EXERCISES FOR CARTESIAN PLANE

Problem 1 *Use graph paper to plot and label the points* $A(-3,-7)$, $B(1.3,-2)$, $C(\pi,\sqrt{10})$, $D(0,8)$, $E(-5.5,0)$, $F(-8,4)$, $G(9.2,-7.8)$ *and* $H(7,5)$*, all in the same coordinate plane.*

Desmos link: https://www.desmos.com/calculator/gs64m6dbzw

Problem 2 *use the points you plotted in Problem* **??** *above, and do the following:*

- *Identify the quadrant or axis in/on which the point lies.*
- *Find the point symmetric to the given point about the x-axis.*
- *Find the point symmetric to the given point about the y-axis.*
- *Find the point symmetric to the given point about the origin.*

The point $A(-3,-7)$

- *in Quadrant*

  Multiple Choice:

  1. *I*
  2. *II*
  3. *III* ✓
  4. *IV*

- *symmetric about x-axis with* $(-3,7)$
- *symmetric about y-axis with* $(3,-7)$
- *symmetric about origin with* $(3,7)$

The point $B(1.3,-2)$

- *in Quadrant*

  Multiple Choice:

  1. *I*
  2. *II*
  3. *III*
  4. *IV* ✓

- *symmetric about x-axis with* $(1.3,2)$
- *symmetric about y-axis with* $(-1.3,-2)$
- *symmetric about origin with* $(-1.3,2)$

The point $C(\pi,\sqrt{10})$

- *in Quadrant*

  Multiple Choice:

  1. *I* ✓

2. *II*
3. *III*
4. *IV*

- *symmetric about x-axis with* $(\pi, -\sqrt{10})$
- *symmetric about y-axis with* $(-\pi, \sqrt{10})$
- *symmetric about origin with* $(-\pi, \sqrt{10})$

The point $D(0, 8)$

- *on the axis*

Multiple Choice:

1. *positive y* ✓
2. *negative y*
3. *positive x*
4. *negative x*

- *symmetric about x-axis with* $(0, -8)$
- *symmetric about y-axis with* $(0, 8)$
- *symmetric about origin with* $(0, -8)$

The point $E(-5.5, 0)$

- *on the axis*

Multiple Choice:

1. *positive y*
2. *negative y*
3. *positive x*
4. *negative x* ✓

- *symmetric about x-axis with* $(-5.5, 0)$
- *symmetric about y-axis with* $(5.5, 0)$
- *symmetric about origin with* $(5.5, 0)$

The point $F(-8, 4)$

- *in Quadrant*

Multiple Choice:

1. *I*
2. *II* ✓
3. *III*

4. *IV*

- *symmetric about x-axis with* $(-8, -4)$
- *symmetric about y-axis with* $(8, 4)$
- *symmetric about origin with* $(8, -4)$

The point $G(9.2, -7.8)$

- *in Quadrant*

Multiple Choice:

1. *I*
2. *II*
3. *III*
4. *IV* ✓

- *symmetric about x-axis with* $(9.2, 7.8)$
- *symmetric about y-axis with* $(-9.2, -7.8)$
- *symmetric about origin with* $(-9.2, 7.8)$

The point $H(7, 5)$

- *in Quadrant*

Multiple Choice:

1. *I* ✓
2. *II*
3. *III*
4. *IV*

- *symmetric about x-axis with* $(7, -5)$
- *symmetric about y-axis with* $(-7, 5)$
- *symmetric about origin with* $(-7, -5)$

Question 3 *In Exercises* **??** *-* **??***, find the distance d between the points and the midpoint M of the line segment which connects them.*

Problem 3.1 $(1, 2)$, $(-3, 5)$

$d = 5$ *units,* $M = (-1, \frac{7}{2})$

Problem 3.2 $(3, -10)$, $(-1, 2)$

$d = 4\sqrt{10}$ *units,* $M = (1, -4)$

Problem 3.3 $\left(\frac{1}{2}, 4\right), \left(\frac{3}{2}, -1\right)$

$d = \sqrt{26}$ *units,* $M = (1, \frac{3}{2})$

Problem 3.4 $\left(-\frac{2}{3}, \frac{3}{2}\right), \left(\frac{7}{3}, 2\right)$

