
1 Essential Vocabulary

Basis A set S of vectors is called a *basis* of $\mathbb{R}^n$ (or a basis of a subspace V of $\mathbb{R}^n$) provided that

1. $\text{span}(S) = \mathbb{R}^n$ (or V)
2. S is linearly independent.

Closed under addition A set V is said to be *closed under addition* if for each element $\mathbf{u} \in V$ and $\mathbf{v} \in V$ the sum $\mathbf{u} + \mathbf{v}$ is also in V .

Closed under scalar multiplication A set V is said to be *closed under scalar multiplication* if for each element $\mathbf{v} \in V$ and for each scalar $k \in \mathbb{R}$ the product $k\mathbf{v}$ is also in V .

Column space of a matrix Let A be an $m \times n$ matrix. The *column space* of A , denoted by $\text{col}(A)$, is the subspace of $\mathbb{R}^m$ spanned by the columns of A .

Coordinate vector with respect to a basis Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$ be an ordered basis. Then the *coordinate vector* $\mathbf{v}$ is the column vector $\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{bmatrix}$ such that $\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_k\mathbf{v}_k$.

Dimension Let V be a subspace of $\mathbb{R}^n$. The *dimension* of V is the number, m , of elements in any basis of V . We write

$$\dim(V) = m$$

Null space of a matrix Let A be an $m \times n$ matrix. The *null space* of A , denoted by $\text{null}(A)$, is the set of all vectors $\mathbf{x}$ in $\mathbb{R}^n$ such that $A\mathbf{x} = \mathbf{0}$. It is a subspace of $\mathbb{R}^n$.

Nullity of a matrix Let A be a matrix. The dimension of the null space of A is called the *nullity* of A .

$$\dim(\text{null}(A)) = \text{nullity}(A)$$

Ordered basis A basis in which the elements appear in a specific fixed order. Establishing an order is necessary because a coordinate vector with respect to a given basis relies on the order in which the basis elements appear.

Rank of a matrix Let A be a matrix. The dimension of the row space of A is called the *rank* of A .

$$\dim(\text{row}(A)) = \text{rank}(A)$$

Rank-Nullity theorem Let A be an $m \times n$ matrix. Then

$$\text{rank}(A) + \text{nullity}(A) = n$$

Row space of a matrix Let A be an $m \times n$ matrix. The *row space* of A , denoted by $\text{row}(A)$, is the subspace of $\mathbb{R}^n$ spanned by the rows of A .

Subspace Suppose that V is a nonempty subset of $\mathbb{R}^n$ that is closed under addition and closed under scalar multiplication. Then V is a *subspace* of $\mathbb{R}^n$.