# Week 3 Section Worksheet

*A worksheet on constructions, the RASS theorem, and inscribed/circumscribed circle to a triangle.*

**Problem 1** ***Warm-up: constructions and congruence.***

(a) *Using only a compass and straightedge, construct the following:*

- *the midpoint of a segment,*
- *the bisector of an angle,*
- *a perpendicular to a line through a given point,*
- *a square on a given segment,*
- *a line parallel to a given line through a given point.*

(b) *Recall the triangle congruence theorems:*

$$\textit{SAS, SSS, AAS/ASA, RASS.}$$

**Problem 2** ***Puzzle.***

*Let $A$, $B$, and $C$ be three non-collinear points.*

*Construct, using only a straightedge and compass, a circle passing through $A$, $B$, and $C$.*

***Remark.*** *This is possible in 7 moves.*

**Problem 3** ***Comparing chords via the center.***

*Let $AB$ and $CD$ be chords of a circle of center $E$.*

- *Consider the line $\ell_1$ perpendicular to $AB$ passing through $E$, and call $F$ the intersection point with $AB$.*
- *Consider the line $\ell_2$ perpendicular to $CD$ passing through $E$, and call $G$ the intersection point with $CD$.*

*Show that*

$$EF = EG \quad \textit{if and only if} \quad AB = CD.$$

Author(s):

**Problem 4** ***Kites and perpendicular diagonals.***

*Let $ABCD$ be a quadrilateral such that*

$$AB = AD \quad \text{and} \quad BC = CD.$$

*Show that the diagonals $AC$ and $BD$ are perpendicular.*

**Problem 5** ***Angle bisectors in a triangle.***

*Show that all the angle bisectors in a triangle intersect at a common point.*

***Idea.*** *Construct the angle bisectors at $\angle A$ and $\angle B$, and let $D$ be their intersection point.*

***Goal.*** *Show that $DC$ bisects $\angle C$.*

***Hint.*** *Construct the perpendicular lines through $D$ to:*

- *$BC$,*
- *$AB$,*
- *$AC$.*

*Using AAS, show that:*

$$FD = ED, \qquad ED = GD.$$

*Conclude using*

$$FD = GD.$$

**Problem 6** ***Bonus: classical constructions.***

*Using only a straightedge and compass, construct:*

- *the circumscribed circle to a given triangle ($par = 7$),*
- *the inscribed circle to a given triangle ($par = 13$).*

*How many moves did you use?*