EXERCISES FOR FUNDAMENTAL TRIGONOMETRIC IDENTITIES

Question 1 *In Exercises* **??** *-* **??***, use the Reciprocal and Quotient Identities (Theorem* **??***) along with the Pythagorean Identities (Theorem* **??***), to find the value of the circular function requested below. (Find the exact value unless otherwise indicated.)*

Problem 1.1 *If* $\sin(\theta) = \frac{\sqrt{5}}{5}$*, find* $\csc(\theta)$*.*

$\csc(\theta) = \sqrt{5}$

Problem 1.2 *If* $\sec(\theta) = -4$*, find* $\cos(\theta)$*.*

$\cos(\theta) = -\frac{1}{4}$

Problem 1.3 *If* $\tan(t) = 3$*, find* $\cot(t)$*.*

$\cot(t) = \frac{1}{3}$

Problem 1.4 *If* θ *is a Quadrant IV angle with* $\cos(\theta) = \frac{5}{13}$*, find* $\sin(\theta)$*.*

$\sin(\theta) = -\frac{12}{13}$

Problem 1.5 *If* θ *is a Quadrant III angle with* $\tan(\theta) = 2$*, find* $\sec(\theta)$*.*

$\sec(\theta) = -\sqrt{5}$.

Problem 1.6 *If* $\frac{\pi}{2} < t < \pi$ *with* $\cot(t) = -2$*, find* $\csc(t)$*.*

$\csc(t) = \sqrt{5}$.

Problem 1.7 *If* $\sec(\theta) = 3$ *and* $\sin(\theta) < 0$*, find* $\tan(\theta)$*.*

$\tan(\theta) = -2\sqrt{2}$.

Problem 1.8 *If* $\sin(\theta) = -\frac{2}{3}$ *but* $\tan(\theta) > 0$*, find* $\cos(\theta)$*.*

$\cos(\theta) = -\frac{\sqrt{5}}{3}$.

Problem 1.9 *If* $0 < t < \frac{\pi}{2}$ *and* $\sin(t) = 0.42$*, find* $\cos(t)$*, rounded to four decimal places.*

$\cos(t) \approx 0.9075$.

Problem 1.10 *If* θ *is Quadrant IV angle with* $\sec(\theta) = 1.17$*, find* $\tan(\theta)$*, rounded to four decimal places.*

$\tan(\theta) \approx -0.6074$.

Problem 1.11 *If* $\pi < t < \frac{3\pi}{2}$ *with* $\cot(t) = 4.2$*, find* $\csc(t)$*, rounded to four decimal places.*

$\csc(t) \approx -4.079$.

Question 2 *In Exercises* **??** *-* **??***, use the Reciprocal and Quotient Identities (Theorem* **??***) along with the Pythagorean Identities (Theorem* **??***), to find the exact values of the remaining circular functions. (Compare your methods with how you solved Exercises* **??** *-* **??** *in Section* **??***.)*

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Problem 2.1 $\sin(\theta) = \frac{3}{5}$ *with* θ *in Quadrant II*

$\cos(\theta) =$

Multiple Choice:

1. $\frac{4}{5}$
2. $-\frac{4}{5}$ ✓
3. $\frac{3}{4}$
4. $-\frac{3}{4}$

$\tan(\theta) =$

Multiple Choice:

1. $\frac{4}{5}$
2. $-\frac{4}{5}$
3. $\frac{3}{4}$
4. $-\frac{3}{4}$ ✓

$\csc(\theta) =$

Multiple Choice:

1. $\frac{5}{3}$ ✓
2. $\frac{5}{4}$
3. $-\frac{5}{4}$
4. $-\frac{4}{3}$

$\sec(\theta) =$

Multiple Choice:

1. $\frac{5}{3}$
2. $\frac{5}{4}$
3. $-\frac{5}{4}$ ✓

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4. $-\frac{4}{3}$

$\cot(\theta) =$

Multiple Choice:

1. $\frac{5}{3}$
2. $\frac{5}{4}$
3. $-\frac{5}{4}$
4. $-\frac{4}{3}$ ✓

Problem 2.2 $\tan(\theta) = \frac{12}{5}$ *with* θ *in Quadrant III*

$\sin(\theta) = -\frac{12}{13}, \cos(\theta) = -\frac{5}{13}, \csc(\theta) = -\frac{13}{12}, \sec(\theta) = -\frac{13}{5}, \cot(\theta) = \frac{5}{12}$

