Exercises for Appendix: Variation

Question 1 *In Exercises **??** - **??**, translate the following into mathematical equations.*

Problem 1.1 *At a constant pressure, the temperature T of an ideal gas is directly proportional to its volume V. (This is Charles's Law)*

$T = kV$

Problem 1.2 *The frequency of a wave f is inversely proportional to the wavelength of the wave λ.*

NOTE: *The character λ is the lower case Greek letter 'lambda.'*

$f = \frac{k}{\lambda}$

Problem 1.3 *The density d of a material is directly proportional to the mass of the object m and inversely proportional to its volume V.*

$d = k\frac{m}{V}$

Problem 1.4 *The square of the orbital period of a planet P is directly proportional to the cube of the semi-major axis of its orbit a. (This is Kepler's Third Law of Planetary Motion)*

$P^2 = ka^3$

Problem 1.5 *The drag of an object traveling through a fluid D varies jointly with the density of the fluid ρ and the square of the velocity of the object ν.*

NOTE: *The characters ρ and ν are the lower case Greek letters 'rho' and 'nu,' respectively.*

$D = k\rho\nu^2$

Problem 1.6 *Suppose two electric point charges, one with charge q and one with charge Q, are positioned r units apart. The electrostatic force F exerted on the charges varies directly with the product of the two charges and inversely with the square of the distance between the charges. (This is Coulomb's Law)*

$F = \frac{kqQ}{r^2}$

NOTE: *Note the similarity to this formula and Newton's Law of Universal Gravitation as discussed in Example **??**.*

Problem 2 *According to this webpage, the frequency f of a vibrating string is given by $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ where T is the tension, μ is the linear mass[1] of the string and L is the length of the vibrating part of the string. Express this relationship using the language of variation.*

Rewriting $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ as $f = \frac{1}{2}\frac{\sqrt{T}}{L\sqrt{\mu}}$ we see that the frequency f varies directly with the square root of the tension and varies inversely with the length and the square root of the linear mass.

Problem 3 *According to the Centers for Disease Control and Prevention www.cdc.gov, a person's Body Mass Index B is directly proportional to his weight W in pounds and inversely proportional to the square of his height h in inches.*

1. *Express this relationship as a mathematical equation.*

   $B = k\frac{W}{h^2}$

[1] Also known as the linear density. It is simply a measure of mass per unit length.

2. *If a person who was* 5 *feet,* 10 *inches tall weighed 235 pounds had a Body Mass Index of 33.7, what is the value of the constant of proportionality?*

   $k = 702.68$

   NOTE: *The CDC uses 703.*

3. *Rewrite the mathematical equation found in part* **??** *to include the value of the constant found in part* **??** *and then find your Body Mass Index.*

   $B = \dfrac{702.68W}{h^2}$

Problem 4 *This exercise refers back to the volume of a right circular cone formula found in Example* **??***.*

1. *First assume that V, h and r are all measured using the same unit of length. Work with your classmates to show that in this case, the k needed for the volume formula $V = khr^2$ has no units on it.*

2. *Now assume that V is measured in milliliters, h is measured in meters and r is measured in yards. Work with your classmates to find the units on k so that the volume formula $V = khr^2$ makes sense.*

Problem 5 *We know that the circumference of a circle varies directly with its radius with 2π as the constant of proportionality. (That is, we know $C = 2\pi r$.) With the help of your classmates, compile a list of other basic geometric relationships which can be seen as variations.*

Problem 6 *Research the Ideal Gas Law $PV = nRT$ to see what sorts of units are used for the constant R. What other formulations of this law did you find in your research?*