$d = \frac{\sqrt{37}}{2}$ *units,* $M = \left(\frac{5}{6}, \frac{7}{4}\right)$

Problem 3.5 $\left(\frac{24}{5}, \frac{6}{5}\right), \left(-\frac{11}{5}, -\frac{19}{5}\right)$

$d = \sqrt{74}$ *units,* $M = \left(\frac{13}{10}, -\frac{13}{10}\right)$

Problem 3.6 $\left(\sqrt{2}, \sqrt{3}\right), \left(-\sqrt{8}, -\sqrt{12}\right)$

$d = 3\sqrt{5}$ *units,* $M = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{3}}{2}\right)$

Problem 3.7 $\left(2\sqrt{45}, \sqrt{12}\right), \left(\sqrt{20}, \sqrt{27}\right)$.

$d = \sqrt{83}$ *units,* $M = \left(4\sqrt{5}, \frac{5\sqrt{3}}{2}\right)$

Problem 3.8 $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$d = 2$ *units,*

$M = (0, 0)$

Question 4 *Let's assume that we are standing at the origin and the positive y-axis points due North while the positive x-axis points due East. Our Sasquatch-o-meter tells us that Sasquatch is 3 miles West and 4 miles South of our current position.*

Problem 4.1 *What are the coordinates of his position?*

$(-3, -4)$

Problem 4.2 *How far away is he from us?*

5 *miles*

Problem 4.3 *If he runs 7 miles due East what would his new position be?*

$(4, -4)$

Problem 5 *Verify the Distance Formula* **??** *for the cases when:*

1. *The points are arranged vertically. (Hint: Use* $P(a, y_0)$ *and* $Q(a, y_1)$*.)*
2. *The points are arranged horizontally. (Hint: Use* $P(x_0, b)$ *and* $Q(x_1, b)$*.)*

3. *The points are actually the same point. (You shouldn't need a hint for this one.)*

Problem 6 *Verify the Midpoint Formula by showing the distance between $P(x_1, y_1)$ and M and the distance between M and $Q(x_2, y_2)$ are both half of the distance between P and Q.*

Question 7 *Show that the points A, B and C below are the vertices of a right triangle.*

Problem 7.1 *$A(-3, 2)$, $B(-6, 4)$, and $C(1, 8)$*

The distance from A to B is $|AB| = \sqrt{13}$, the distance from A to C is $|AC| = \sqrt{52}$, and the distance from B to C is $|BC| = \sqrt{65}$. Since $\left(\sqrt{13}\right)^2 + \left(\sqrt{52}\right)^2 = \left(\sqrt{65}\right)^2$, we are guaranteed by the converse of the Pythagorean Theorem that the triangle is a right triangle.

Problem 7.2 *$A(-3, 1)$, $B(4, 0)$ and $C(0, -3)$*

Hint: *Show that $|AC|^2 + |BC|^2 = |AB|^2$*

Problem 8 *Find a point $D(x, y)$ such that the points $A(-3, 1)$. $B(4, 0)$, $C(0, -3)$ and D are the corners of a square. Justify your answer.*

Problem 9 *Suppose the distance between $C(h, k)$ and $P(x, y)$ is r. Use the distance formula to show*

$$(x - h)^2 + (y - k)^2 = r^2$$

*We will see this formula (and its cousins) in Chapter **??**.*

Problem 10 *Let $P(x, y)$ be a point in the plane and let Q be the result of reflecting P about the x-axis, y-axis, or origin. Show the distance from the origin to P is the same as the distance from the origin to Q.*

Problem 11 *Let $O(0, 0)$ (that is, O is the origin), $P(-2, 1)$, $Q(-4, 2)$, and $R(6, -3)$.*

1. *Find the distance from O to P and from O to Q. What do you notice?*
2. *Find the distance from O to P and from O to R. What do you notice?*
3. *For a generic point $P(x, y)$, let $Q(kx, ky)$ be the point obtained from P by multiplying both the x and y coordinates of P by the same number, k. Show the distance from O to Q is exactly $|k|$ times the distance from O to P. Explain what these results mean geometrically. (We'll revisit this in Theorem **??** in Section **??**.)*