Problem 2.3 $\csc(\theta) = \frac{25}{24}$ *with* θ *in Quadrant I*

$\sin(\theta) = \frac{24}{25}$

$\cos(\theta) = \frac{7}{25}$

$\tan(\theta) = \frac{24}{7}$

$\sec(\theta) = \frac{25}{7}$

$\cot(\theta) = \frac{7}{24}$

Problem 2.4 $\sec(\theta) = 7$ *with* θ *in Quadrant IV*

$\sin(\theta) = \frac{-4\sqrt{3}}{7}, \cos(\theta) = \frac{1}{7}, \tan(\theta) = -4\sqrt{3}, \csc(\theta) = -\frac{7\sqrt{3}}{12}, \sec(\theta) = 7, \cot(\theta) = -\frac{\sqrt{3}}{12}$

Problem 2.5 $\csc(\theta) = -\frac{10\sqrt{91}}{91}$ *with* θ *in Quadrant III*

$\sin(\theta) = -\frac{\sqrt{91}}{10}$

$\cos(\theta) = -\frac{3}{10}$

$\tan(\theta) = \frac{\sqrt{91}}{3}$

$\csc(\theta) = -\frac{10\sqrt{91}}{91}$

$\sec(\theta) = -\frac{10}{3}$

$\cot(\theta) = \frac{3\sqrt{91}}{91}$

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EXERCISES FOR FUNDAMENTAL TRIGONOMETRIC IDENTITIES

Problem 2.6 $\cot(\theta) = -23$ *with* θ *in Quadrant II*

$$\sin(\theta) = \frac{\sqrt{530}}{530}, \cos(\theta) = -\frac{23\sqrt{530}}{530}, \tan(\theta) = -\frac{1}{23}, \csc(\theta) = \sqrt{530}, \sec(\theta) = -\frac{\sqrt{530}}{23}, \cot(\theta) = -23$$

Problem 2.7 $\tan(\theta) = -2$ *with* θ *in Quadrant IV.*

$\sin(\theta) = -\frac{2\sqrt{5}}{5}$

$\cos(\theta) = \frac{\sqrt{5}}{5}$

$\tan(\theta) = -2$

$\csc(\theta) = -\frac{\sqrt{5}}{2}$

$\sec(\theta) = \sqrt{5}$

$\cot(\theta) = -\frac{1}{2}$

Problem 2.8 $\sec(\theta) = -4$ *with* θ *in Quadrant II.*

$$\sin(\theta) = \frac{\sqrt{15}}{4}, \cos(\theta) = -\frac{1}{4}, \tan(\theta) = -\sqrt{15}, \csc(\theta) = \frac{4\sqrt{15}}{15}, \sec(\theta) = -4, \cot(\theta) = -\frac{\sqrt{15}}{15}$$

Problem 2.9 $\cot(\theta) = \sqrt{5}$ *with* θ *in Quadrant III.*

$\sin(\theta) = -\frac{\sqrt{6}}{6}$

$\cos(\theta) = -\frac{\sqrt{30}}{6}$

$\tan(\theta) = \frac{\sqrt{5}}{5}$

$\csc(\theta) = -\sqrt{6}$

$\sec(\theta) = -\frac{\sqrt{30}}{5}$

$\cot(\theta) = \sqrt{5}$

Problem 2.10 $\cos(\theta) = \frac{1}{3}$ *with* θ *in Quadrant I.*

$$\sin(\theta) = \frac{2\sqrt{2}}{3}, \cos(\theta) = \frac{1}{3}, \tan(\theta) = 2\sqrt{2}, \csc(\theta) = \frac{3\sqrt{2}}{4}, \sec(\theta) = 3, \cot(\theta) = \frac{\sqrt{2}}{4}$$

Problem 2.11 $\cot(t) = 2$ *with* $0 < t < \frac{\pi}{2}$.

$\sin(t) = \frac{\sqrt{5}}{5}$

$\cos(t) = \frac{2\sqrt{5}}{5}$

$\tan(t) = \frac{1}{2}$

$\csc(t) = \sqrt{5}$

$\sec(t) = \frac{\sqrt{5}}{2}$

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$\cot(t) = 2$

Problem 2.12 $\csc(t) = 5$ *with* $\frac{\pi}{2} < t < \pi$.

$\sin(t) = \frac{1}{5}, \cos(t) = -\frac{2\sqrt{6}}{5}, \tan(t) = -\frac{\sqrt{6}}{12}, \csc(t) = 5, \sec(t) = -\frac{5\sqrt{6}}{12}, \cot(t) = -2\sqrt{6}$

Problem 2.13 $\tan(t) = \sqrt{10}$ *with* $\pi < t < \frac{3\pi}{2}$.

