
1 Essential Vocabulary

Composition of linear transformations Let U , V and W be vector spaces, and let $T : U \rightarrow V$ and $S : V \rightarrow W$ be linear transformations. The *composition* of S and T is the transformation $S \circ T : U \rightarrow W$ given by

$$(S \circ T)(\mathbf{u}) = S(T(\mathbf{u}))$$

The matrix of a composition is the product of the matrices corresponding to the transformations in the composition, in the same order.

Image of a linear transformation Let V and W be vector spaces, and let $T : V \rightarrow W$ be a linear transformation. The *image* of T , denoted by $\text{im}(T)$, is the set

$$\text{im}(T) = \{T(\mathbf{v}) : \mathbf{v} \in V\}$$

In other words, the image of T consists of individual images of all vectors of V .

Inverse of a linear transformation Let V and W be vector spaces, and let $T : V \rightarrow W$ be a linear transformation. A transformation $S : W \rightarrow V$ that satisfies $S \circ T = \text{id}_V$ and $T \circ S = \text{id}_W$ is called an *inverse* of T . If T has an inverse, T is called *invertible*.

Kernel of a linear transformation Let V and W be vector spaces, and let $T : V \rightarrow W$ be a linear transformation. The *kernel* of T , denoted by $\ker(T)$, is the set

$$\ker(T) = \{\mathbf{v} : T(\mathbf{v}) = \mathbf{0}\}$$

In other words, the kernel of T consists of all vectors of V that map to $\mathbf{0}$ in W .

Linear transformation A transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is called a *linear transformation* if the following are true for all vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, and scalars k .

$$T(k\mathbf{u}) = kT(\mathbf{u})$$

$$T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$$

Standard matrix of a linear transformation Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation. Then the matrix

$$A = \begin{bmatrix} | & | & & | \\ T(\mathbf{e}_1) & T(\mathbf{e}_2) & \dots & T(\mathbf{e}_n) \\ | & | & & | \end{bmatrix}$$

is known as the *standard matrix of the linear transformation* T .
