Exercises for Trigonometric Equations and Inequalities

Question 1 *In Exercises* **??** - **??**, *find all of the exact solutions of the equation and then list those solutions which are in the interval* $[0, 2\pi)$.

Problem 1.1 $\sin(5\theta) = 0$

$\theta = \frac{\pi k}{5}$; $\theta = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}, \frac{9\pi}{5}$

Problem 1.2 $\cos(3t) = \frac{1}{2}$

$t = \frac{\pi}{9} + \frac{2\pi k}{3}$ *or* $t = \frac{5\pi}{9} + \frac{2\pi k}{3}$; $t = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$

Problem 1.3 $\sin(-2x) = \frac{\sqrt{3}}{2}$

$x = \frac{2\pi}{3} + \pi k$ *or* $x = \frac{5\pi}{6} + \pi k$; $x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

Problem 1.4 $\tan(6\theta) = 1$

$\theta = \frac{\pi}{24} + \frac{\pi k}{6}$; $\theta = \frac{\pi}{24}, \frac{5\pi}{24}, \frac{3\pi}{8}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{7\pi}{8}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{11\pi}{8}, \frac{37\pi}{24}, \frac{41\pi}{24}, \frac{15\pi}{8}$

Problem 1.5 $\csc(4t) = -1$

$t = \frac{3\pi}{8} + \frac{\pi k}{2}$; $t = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

Problem 1.6 $\sec(3x) = \sqrt{2}$

$x = \frac{\pi}{12} + \frac{2\pi k}{3}$ *or* $x = \frac{7\pi}{12} + \frac{2\pi k}{3}$; $x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}$

Problem 1.7 $\cot(2\theta) = -\frac{\sqrt{3}}{3}$

$\theta = \frac{\pi}{3} + \frac{\pi k}{2}$; $\theta = \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$

Problem 1.8 $\cos(9t) = 9$

No solution

Problem 1.9 $\sin\left(\frac{x}{3}\right) = \frac{\sqrt{2}}{2}$

$x = \frac{3\pi}{4} + 6\pi k$ *or* $x = \frac{9\pi}{4} + 6\pi k$; $x = \frac{3\pi}{4}$

Problem 1.10 $\cos\left(\theta + \frac{5\pi}{6}\right) = 0$

$\theta = -\frac{\pi}{3} + \pi k$; $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

Problem 1.11 $\sin\left(2t - \frac{\pi}{3}\right) = -\frac{1}{2}$

$t = \frac{3\pi}{4} + \pi k$ *or* $t = \frac{13\pi}{12} + \pi k$; $t = \frac{\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{7\pi}{4}$

Problem 1.12 $2\cos\left(x+\frac{7\pi}{4}\right)=\sqrt{3}$

$x=-\frac{19\pi}{12}+2\pi k$ *or* $x=\frac{\pi}{12}+2\pi k;\ x=\frac{\pi}{12},\frac{5\pi}{12}$

Problem 1.13 $\csc(\theta)=0$

No solution

Problem 1.14 $\tan(2t-\pi)=1$

$t=\frac{5\pi}{8}+\frac{\pi k}{2};\ t=\frac{\pi}{8},\frac{5\pi}{8},\frac{9\pi}{8},\frac{13\pi}{8}$

Problem 1.15 $\tan^2(x)=3$

$x=\frac{\pi}{3}+\pi k$ *or* $x=\frac{2\pi}{3}+\pi k;\ x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$

Problem 1.16 $\sec^2(\theta)=\frac{4}{3}$

$\theta=\frac{\pi}{6}+\pi k$ *or* $\theta=\frac{5\pi}{6}+\pi k;\ \theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$

Problem 1.17 $\cos^2(t)=\frac{1}{2}$

$t=\frac{\pi}{4}+\frac{\pi k}{2};\ t=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

Problem 1.18 $\sin^2(x)=\frac{3}{4}$

$x=\frac{\pi}{3}+\pi k$ *or* $x=\frac{2\pi}{3}+\pi k;\ x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$

