
1 Essential Vocabulary

Augmented matrix Every linear system

$$\begin{array}{cccccc} a_{11}x_1 & + & a_{12}x_2 & + & \dots & + & a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & \dots & + & a_{2n}x_n & = & b_2 \\ & & & & \vdots & & & & \\ a_{m1}x_1 & + & a_{m2}x_2 & + & \dots & + & a_{mn}x_n & = & b_m \end{array}$$

can be written in the *augmented matrix form* as follows:

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{array} \right]$$

The array to the left of the vertical bar is called the *coefficient matrix* of the linear system and is often given a capital letter name, like A . The vertical array to the right of the bar is called a *constant vector*.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

We will sometimes use the following notation to represent an augmented matrix.

$$\left[A \mid \mathbf{b} \right]$$

Back substitution When a matrix is in row-echelon form, we can compute the solution to the system by starting from the last equation and working backwards. This process is known as *back substitution*.

Basic variable See Leading variable

Coefficient matrix A *coefficient matrix* is a matrix whose entries are the coefficients of a system of linear equations. For the system

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the *coefficient matrix* is $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$.

Consistent system A system of equations that has at least one solution.

Convergence when the iterates of an iterative method approach a solution

Divergence when the iterates of an iterative method fail to approach a solution

Elementary row operations The following three operations performed on a linear system are called *elementary row operations*.

1. Switching the order of equations (rows) i and j :

$$R_i \leftrightarrow R_j$$

2. Multiplying both sides of equation (row) i by the same non-zero constant, k , and replacing equation i with the result:

$$kR_i \rightarrow R_i$$

3. Adding k times equation (row) i to equation (row) j , and replacing equation j with the result:

$$R_j + kR_i \rightarrow R_j$$

Equivalent linear systems Linear systems are called *equivalent* if they have the same solution set.

Free variable When a linear system is in row-echelon form, the variables corresponding to columns that do not have any leading coefficients (if there are any) are known as *free variables*.

Gauss-Jordan elimination The process of using the elementary row operations on a matrix to transform it into reduced row-echelon form is called *Gauss-Jordan elimination*.

Gauss-Seidel Method An iterative method for solving linear systems that is a refinement of the Jacobi method, where we use computed values of variables alternately for quicker convergence.

Gaussian elimination The process of using the elementary row operations on a matrix to transform it into row-echelon form is called *Gaussian Elimination*.

Inconsistent system A system of equations that has no solution.

Iterative method A technique where we repeat the same procedure (called an *iteration*) many times (usually using a computer), and we obtain approximate solutions which we hope “converge to” the actual solution.

Jacobi’s method An iterative method for solving a system of equations where one variable is isolated in each equation in order to compute the coordinate of the next iterate.

Leading entry (leading 1) The first non-zero entry in a row of a matrix (when read from left to right) is called the *leading entry*. When the leading entry is 1, we refer to it as a *leading 1*.

Leading variable (or Basic variable) When a coefficient matrix is in row echelon form, a *leading variable* is a variable corresponding to a column of the matrix with at least one leading entry.

Linear equation A *linear equation* in variables $x_1, \dots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$$

where $a_1, \dots, a_n$ and b are constants.

Pivot In Gaussian elimination, an entry chosen to become a leading coefficient used to get zeros in the remaining rows.

Rank of a matrix The *rank* of matrix A , denoted by $\text{rank}(A)$, is the number of nonzero rows that remain after we reduce A to row-echelon form by elementary row operations.

Reduced row echelon form A matrix that is already in *row-echelon* form is said to be in *reduced row-echelon form* if:

1. Each leading entry is 1
2. All entries *above* and below each leading 1 are 0

Row echelon form A matrix is said to be in *row-echelon form* if:

1. All entries below each leading entry are 0.
 2. Each leading entry is in a column to the right of the leading entries in the rows above it.
 3. All rows of zeros, if there are any, are located below non-zero rows.
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Row equivalent matrices Two matrices A and B are said to be row equivalent if there is a sequence of elementary row operations that converts A to B .

System of linear equations A finite set of linear (degree 1) equations each with the same variables.