$\sin(t) = -\frac{\sqrt{110}}{11}$

$\cos(t) = -\frac{\sqrt{11}}{11}$

$\tan(t) = \sqrt{10}$

$\csc(t) = -\frac{\sqrt{110}}{10}$

$\sec(t) = -\sqrt{11}$

$\cot(t) = \frac{\sqrt{10}}{10}$

Problem 2.14 $\sec(t) = 2\sqrt{5}$ *with* $\frac{3\pi}{2} < t < 2\pi$.

$\sin(t) = -\frac{\sqrt{95}}{10}, \cos(t) = \frac{\sqrt{5}}{10}, \tan(t) = -\sqrt{19}, \csc(t) = -\frac{2\sqrt{95}}{19}, \sec(t) = 2\sqrt{5}, \cot(t) = -\frac{\sqrt{19}}{19}$

Problem 3 *Skippy claims* $\cos(\theta) + \sin(\theta) = 1$ *is an identity because when* $\theta = 0$*, the equation is true. Is Skippy correct? Explain.*

No, Skippy is not correct. In order to be an identity, an equation must hold for all applicable angles. For example, $\cos(\theta) + \sin(\theta) = 1$ *does not hold when* $\theta = \pi$.

Question 4 *In Exercises* **??** - **??**, *verify the identity. Assume that all quantities are defined.*

Problem 4.1 $\cos(\theta)\sec(\theta) = 1$

Problem 4.2 $\tan(t)\cos(t) = \sin(t)$

Problem 4.3 $\sin(\theta)\csc(\theta) = 1$

Problem 4.4 $\tan(t)\cot(t) = 1$

Problem 4.5 $\csc(x)\cos(x) = \cot(x)$

Problem 4.6 $\frac{\sin(t)}{\cos^2(t)} = \sec(t)\tan(t)$

Problem 4.7 $\frac{\cos(\theta)}{\sin^2(\theta)} = \csc(\theta)\cot(\theta)$

Problem 4.8 $\frac{1 + \sin(x)}{\cos(x)} = \sec(x) + \tan(x)$

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Problem 4.9 $\dfrac{1-\cos(\theta)}{\sin(\theta)} = \csc(\theta) - \cot(\theta)$

Problem 4.10 $\dfrac{\cos(t)}{1-\sin^2(t)} = \sec(t)$

Problem 4.11 $\dfrac{\sin(x)}{1-\cos^2(x)} = \csc(x)$

Problem 4.12 $\dfrac{\sec(t)}{1+\tan^2(t)} = \cos(t)$

Problem 4.13 $\dfrac{\csc(\theta)}{1+\cot^2(\theta)} = \sin(\theta)$

Problem 4.14 $\dfrac{\tan(x)}{\sec^2(x)-1} = \cot(x)$

Problem 4.15 $\dfrac{\cot(t)}{\csc^2(t)-1} = \tan(t)$

Problem 4.16 $4\cos^2(\theta) + 4\sin^2(\theta) = 4$

Problem 4.17 $9 - \cos^2(t) - \sin^2(t) = 8$

Problem 4.18 $\tan^3(t) = \tan(t)\sec^2(t) - \tan(t)$

Problem 4.19 $\sin^5(x) = \left(1-\cos^2(x)\right)^2 \sin(x)$

Problem 4.20 $\sec^{10}(t) = \left(1+\tan^2(t)\right)^4 \sec^2(t)$

Problem 4.21 $\cos^2(x)\tan^3(x) = \tan(x) - \sin(x)\cos(x)$

Problem 4.22 $\sec^4(t) - \sec^2(t) = \tan^2(t) + \tan^4(t)$

Problem 4.23 $\dfrac{\cos(\theta)+1}{\cos(\theta)-1} = \dfrac{1+\sec(\theta)}{1-\sec(\theta)}$

Problem 4.24 $\dfrac{\sin(t)+1}{\sin(t)-1} = \dfrac{1+\csc(t)}{1-\csc(t)}$

Problem 4.25 $\dfrac{1-\cot(x)}{1+\cot(x)} = \dfrac{\tan(x)-1}{\tan(x)+1}$

Problem 4.26 $\dfrac{1-\tan(t)}{1+\tan(t)} = \dfrac{\cos(t)-\sin(t)}{\cos(t)+\sin(t)}$

Problem 4.27 $\tan(\theta) + \cot(\theta) = \sec(\theta)\csc(\theta)$

Problem 4.28 $\csc(t) - \sin(t) = \cot(t)\cos(t)$

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Problem 4.29 $\cos(x) - \sec(x) = -\tan(x)\sin(x)$

