
1 Essential Vocabulary

Linear combination of vectors A vector $\mathbf{v}$ is said to be a *linear combination* of vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ if

$$\mathbf{v} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \dots + a_n\mathbf{v}_n$$

for some scalars $a_1, a_2, \dots, a_n$.

Linearly dependent vectors Let $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ be vectors of $\mathbb{R}^n$. We say that the set $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$ is *linearly independent* if the only solution to

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_k\mathbf{v}_k = \mathbf{0}$$

is the *trivial solution* $c_1 = c_2 = \dots = c_k = 0$.

If, in addition to the trivial solution, a *non-trivial solution* (not all $c_1, c_2, \dots, c_k$ are zero) exists, then we say that the set $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$ is *linearly dependent*.

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Redundant vectors Let $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$ be a set of vectors in $\mathbb{R}^n$. If we can remove one vector without changing the span of this set, then that vector is *redundant*. In other words, if

$$\text{span}(\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k) = \text{span}(\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{j-1}, \mathbf{v}_{j+1}, \dots, \mathbf{v}_k)$$

we say that $\mathbf{v}_j$ is a redundant element of $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$, or simply redundant.

Span of a set of vectors Let $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p$ be vectors in $\mathbb{R}^n$. The set S of all linear combinations of $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p$ is called the *span* of $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p$. We write

$$S = \text{span}(\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p)$$

and we say that vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p$ *span* S . Any vector in S is said to be *in the span* of $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p$. The set $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p\}$ is called a *spanning set* for S .
