
1 Essential Vocabulary

Algebraic multiplicity of an eigenvalue The multiplicity of an eigenvalue as a root of the characteristic equation.

Characteristic equation The equation

$$\det(A - \lambda I) = 0$$

is called the *characteristic equation* of A .

Characteristic polynomial The polynomial

$$\det(A - \lambda I)$$

is called the *characteristic polynomial* of A .

Diagonalizable matrix Let A be an $n \times n$ matrix. Then A is said to be *diagonalizable* if there exists an invertible matrix P such that

$$P^{-1}AP = D$$

where D is a diagonal matrix. In other words, a matrix A is diagonalizable if it is similar to a diagonal matrix, $A \sim D$.

Eigenspace If λ is an eigenvalue of an $n \times n$ matrix, the set of all eigenvectors associated to λ along with the zero vector is the *eigenspace* associated to λ . The eigenspace is a subspace of $\mathbb{R}^n$.

Eigenvalue Let A be an $n \times n$ matrix. We say that a scalar λ is an *eigenvalue* of A if

$$A\mathbf{x} = \lambda\mathbf{x}$$

for some nonzero vector $\mathbf{x}$. We say that $\mathbf{x}$ is an *eigenvector* of A associated with the eigenvalue λ .

Eigenvalue decomposition If λ is an eigenvalue of an $n \times n$ matrix, the set of all eigenvectors associated to λ along with the zero vector is the *eigenspace* associated to λ . The eigenspace is a subspace of $\mathbb{R}^n$.

Eigenvector Let A be an $n \times n$ matrix. We say that a non-zero vector $\mathbf{x}$ is an *eigenvector* of A if

$$A\mathbf{x} = \lambda\mathbf{x}$$

for some scalar λ . We say that λ is an *eigenvalue* of A associated with the eigenvector $\mathbf{x}$.

Geometric multiplicity of an eigenvalue The *geometric multiplicity* of an eigenvalue λ is the dimension of the corresponding eigenspace S_λ .

Gershgorin disk A circle in the complex plane which has a diagonal entry of a matrix as its center and the sum of the absolute values of the other entries in that row (or column) as its radius.

Gershgorin's Theorem *Gershgorin's theorem* says that the n eigenvalues of an $n \times n$ matrix can be found in the region in the complex plane consisting of the n Gershgorin disks.

Power method (and its variants) The *power method* is an iterative method for computing the dominant eigenvalue of a matrix. Its variants can compute the smallest eigenvalue or the eigenvalue closest to some target.

Properties of similar matrices Similar matrices must have the same...

1. determinant,
2. rank,
3. trace,
4. characteristic polynomial,
and
5. eigenvalues.

Similar matrices If A and B are $n \times n$ matrices, we say that A and B are *similar*, if $B = P^{-1}AP$ for some invertible matrix P . In this case we write $A \sim B$.