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# 1 Essential Vocabulary

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Column matrix (or column vector) A matrix with  $m$  rows and only 1 column.

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Diagonal matrix A matrix  $A = [a_{ij}]$  where  $a_{ij} = 0$  whenever  $i \neq j$

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Dimensions of a matrix An  $m \times n$  matrix is a matrix with  $m$  rows and  $n$  columns.

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Elementary matrix A matrix formed by applying one elementary row operation to the identity matrix.

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Homogeneous system A system of linear equations is called *homogeneous* if the system can be written in the form

$$\begin{array}{ccccccccc} a_{11}x_1 & + & a_{12}x_2 & + & \dots & + & a_{1n}x_n & = & 0 \\ a_{21}x_1 & + & a_{22}x_2 & + & \dots & + & a_{2n}x_n & = & 0 \\ & & & & \vdots & & & & \\ a_{m1}x_1 & + & a_{m2}x_2 & + & \dots & + & a_{mn}x_n & = & 0 \end{array},$$

or as a matrix equation as  $A\mathbf{x} = \mathbf{0}$ .

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Identity matrix A square matrix with ones as diagonal entries and zeros for the remaining entries.

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Inverse of a square matrix Let  $A$  be an  $n \times n$  matrix. An  $n \times n$  matrix  $B$  is called an *inverse* of  $A$  if

$$AB = BA = I$$

where  $I$  is an  $n \times n$  identity matrix. If such an inverse matrix exists, we say that  $A$  is *invertible*. If an inverse does not exist, we say that  $A$  is not invertible. The inverse of  $A$  is denoted by  $A^{-1}$ .

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LU factorization A factorization  $A = LU$  where  $L$  is lower triangular and  $U$  is upper triangular with ones on the diagonal (called unit upper triangular). It is useful for solving  $Ax = b$ .

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Matrix A rectangular array of numbers. It has  $m$  rows and  $n$  columns for some positive integers  $m$  and  $n$ .

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Matrix addition Let  $A = [a_{ij}]$  and  $B = [b_{ij}]$  be two  $m \times n$  matrices. Then the *sum of matrices*  $A$  and  $B$ , denoted by  $A + B$ , is an  $m \times n$  matrix given by

$$A + B = [a_{ij} + b_{ij}]$$

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Matrix factorization Representing a matrix as a product of two or more matrices.

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Matrix multiplication Let  $A$  be an  $m \times n$  matrix whose rows are vectors  $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n$ . Let  $B$  be an  $n \times p$  matrix with columns  $\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_p$ . Then the matrix product  $AB$  is an  $m \times p$  matrix with entries given by the dot products

$$AB = \begin{bmatrix} - & \mathbf{r}_1 & - \\ - & \mathbf{r}_2 & - \\ & \vdots & \\ - & \mathbf{r}_i & - \\ & \vdots & \\ - & \mathbf{r}_m & - \end{bmatrix} \begin{bmatrix} | & | & & | & & | \\ \mathbf{b}_1 & \mathbf{b}_2 & \dots & \mathbf{b}_j & \dots & \mathbf{b}_p \\ | & | & & | & & | \end{bmatrix} =$$
$$= \begin{bmatrix} \mathbf{r}_1 \cdot \mathbf{b}_1 & \mathbf{r}_1 \cdot \mathbf{b}_2 & \dots & \mathbf{r}_1 \cdot \mathbf{b}_j & \dots & \mathbf{r}_1 \cdot \mathbf{b}_p \\ \mathbf{r}_2 \cdot \mathbf{b}_1 & \mathbf{r}_2 \cdot \mathbf{b}_2 & \dots & \mathbf{r}_2 \cdot \mathbf{b}_j & \dots & \mathbf{r}_2 \cdot \mathbf{b}_p \\ \vdots & \vdots & & \vdots & & \vdots \\ \mathbf{r}_i \cdot \mathbf{b}_1 & \mathbf{r}_i \cdot \mathbf{b}_2 & \dots & \mathbf{r}_i \cdot \mathbf{b}_j & \dots & \mathbf{r}_i \cdot \mathbf{b}_p \\ \vdots & \vdots & & \vdots & & \vdots \\ \mathbf{r}_m \cdot \mathbf{b}_1 & \mathbf{r}_m \cdot \mathbf{b}_2 & \dots & \mathbf{r}_m \cdot \mathbf{b}_j & \dots & \mathbf{r}_m \cdot \mathbf{b}_p \end{bmatrix}$$

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Matrix powers If  $A$  is a square matrix then we can define  $A^k$  to be the result of multiplying  $A$  by itself  $k$  times.

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Negative of a matrix The additive inverse of a matrix, formed by multiplying the matrix by the scalar  $-1$ .

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Nonsingular matrix A square matrix  $A$  is said to be *nonsingular* provided that  $\text{rref}(A) = I$ . Otherwise we say that  $A$  is *singular*.

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Partitioned matrices - block multiplication Subdividing a matrix into submatrices using imaginary horizontal and vertical lines - used to multiply matrices more efficiently.

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Permutation matrix A matrix formed by permuting the rows of the identity matrix.

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Properties of matrix algebra Addition and scalar multiplication of matrices have many nice properties. Matrix multiplication is not commutative, but the associative and distributive laws hold. The inverse and transpose operations of a product follow the "shoes and socks" rule.

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Row matrix A matrix with only 1 row and  $n$  columns.

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Scalar multiple of a matrix If  $A = \begin{bmatrix} a_{ij} \end{bmatrix}$  and  $k$  is a scalar, then  $kA = \begin{bmatrix} ka_{ij} \end{bmatrix}$ .

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**Singular matrix** A square matrix  $A$  is said to be *singular* provided that  $\text{rref}(A)$  is NOT the identity matrix. If instead  $\text{rref}(A) = I$ , we say that  $A$  is *nonsingular*.

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**Square matrix** A matrix with the same number of rows and columns.

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**Symmetric matrix** An  $n \times n$  matrix  $A$  is said to be *symmetric* if  $A = A^T$ . It is said to be *skew symmetric* if  $A = -A^T$ .

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**Transpose of a matrix** Let  $A = [a_{ij}]$  be an  $m \times n$  matrix. Then the *transpose of  $A$* , denoted by  $A^T$ , is the  $n \times m$  matrix given by switching the rows and columns:

$$A^T = [a_{ij}]^T = [a_{ji}]$$

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**Zero matrix** The  $m \times n$  *zero matrix* is the  $m \times n$  matrix having every entry equal to zero. The zero matrix is denoted by  $O$ .

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