
Exercise 1 Let V denote the set of ordered triples (x, y, z) and define addition in V as in $\mathbb{R}^3$. For each of the following definitions of scalar multiplication, select all properties of scalar multiplication for vector spaces that hold and decide whether V is a vector space.

1.

$$a(x, y, z) = (ax, y, az)$$

Select all properties of scalar multiplication that hold true.

Select All Correct Answers:

- (i) Closure under scalar multiplication
- (ii) Distributive Property over Vector Addition: $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$
- (iii) Distributive Property over Scalar Addition: $(k + p)\mathbf{u} = k\mathbf{u} + p\mathbf{u}$
- (iv) Associative Property for Scalar Multiplication: $k(p\mathbf{u}) = (kp)\mathbf{u}$
- (v) Multiplication by 1: $1\mathbf{u} = \mathbf{u}$

Is V a vector space?

Multiple Choice:

- (i) V is a vector space.
- (ii) V is not a vector space.

2.

$$a(x, y, z) = (ax, 0, az)$$

Select all properties of scalar multiplication that hold true.

Select All Correct Answers:

- (i) Closure under scalar multiplication
- (ii) Distributive Property over Vector Addition: $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$
- (iii) Distributive Property over Scalar Addition: $(k + p)\mathbf{u} = k\mathbf{u} + p\mathbf{u}$
- (iv) Associative Property for Scalar Multiplication: $k(p\mathbf{u}) = (kp)\mathbf{u}$
- (v) Multiplication by 1: $1\mathbf{u} = \mathbf{u}$

Is V a vector space?

Multiple Choice:

- (i) V is a vector space.
- (ii) V is not a vector space.

3.

$$a(x, y, z) = (0, 0, 0)$$

Select all properties of scalar multiplication that hold true.

Select All Correct Answers:

- (i) Closure under scalar multiplication
- (ii) Distributive Property over Vector Addition: $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$
- (iii) Distributive Property over Scalar Addition: $(k + p)\mathbf{u} = k\mathbf{u} + p\mathbf{u}$
- (iv) Associative Property for Scalar Multiplication: $k(p\mathbf{u}) = (kp)\mathbf{u}$
- (v) Multiplication by 1: $1\mathbf{u} = \mathbf{u}$

Is V a vector space?

Multiple Choice:

- (i) V is a vector space.
- (ii) V is not a vector space.

4.

$$a(x, y, z) = (2ax, 2ay, 2az)$$

Select all properties of scalar multiplication that hold true.

Select All Correct Answers:

- (i) Closure under scalar multiplication
- (ii) Distributive Property over Vector Addition: $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$
- (iii) Distributive Property over Scalar Addition: $(k + p)\mathbf{u} = k\mathbf{u} + p\mathbf{u}$
- (iv) Associative Property for Scalar Multiplication: $k(p\mathbf{u}) = (kp)\mathbf{u}$
- (v) Multiplication by 1: $1\mathbf{u} = \mathbf{u}$

Is V a vector space?

Multiple Choice:

- (i) V is a vector space.
- (ii) V is not a vector space.

Source

[Nicholson] W. Keith Nicholson, *Linear Algebra with Applications*, Lyryx 2021, Open Edition, Exercises 6.1.1.