Problem 12 *In this exercise, we explore some of the properties of distance. For brevity, we'll adopt the notation 'd(P, Q)' to denote the distance between points P and Q.*

1. *(Non-negative Property) Explain why $d(P, Q) \geq 0$ for any two points in the plane.*
2. *(Symmetric Property) Explain why $d(P, Q) = d(Q, P)$ for any two points in the plane.*
3. *(Identity Property) Show that $d(P, Q) = 0$ if and only if P and Q are the same point.*

   ***NOTE:** The phrase 'if and only if' means you need to show two things:*

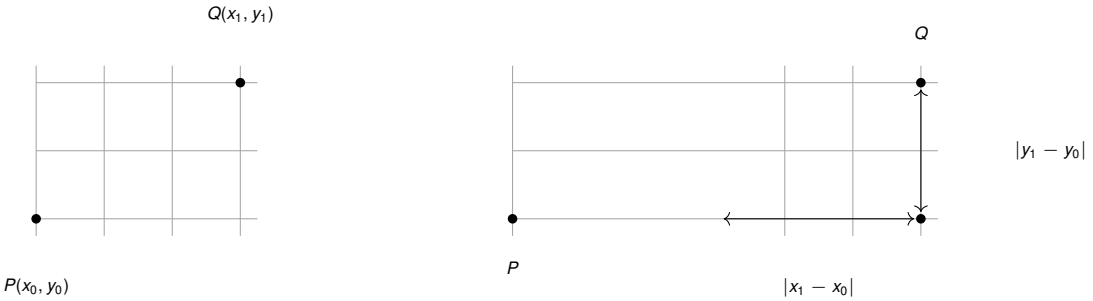
   - *If P and Q are the same point, then $d(P, Q) = 0$.*
   - *If $d(P, Q) = 0$, then P and Q are the same point.*

4. *(Triangle Inequality) The Triangle Inequality says that for any triangle, the sum of the lengths of two sides of a triangle always exceeds the length of the third. Use the Triangle Inequality to show that for any three points P, Q, and R,*

$$d(P,R) \leq d(P,Q) + d(Q,R)$$

Under what conditions does $d(P,R) = d(P,Q) + d(Q,R)$?

Problem 13 *(Another way to measure distance.) In this text, we defined the distance between two points as the length of the line segment connecting the two points. Depending on the situation, however, there may be better ways to describe how far one location is from another. Consider the situation below on the left. Suppose P and Q are locations on a city grid, and a taxi is hailed at point P to travel to point Q. In this situation, diagonal movement is impossible,[1] so the taxi is limited to traveling horizontally and vertically.*



From the diagram, we see the horizontal distance is $|x_1 - x_0|$ and the vertical distance is $|y_1 - y_0|$, so the total distance the taxi needs to travel to get from P to Q is given by:

$$d_T = |x_1 - x_0| + |y_1 - y_0|$$

We call d_T the 'taxi distance' from P to Q.

1. *Let $P(-2,3)$ and $Q(4,2)$. Find the distance, d from P to Q and the taxi distance, d_T from P to Q. Repeat this exercise with several points of your own choosing. Which is larger, d or d_T?*
2. *Using the notation of Exercise **??**, show that $d(P,Q) \leq d_T(P,Q)$ for any two points P and Q in the plane. (The Triangle Inequality is useful once again here.) Under what conditions is $d(P,Q) = d_T(P,Q)$?*
3. *Repeat Exercise **??** with the taxi distance, d_T. (You may need to skip ahead to Exercise **??** in Section **??** to verify the Triangle Inequality piece.)*
4. *Think about ways to define a 'midpoint' using the taxi distance. What would your formula be? To help you get started, play around with the origin $(0,0)$ as one point and the point $(4,2)$ as the other.*

Problem 14 *The world is not flat.[2] Thus the Cartesian Plane cannot possibly be the end of the story. Discuss with your classmates how you would extend Cartesian Coordinates to represent the three dimensional world. What would the Distance and Midpoint formulas look like, assuming those concepts make sense at all?*

[1] Maybe 'discouraged' or 'difficult' would be better word choices.

[2] There are those who disagree with this statement. Look them up on the Internet some time when you're bored.