
1 Review for Exam 1

Exercise 1 Select all pairs of parallel vectors.

Select All Correct Answers:

1. $\begin{bmatrix} -3 \\ -6 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 10 \\ -5 \end{bmatrix}$

2. $\begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$

3. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

4. $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -8 \\ 0 \\ 4 \end{bmatrix}$

Exercise 2 Find Q such that $\overrightarrow{PQ} = \mathbf{v}$ if

$$P(-1, 2, 2), \mathbf{v} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$
$$Q(\dots, \dots, \dots)$$

Exercise 3 Find a unit vector $\mathbf{u}$ in the direction of $\begin{bmatrix} 7 \\ -1 \\ 5 \end{bmatrix}$

$$\mathbf{u} = \dots \dots \begin{bmatrix} 7 \\ -1 \\ 5 \end{bmatrix}$$

Enter the exact answer. No decimal approximations.

Exercise 4 Select all pairs of orthogonal vectors.

Select All Correct Answers:

$$1. \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$2. \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -6 \\ 4 \end{bmatrix}$$

$$3. \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$$

$$4. \begin{bmatrix} -1 \\ 3 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \\ -1 \end{bmatrix}$$

Exercise 5 Use the augmented matrix notation and elementary row operations to find all solutions (if any) to the system.

$$\begin{array}{rcl} 3x & - & y = 0 \\ 2x & - & 3y = 1 \end{array}$$

(... ..)

Enter your answers using fractions. If the system is inconsistent, type DNE in each answer cell.

Exercise 6 Use the augmented matrix notation and elementary row operations to find all solutions (if any) to the system.

$$\begin{array}{rcl} 3x & - & y = 2 \\ -6x & + & 2y = -4 \end{array}$$

(... .., t)

Enter your answers using fractions.

Exercise 7 Use the augmented matrix notation and elementary row operations to find all solutions (if any) to the system.

$$\begin{array}{rcl} -2x & + & 3y + 3z = -9 \\ 3x & - & 4y + z = 5 \\ -5x & + & 7y + 2z = -14 \end{array}$$

(... ..,, t)

If the system is inconsistent, type DNE in each answer cell.

Exercise 8 Use the augmented matrix notation and elementary row operations to find all solutions (if any) to the system.

$$\begin{array}{rrcr} x & + & 2y & - & z & = & 2 \\ 2x & + & 5y & - & 3z & = & 1 \\ x & + & 4y & - & 3z & = & 3 \end{array}$$

(... ..,,)

If the system is inconsistent, type DNE in each answer cell.

Exercise 9 Use the augmented matrix notation and elementary row operations to find all solutions (if any) to the system.

$$\begin{array}{rrcr} 3x & - & 2y & + & z & = & -2 \\ x & - & y & + & 3z & = & 5 \\ -x & + & y & + & z & = & -1 \end{array}$$

(... ..,,)

If the system is inconsistent, type DNE in each answer cell.

Exercise 10 The augmented matrix of a system of linear equations has been carried to the following by row operations. Solve the system.

$$\left[\begin{array}{cccccc|c} 1 & -2 & 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 5 & 0 & -3 & -1 \\ 0 & 0 & 0 & 0 & 1 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

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Exercise 11 The augmented matrix of a system of linear equations has been carried to the following by row operations. Solve the system.

$$\left[\begin{array}{ccccc|c} 1 & -1 & 2 & 4 & 6 & 2 \\ 0 & 1 & 2 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

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Exercise 12 Consider the following augmented matrix in which * denotes an arbitrary number and ■ denotes a nonzero number.

$$\left[\begin{array}{cccc|c} \blacksquare & * & * & * & * \\ 0 & \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & \blacksquare & * \\ 0 & 0 & 0 & 0 & \blacksquare \end{array} \right]$$

Which of the following is true? Select all that apply.

Select All Correct Answers:

1. The corresponding system of equations is consistent.
2. The corresponding system of equations is inconsistent.
3. The rank of the augmented matrix is 4.
4. The rank of the augmented matrix is 5.
5. The rank of the augmented matrix is 6.
6. There are no free variables.
7. There is one free variable.
8. There are two free variables.
9. The corresponding system has a unique solution.
10. The corresponding system has infinitely many solutions.

Exercise 13 Consider an augmented matrix of the form

$$\left[A \mid \mathbf{b} \right]$$

corresponding to a system of linear equations.

True or False?

1. If there is more than one solution, $\left[A \mid \mathbf{b} \right]$ has a row of zeros.

Multiple Choice:

- (i) True
- (ii) False
2. If $\left[A \mid \mathbf{b} \right]$ has a row of zeros, there is more than one solution.

Multiple Choice:

- (i) *True*
- (ii) *False*

3. If there is no solution, $\text{rref}(A)$ has a row of zeros.

Multiple Choice:

- (i) *True*
- (ii) *False*

4. If $\text{rref}(A)$ has a row of zeros, there is no solution.

Multiple Choice:

- (i) *True*
- (ii) *False*

5. There is NO system that is inconsistent for every $\mathbf{b}$.

Multiple Choice:

- (i) *True*
- (ii) *False*

6. If the system is consistent for some choice of $\mathbf{b}$, it is consistent for every choice of $\mathbf{b}$.

Multiple Choice:

- (i) *True*
- (ii) *False*

Exercise 14 Express $\begin{bmatrix} 127 \\ -26 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 101 \\ 50 \end{bmatrix}$ and $\begin{bmatrix} 25 \\ 42 \end{bmatrix}$.

$$\dots\dots \begin{bmatrix} 101 \\ 50 \end{bmatrix} + \dots\dots \begin{bmatrix} 25 \\ 42 \end{bmatrix} = \begin{bmatrix} 127 \\ -26 \end{bmatrix}$$

Exercise 15 Suppose $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$. What are the implications of the statement: $\mathbf{w}$ is in the span of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. Select all that apply.