Problem 4.30 $\cos(x)(\tan(x) + \cot(x)) = \csc(x)$

Problem 4.31 $\sin(t)(\tan(t) + \cot(t)) = \sec(t)$

Problem 4.32 $\dfrac{1}{1-\cos(\theta)} + \dfrac{1}{1+\cos(\theta)} = 2\csc^2(\theta)$

Problem 4.33 $\dfrac{1}{\sec(t)+1} + \dfrac{1}{\sec(t)-1} = 2\csc(t)\cot(t)$

Problem 4.34 $\dfrac{1}{\csc(x)+1} + \dfrac{1}{\csc(x)-1} = 2\sec(x)\tan(x)$

Problem 4.35 $\dfrac{1}{\csc(t)-\cot(t)} - \dfrac{1}{\csc(t)+\cot(t)} = 2\cot(t)$

Problem 4.36 $\dfrac{\cos(\theta)}{1-\tan(\theta)} + \dfrac{\sin(\theta)}{1-\cot(\theta)} = \sin(\theta) + \cos(\theta)$

Problem 4.37 $\dfrac{1}{\sec(t)+\tan(t)} = \sec(t) - \tan(t)$

Problem 4.38 $\dfrac{1}{\sec(x)-\tan(x)} = \sec(x) + \tan(x)$

Problem 4.39 $\dfrac{1}{\csc(t)-\cot(t)} = \csc(t) + \cot(t)$

Problem 4.40 $\dfrac{1}{\csc(\theta)+\cot(\theta)} = \csc(\theta) - \cot(\theta)$

Problem 4.41 $\dfrac{1}{1-\sin(x)} = \sec^2(x) + \sec(x)\tan(x)$

Problem 4.42 $\dfrac{1}{1+\sin(t)} = \sec^2(t) - \sec(t)\tan(t)$

Problem 4.43 $\dfrac{1}{1-\cos(\theta)} = \csc^2(\theta) + \csc(\theta)\cot(\theta)$

Problem 4.44 $\dfrac{1}{1+\cos(x)} = \csc^2(x) - \csc(x)\cot(x)$

Problem 4.45 $\dfrac{\cos(t)}{1+\sin(t)} = \dfrac{1-\sin(t)}{\cos(t)}$

Problem 4.46 $\csc(\theta) - \cot(\theta) = \dfrac{\sin(\theta)}{1+\cos(\theta)}$

Problem 4.47 $\dfrac{1-\sin(x)}{1+\sin(x)} = (\sec(x) - \tan(x))^2$

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Question 5 *In Exercises* **??** *-* **??**, *verify the identity. You may need to consult Sections* **??** *and* **??** *for a review of the properties of absolute value and logarithms before proceeding.*

Problem 5.1 $\ln|\sec(x)| = -\ln|\cos(x)|$

Problem 5.2 $-\ln|\csc(x)| = \ln|\sin(x)|$

Problem 5.3 $-\ln|\sec(x) - \tan(x)| = \ln|\sec(x) + \tan(x)|$

Problem 5.4 $-\ln|\csc(x) + \cot(x)| = \ln|\csc(x) - \cot(x)|$

Problem 6 *1. What indeterminate form is present in the limit*

$$\lim_{\theta\to 0}\frac{1-\cos(\theta)}{\theta}?$$

As $\theta \to 0$, $\dfrac{1-\cos(\theta)}{\theta} \to \dfrac{0}{0}$.

2. Graph $f(\theta) = \dfrac{1-\cos(\theta)}{\theta}$ *near* $\theta = 0$. *What appears to be*

$$\lim_{\theta\to 0}\frac{1-\cos(\theta)}{\theta}?$$

The graph of $f(\theta) = \dfrac{1-\cos(\theta)}{\theta}$ *approaches* $(0,0)$, *so* $\lim\limits_{\theta\to 0}\dfrac{1-\cos(\theta)}{\theta}$ *appears to be* 0.

3. Verify the identity: $\dfrac{1-\cos(\theta)}{\theta} = \dfrac{\sin(\theta)}{1+\cos(\theta)}\dfrac{\sin(\theta)}{\theta}$.

4. Use the fact[1] *that*

$$\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta} = 1$$

along with part **??** *to help you find* $\lim\limits_{\theta\to 0}\dfrac{1-\cos(\theta)}{\theta}$.

$$\lim_{\theta\to 0}\frac{1-\cos(\theta)}{\theta} = \lim_{\theta\to 0}\frac{\sin(\theta)}{1+\cos(\theta)}\frac{\sin(\theta)}{\theta} = \left(\frac{0}{1+\cos(0)}\right)(1) = \left(\frac{0}{2}\right)(1) = 0.$$

[1] See Exercise **??** in Section **??**.

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