webpage](#), select OCTAVE as the language, enter your code, and press EVALUATE.

To "save" or share your online code, click on the *Share* button, select *Permalink*, then copy the address directly from the browser window. You can store this link to access your work later or share this link with others. You will need to get a new Permalink every time you modify the code.

Octave Tutorial

When solving systems by hand, you utilized Gauss-Jordan elimination to find rref. Having the reduced row echelon form available made it easy to write out the solution(s) or determine that the system is inconsistent. We can find rref and the rank of a matrix using Octave as follows.

Template 1. Consider the system

$$\begin{array}{cccccccl} x & - & y & & & & = & 0 \\ 2x & - & 2y & + & z & + & 2w & = & 4 \\ & & y & & & + & w & = & 0 \\ & & & & 2z & + & w & = & 5 \end{array} \quad (1)$$

(You may remember this system from [Augmented Matrix Notation and Elementary Row Operations](#).) We will solve this system by finding the reduced row echelon form of the corresponding augmented matrix in Octave.

```
% Define the augmented matrix
% corresponding to the coefficient matrix A and vector b
A_b = [1 -1 0 0 0;
       2 -2 1 2 4;
       0 1 0 1 0;
       0 0 2 1 5];

% Find rref of A_b
rref(A_b)

% Find the rank of A_b
rank(A_b)
```

[Link to code.](#)

If you start with a matrix A and a separate vector $\mathbf{b}$, you can create an augmented matrix without re-typing as follows:

```
% Define the coefficient matrix A
A = [1 -1 0 0;
     2 -2 1 2;
     0 1 0 1;
     0 0 2 1];

% Define vector b
b = [0;4;0;5];

% Create an augmented matrix
A_b=[A b]
```

[Link to code.](#)

Template 2. We can access individual rows, columns, and entries of a matrix. Let

$$A = \begin{bmatrix} 2 & -1 & 3 & -1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 1 & -2 \end{bmatrix}$$

```
% Define a matrix
A=[2 -1 3 -1; 1 0 2 1; 1 -1 1 -2]

% Third row of A
row_3=A(3,:)

% Second column of A
col_2=A(:,2)

% Second row third entry of A
a_23=A(2,3)
```

[Link to code.](#)

Example 1. Use Octave code to show one step of Gaussian elimination by subtracting half of the first row from the second row of the given matrix A .

Explanation. % Define matrix A

```
A=[2 -1 3 -1; 1 0 2 1; 1 -1 1 -2]
```

```
% Replace the second row of A with second row minus half the first row.
A(2,:)=-1/2*A(1,:)+A(2,:)
```

[Link to code.](#)

Octave Exercises

Problem 1 Given the code below, write out the system of equations and the corresponding solution(s).

```
% Coefficient matrix A
A=[2 -4 1 1 3;
0 2 -1 4 -1;
2 -2 1 1 4];

% Column vector b
b=[-2; 1; 4];

% Augmented matrix A_b
A_b=[A b];

% Reduced row-echelon form
rref_A_b=rref(A_b)
```

[Link to code.](#)

Problem 2 Solve the following system using Octave.

$$\begin{array}{rrrrrrrrcl} x_1 & - & x_2 & + & 2x_3 & & + & x_5 & = & -1 \\ -x_1 & & & & -x_3 & + & x_4 & & = & 1 \\ 2x_1 & + & 2x_2 & & & - & x_4 & + & 2x_5 & = & 1 \\ x_1 & & & & & + & 2x_4 & - & x_5 & = & 0 \end{array} \quad (2)$$

Problem 3 Solve the following system using Octave.

$$\begin{array}{rrrrrrrrcl} 3x_1 & + & x_2 & + & 4x_3 & - & x_4 & + & 2x_5 & = & 10 \\ x_1 & & & + & 2x_3 & - & x_4 & + & x_5 & = & 3 \\ & & x_2 & + & x_3 & + & x_4 & + & 3x_5 & = & 2 \\ 2x_1 & - & x_2 & - & x_3 & & & + & 3x_5 & = & -2 \\ 4x_1 & + & x_2 & - & 5x_3 & + & 2x_4 & + & x_5 & = & 1 \end{array} \quad (3)$$

Problem 4 Suppose the graph of function f of the form $f(x) = ax^3 + bx^2 + cx + d$ passes through points $(-2, 1)$, $(0, 4)$, $(1, -2)$ and $(4, 5)$. Set up a system of linear equations to find the coefficients a , b , c and d . Update the coefficients in the GeoGebra interactive below to sketch the graph.

GeoGebra link: <https://www.geogebra.org/m/xry6un6h>

Problem 5 The following code generates a random 20×3 matrix A whose entries are integers -1 , 0 , and 1 , and a random 20×1 vector b whose components are integers -1 , 0 , and 1 .

```
% A is a randomly generated 20 by 3 matrix whose entries are -1, 0, 1.
A=randi([-1,1],20,3);

% b is a randomly generated column vector whose entries are -1, 0, 1.
b=randi([-1,1],20,1);

% Augmented matrix A_b
A_b=[A b];

% Reduced row-echelon form
rref_A_b=rref(A_b)

% Rank of A_b
rank(A_b)
```

[Link to code.](#)

Run the code several times to generate several random combinations of A and $\mathbf{b}$ and find the rank and the reduced row-echelon form of $[A|\mathbf{b}]$. Are you getting the answers you were expecting? Answer the following questions based on your theoretical knowledge.

What are the possibilities for the rank of such $[A|\mathbf{b}]$?

Multiple Choice:

1. The rank can be any positive integer less than or equal to 20.
2. The rank can be any positive integer less than or equal to 3.
3. The rank is always equal to 4.
4. The rank can be any non-negative integer less than or equal to 4.

What are the possibilities for the number of solutions of such $[A|\mathbf{b}]$?

Multiple Choice:

1. This system always has a unique solution.
2. This system may have no solutions, infinitely many solutions or a unique solution.
3. This system will always have infinitely many solutions.
4. This system will always be infeasible.

There is a good chance that your experiments have led you to believe that $\text{rank}[A|\mathbf{b}] = 4$. Can you manually construct $[A|\mathbf{b}]$, satisfying the condition of the problem, with a smaller rank? How many solutions does your system have? Why do you think it is so rare to randomly generate a system $[A|\mathbf{b}]$ of rank less than 4?

Further Considerations

Issues related to computational aspects of linear algebra are outside the scope of this text. However, we want the reader to be aware of some potential problems that may arise.

Warning 1. The `rref` function utilizes a modified version of the Gauss-Jordan elimination algorithm ([Reference Link](#)). For some matrices, implementing this algorithm directly leads to round-off errors and other problems that are outside of the scope of this text. We illustrate what can happen in the next example.

Example 2. We will use a 12×12 Hilbert matrix to illustrate how the `rref` function can fail. You can learn more about Hilbert matrices [here](#).

```
% Use a 12 by 12 Hilbert matrix
A=hilb(12);

% Theoretical rank of A is 12, but the following computation shows a rank of 11
rank(A)

% rref of A should be the identity matrix, but the following result contains a row of zeros
rref(A)
```

[Link to code.](#)

Acknowledgements

The inspiration for Example ?? came from [this Stack Overflow](#) comment.