

EXERCISES FOR APPENDIX: COMPLEX NUMBERS

Question 1 In Exercises ?? - ??, use the given complex numbers z and w to find and simplify the following.

- $z + w$
- zw
- z^2
- $\frac{1}{z}$
- $\frac{z}{w}$
- $\frac{w}{z}$
- $\bar{z}$
- $z\bar{z}$
- $(\bar{z})^2$

Problem 1.1 $z = 2 + 3i$, $w = 4i$

- $z + w = 2 + 7i$
- $zw = -12 + 8i$
- $z^2 = -5 + 12i$
- $\frac{1}{z} = \frac{2}{13} - \frac{3}{13}i$
- $\frac{z}{w} = \frac{3}{4} - \frac{1}{2}i$
- $\frac{w}{z} = \frac{12}{13} + \frac{8}{13}i$
- $\bar{z} = 2 - 3i$
- $z\bar{z} = 13$
- $(\bar{z})^2 = -5 - 12i$

Problem 1.2 $z = 1 + i$, $w = -i$

- $z + w = 1$
- $zw = 1 - i$
- $z^2 = 2i$
- $\frac{1}{z} = \frac{1}{2} - \frac{1}{2}i$
- $\frac{z}{w} = -1 + i$
- $\frac{w}{z} = -\frac{1}{2} - \frac{1}{2}i$
- $\bar{z} = 1 - i$

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- $z\bar{z} = 2$
- $(\bar{z})^2 = -2i$

Problem 1.3 $z = i, w = -1 + 2i$

- $z + w = -1 + 3i$
- $zw = -2 - i$
- $z^2 = -1$
- $\frac{1}{z} = -i$
- $\frac{z}{w} = \frac{2}{5} - \frac{1}{5}i$
- $\frac{w}{z} = 2 + i$
- $\bar{z} = -i$
- $z\bar{z} = 1$
- $(\bar{z})^2 = -1$

Problem 1.4 $z = 4i, w = 2 - 2i$

- $z + w = 2 + 2i$
- $zw = 8 + 8i$
- $z^2 = -16$
- $\frac{1}{z} = -\frac{1}{4}i$
- $\frac{z}{w} = -1 + i$
- $\frac{w}{z} = -\frac{1}{2} - \frac{1}{2}i$
- $\bar{z} = -4i$
- $z\bar{z} = 16$
- $(\bar{z})^2 = -16$

Problem 1.5 $z = 3 - 5i, w = 2 + 7i$

- $z + w = 5 + 2i$
- $zw = 41 + 11i$
- $z^2 = -16 - 30i$
- $\frac{1}{z} = \frac{3}{34} + \frac{5}{34}i$
- $\frac{z}{w} = -\frac{29}{53} - \frac{31}{53}i$

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- $\frac{w}{z} = -\frac{29}{34} + \frac{31}{34}i$
- $\bar{z} = 3 + 5i$
- $z\bar{z} = 34$
- $(\bar{z})^2 = -16 + 30i$

Problem 1.6 $z = -5 + i$, $w = 4 + 2i$

- $z + w = -1 + 3i$
- $zw = -22 - 6i$
- $z^2 = 24 - 10i$
- $\frac{1}{z} = -\frac{5}{26} - \frac{1}{26}i$
- $\frac{z}{w} = -\frac{9}{10} + \frac{7}{10}i$
- $\frac{w}{z} = -\frac{9}{13} - \frac{7}{13}i$
- $\bar{z} = -5 - i$
- $z\bar{z} = 26$
- $(\bar{z})^2 = 24 + 10i$

Problem 1.7 $z = \sqrt{2} - i\sqrt{2}$, $w = \sqrt{2} + i\sqrt{2}$

- $z + w = 2\sqrt{2}$
- $zw = 4$
- $z^2 = -4i$
- $\frac{1}{z} = \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}i$
- $\frac{z}{w} = -i$
- $\frac{w}{z} = i$
- $\bar{z} = \sqrt{2} + i\sqrt{2}$
- $z\bar{z} = 4$
- $(\bar{z})^2 = 4i$

