# Mock Midterm Exam

*A worksheet to help you prepare for the midterm.*

**Problem 1** *For each question, circle the unique correct answer. No justification is required.*

(i) *Which of the following statements is true?*

- (a) *If two lines are parallel, then they intersect at exactly one point.*
- (b) *If two lines are both perpendicular to a third line, then they are parallel.*
- (c) *If two lines intersect, then they are parallel.*
- (d) *If two lines are not parallel, then they are perpendicular.*

(ii) *In the six parts of a Euclidean proposition, which part usually states what is given?*

- (a) *Exposition.*
- (b) *Specification.*
- (c) *Construction.*
- (d) *Conclusion.*

(iii) *Which of the following assertions is* ***false****?*

- (a) *Vertical angles are equal.*
- (b) *The perpendicular bisector of a segment passes through its midpoint.*
- (c) *If two lines are parallel, then alternate interior angles are equal.*
- (d) *If two segments have the same length, then they lie on the same line.*

**Problem 2** *This exercise contains four "Euclid-style" proof questions.*

(i) *Let $ABCD$ be a quadrilateral such that*

$$AB \parallel CD \qquad \text{and} \qquad AB = CD.$$

*Prove that*

$$AD \parallel BC \qquad \text{and} \qquad AD = BC.$$

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(ii) *Let $ABCD$ be a quadrilateral such that*

$$AB = AD \qquad \text{and} \qquad CB = CD.$$

*Assume that the diagonals $AC$ and $BD$ meet at a point $E$. Prove that*

$$AC \perp BD.$$

(iii) *Let $AB$ be a diameter of a circle, and let $C$ be another point on the circle. Prove that*

$$\angle ACB$$

*is a right angle.*

(iv) *Let $ABCD$ be a cyclic quadrilateral (i.e. four points on a circle) such that*

$$AB = CD.$$

*Prove that*

$$AD \parallel BC.$$

***Hint.*** *Draw the diagonals $AC$ and $BD$, and let $E$ be their point of intersection. First show that the triangles $\triangle CDE$ and $\triangle ABE$ are congruent. Then show that*

$$\angle DAE = \angle BCE,$$

*and conclude.*

**Problem 3** *Using only straightedge and compass, perform the following constructions and state how many moves you used. No justification is required.*

(i) *Construct the midpoint of a given segment $AB$.*

(ii) *Construct a square on a given segment $AB$.*

(iii) *Construct the perpendicular to a given line $l$ passing through a given point $P$ not on the line.*

(iv) *Construct a circle passing through three given non-collinear points $A$, $B$, and $C$.*

(v) *Construct a line tangent to a given circle and passing through a given point $A$ outside the circle.*

**Problem 4** *Assume the incidence axioms **I1**, **I2**, and **I3**. Prove the following.*

(i) *Two distinct lines intersect in at most one point.*

(ii) *If $P$ is a point and $l$ is a line not passing through $P$, then there exist at least two distinct lines through $P$ which meet $l$.*

(iii) *There exist at least three distinct lines.*