Select All Correct Answers:

1. Vector $\mathbf{w}$ a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$
2. The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\}$ contains redundant vectors.
3. Vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}$ are linearly independent.

4. Vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}$ are linearly dependent.
5. Vector $\mathbf{v}_1$ can be written as a linear combination of $\mathbf{v}_2, \mathbf{v}_3, \mathbf{w}$.
6. The equation $a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3 + d\mathbf{w} = \mathbf{0}$ has a non-trivial solution.

Exercise 16 Express $\begin{bmatrix} -1 \\ 4 \\ 5 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$.

If this is not possible, enter DNE in each answer cell.

$$\begin{bmatrix} -1 \\ 4 \\ 5 \end{bmatrix} = \dots \dots \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} + \dots \dots \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix} + \dots \dots \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

Exercise 17 Suppose three planes given by the equations below intersect at a single point.

$$\begin{aligned} a_1x + a_2y + a_3z &= 0 \\ b_1x + b_2y + b_3z &= 0 \\ c_1x + c_2y + c_3z &= 0 \end{aligned}$$

What can be said about the vectors $\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$, $\begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$ and $\begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix}$?

Select all that apply.

Select All Correct Answers:

1. Nothing can be deduced from the information given.
2. The three vectors are linearly independent.
3. The three vectors are linearly dependent.
4. The three vectors are pairwise orthogonal.
5. The first vector can be written as a linear combination of the other two.
6. The equation $\alpha \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} + \beta \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} + \omega \begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix} = \mathbf{0}$ has only the trivial solution.

Exercise 18 Use the navigation bar arrows in the interactive below to advance the construction steps. Use the final result to express $\mathbf{w}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

GeoGebra link: <https://www.geogebra.org/m/mafugdge>

$$\mathbf{w} = \dots \mathbf{u} + \dots \mathbf{v}$$

Exercise 19 True or False?

1. If $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent, then $\{\mathbf{x}, \mathbf{y}, \mathbf{x} + \mathbf{y}\}$ is linearly independent.

Multiple Choice:

- (i) True
- (ii) False

2. If $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent, then $\{\mathbf{y}, \mathbf{z}\}$ is linearly independent.

Multiple Choice:

- (i) True
- (ii) False

3. If $\{\mathbf{y}, \mathbf{z}\}$ is linearly dependent, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent.

Multiple Choice:

- (i) True
- (ii) False

4. If $a\mathbf{x} + b\mathbf{y} + c\mathbf{z} = \mathbf{0}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent.

Multiple Choice:

- (i) True
- (ii) False

Exercise 20 Find the inverse of the following matrix.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -5 & 7 & -11 \\ -2 & 3 & -5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix}$$

Exercise 21 Consider the system of equations:

$$\begin{array}{rcrcrcrcrcl} x & - & 3y & + & z & = & 2 \\ -3x & - & 4y & + & z & = & 0 \\ & & 2y & - & z & = & 1 \end{array}$$

Let A be the coefficient matrix corresponding to this system, and let $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

Suppose that we find that $(1, -2, -5)$ is a unique solution to this system. What does this tell us? Select ALL that apply.

Select All Correct Answers:

1. A is invertable.
2. Vector $\mathbf{b}$ is in the span of the columns of A .
3. Columns of A are linearly dependent.
4. Equation $A\mathbf{x} = \mathbf{b}$ has a unique solution.
5. Vector $\mathbf{b}$ can be written as a linear combination of the columns of A .

6. $A\mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}$

Exercise 22 Let $A = \begin{bmatrix} | & | & | \\ \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \\ | & | & | \end{bmatrix}$ be a 3×3 matrix with columns $\mathbf{c}_1$, $\mathbf{c}_2$ and $\mathbf{c}_3$.

Suppose $A \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Which of the following can we conclude from the given information? Select ALL that apply.

Select All Correct Answers:

1. $\mathbf{c}_2 + \mathbf{c}_3 = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

2. $-\mathbf{c}_1 - \mathbf{c}_2 + 2\mathbf{c}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

3. A is non-singular.

4. $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ is in $\text{span}(\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3)$.

5. Vector $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ can be written as a linear combination of the columns of A .

Exercise 23 Suppose A is a non-singular 3×3 matrix. Let $A^{-1} = \begin{bmatrix} | & | & | \\ \mathbf{d}_1 & \mathbf{d}_2 & \mathbf{d}_3 \\ | & | & | \end{bmatrix}$.

Suppose $A \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

Express $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ as a linear combination of the columns of A^{-1} .

$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \dots \mathbf{d}_1 + \dots \mathbf{d}_2 + \dots \mathbf{d}_3$$

Exercise 24 Let A be a 2×5 matrix. Compute the dimensions of each product.

$A^T A$ is a $\dots \times \dots$ matrix.

AA^T is a $\dots \times \dots$ matrix.

Exercise 25 Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$. Find elementary matrices E_1 and E_2 such that $C = E_2 E_1 A$.

$$E_1 = \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}; \quad E_2 = \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}$$