Problem 1.8 $z = 1 - i\sqrt{3}$, $w = -1 - i\sqrt{3}$

- $z + w = -2i\sqrt{3}$
- $zw = -4$
- $z^2 = -2 - 2i\sqrt{3}$

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- $\frac{1}{z} = \frac{1}{4} + \frac{\sqrt{3}}{4}i$
- $\frac{z}{w} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$
- $\frac{w}{z} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$
- $\bar{z} = 1 + i\sqrt{3}$
- $z\bar{z} = 4$
- $(\bar{z})^2 = -2 + 2i\sqrt{3}$

Problem 1.9 $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$, $w = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

- $z + w = i\sqrt{3}$
- $zw = -1$
- $z^2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$
- $\frac{1}{z} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$
- $\frac{z}{w} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$
- $\frac{w}{z} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$
- $\bar{z} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$
- $z\bar{z} = 1$
- $(\bar{z})^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Problem 1.10 $z = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, $w = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

- $z + w = -\sqrt{2}$
- $zw = 1$
- $z^2 = -i$
- $\frac{1}{z} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$
- $\frac{z}{w} = -i$
- $\frac{w}{z} = i$
- $\bar{z} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

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- $z\bar{z} = 1$
- $(\bar{z})^2 = i$

Question 2 In Exercises ?? - ??, simplify the quantity.

Problem 2.1 $\sqrt{-49}$
 $7i$

Problem 2.2 $\sqrt{-9} = 3i$

Problem 2.3 $\sqrt{-25}\sqrt{-4}$
 $10i^2 = -10$

Problem 2.4 $\sqrt{(-25)(-4)} = 10$

Problem 2.5 $\sqrt{-9}\sqrt{-16} = -12$

Problem 2.6 $\sqrt{(-9)(-16)}$
 $\sqrt{144} = 12$

Problem 2.7 $\sqrt{-(-9)}$
 $\sqrt{9} = 3$

Problem 2.8 $-\sqrt{(-9)} = -3i$

Question 3 We know that $i^2 = -1$ which means $i^3 = i^2 \cdot i = (-1) \cdot i = -i$ and $i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$. In Exercises ?? - ??, use this information to simplify the given power of i .

Problem 3.1 i^5
 $i^5 = i^4 \cdot i = 1 \cdot i = i$

Problem 3.2 i^6
 $i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$

Problem 3.3 $i^7 = -i$

Problem 3.4 $i^8 = 1$

Problem 3.5 i^{15}
 $i^{15} = (i^4)^3 \cdot i^3 = 1 \cdot (-i) = -i$

Problem 3.6 $i^{26} = -1$

Problem 3.7 i^{117}
 $i^{117} = (i^4)^{29} \cdot i = 1 \cdot i = i$

Problem 3.8 $i^{304} = 1$

EXERCISES FOR APPENDIX: COMPLEX NUMBERS

Question 4 In Exercises ?? - ??, find all complex solutions.

Problem 4.1 $3x^2 + 6 = 4x$

$$x = \frac{2 \pm i\sqrt{14}}{3}$$

Problem 4.2 $15t^2 + 2t + 5 = 3t(t^2 + 1)$

$$t = 5, \pm \frac{i\sqrt{3}}{3}$$

Problem 4.3 $3y^2 + 4 = y^4$

$$y = \pm 2, \pm i$$

Problem 4.4 $\frac{2}{1-w} = w$

$$w = \frac{1 \pm i\sqrt{7}}{2}$$

Problem 4.5 $\frac{y}{3} - \frac{3}{y} = y$

$$y = \pm \frac{3i\sqrt{2}}{2}$$

Problem 4.6 $\frac{x^3}{2x-1} = \frac{x}{3}$

$$x = 0, \frac{1 \pm i\sqrt{2}}{3}$$

Problem 4.7 $x = \frac{2}{\sqrt{5}-x}$

$$x = \frac{\sqrt{5} \pm i\sqrt{3}}{2}$$

Problem 4.8 $\frac{5y^4 + 1}{y^2 - 1} = 3y^2$

$$y = \pm i, \pm \frac{i\sqrt{2}}{2}$$

Problem 4.9 $z^4 = 16$

$$z = \pm 2, \pm 2i$$

Problem 5 Multiply and simplify: $(x - [3 - i\sqrt{23}])(x - [3 + i\sqrt{23}])$

$$x^2 - 6x + 32$$