
Exercise 1 True or False? If False, you should come up with a counterexample. If True, can you give a proof?

1. $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, given by $T(x, y) = (x, -y)$, is a linear transformation.

Multiple Choice:

- (i) True
- (ii) False

2. $T : \mathbb{R}^n \rightarrow \mathbb{R}$, given by $T(\mathbf{x}) = \mathbf{x} \cdot \mathbf{z}$ for some fixed vector $\mathbf{z} \in \mathbb{R}^n$, is a linear transformation.

Multiple Choice:

- (i) True
- (ii) False

3. $T : \mathbb{R} \rightarrow \mathbb{R}$, given by $T(x) = x^2$, is a linear transformation.

Multiple Choice:

- (i) True
- (ii) False

4. Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation and let $\mathbf{v}_1, \dots, \mathbf{v}_k$ denote vectors in $\mathbb{R}^n$. If $\{T(\mathbf{v}_1), \dots, T(\mathbf{v}_k)\}$ is linearly independent, then $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ is also linearly independent.

Multiple Choice:

- (i) True
- (ii) False

5. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation and suppose $\mathbf{v}_1, \mathbf{v}_2$ denote vectors in $\mathbb{R}^2$. If $\{\mathbf{v}_1, \mathbf{v}_2\}$ is linearly independent, then $\{T(\mathbf{v}_1), T(\mathbf{v}_2)\}$ is also linearly independent.

Multiple Choice:

- (i) True
- (ii) False

Source

[Nicholson] W. Keith Nicholson, *Linear Algebra with Applications*, Lyryx 2021, Open Edition, Exercises 7.1.1, 7.1.2, and 7.1.17,