Question 2 *In Exercises* **??** - **??**, *solve the equation, giving the exact solutions which lie in* $[0,2\pi)$

Problem 2.1 $\sin(\theta)=\cos(\theta)$

$\theta=\frac{\pi}{4},\frac{5\pi}{4}$

Problem 2.2 $\sin(2t)=\sin(t)$

$t=0,\frac{\pi}{3},\pi,\frac{5\pi}{3}$

Problem 2.3 $\sin(2x)=\cos(x)$

Select All Correct Answers:

1. $x=\frac{\pi}{6}$ ✓
2. $x=\frac{\pi}{3}$
3. $x=\frac{\pi}{2}$ ✓

4. $x = \dfrac{2\pi}{3}$
5. $x = \dfrac{5\pi}{6}$ ✓
6. $x = \pi$
7. $x = \dfrac{3\pi}{2}$ ✓

Problem 2.4 $\cos(2\theta) = \sin(\theta)$

$\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$

Problem 2.5 $\cos(2t) = \cos(t)$

Select All Correct Answers:

1. $t = 0$ ✓
2. $t = \dfrac{\pi}{3}$
3. $t = \dfrac{2\pi}{3}$ ✓
4. $t = \pi$
5. $t = \dfrac{4\pi}{3}$ ✓
6. $t = \dfrac{5\pi}{3}$

Problem 2.6 $\cos(2x) = 2 - 5\cos(x)$

$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

Problem 2.7 $3\cos(2\theta) + \cos(\theta) + 2 = 0$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \arccos\left(\dfrac{1}{3}\right), 2\pi - \arccos\left(\dfrac{1}{3}\right)$

Problem 2.8 $\cos(2t) = 5\sin(t) - 2$

Select All Correct Answers:

1. $t = \dfrac{\pi}{6}$ ✓
2. $t = \dfrac{\pi}{3}$
3. $t = \dfrac{\pi}{2}$
4. $t = \dfrac{2\pi}{3}$
5. $t = \dfrac{5\pi}{6}$ ✓
6. $t = \pi$

7. $t = \dfrac{3\pi}{2}$

Problem 2.9 $3\cos(2x) = \sin(x) + 2$

$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}, \arcsin\left(\dfrac{1}{3}\right), \pi - \arcsin\left(\dfrac{1}{3}\right)$

Problem 2.10 $2\sec^2(\theta) = 3 - \tan(\theta)$

$\theta = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}, \arctan\left(\dfrac{1}{2}\right), \pi + \arctan\left(\dfrac{1}{2}\right)$

Problem 2.11 $\tan^2(t) = 1 - \sec(t)$

Select All Correct Answers:

1. $t = 0$ ✓
2. $t = \dfrac{\pi}{3}$
3. $t = \dfrac{2\pi}{3}$ ✓
4. $t = \pi$
5. $t = \dfrac{4\pi}{3}$ ✓
6. $t = \dfrac{5\pi}{3}$

Problem 2.12 $\cot^2(x) = 3\csc(x) - 3$

Select All Correct Answers:

1. $x = \dfrac{\pi}{6}$ ✓
2. $x = \dfrac{\pi}{3}$
3. $x = \dfrac{\pi}{2}$ ✓
4. $x = \dfrac{2\pi}{3}$
5. $x = \dfrac{5\pi}{6}$ ✓
6. $x = \pi$

Problem 2.13 $\sec(\theta) = 2\csc(\theta)$

$\theta = \arctan(2), \pi + \arctan(2)$

Problem 2.14 $\cos(t)\csc(t)\cot(t) = 6 - \cot^2(t)$

$t = \dfrac{\pi}{6}, \dfrac{7\pi}{6}, \dfrac{5\pi}{6}, \dfrac{11\pi}{6}$

Problem 2.15 $\sin(2x) = \tan(x)$

$x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Problem 2.16 $\cot^4(\theta) = 4\csc^2(\theta) - 7$

$\theta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{6}$

Problem 2.17 $\cos(2t) + \csc^2(t) = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2}$

Problem 2.18 $\tan^3(x) = 3\tan(x)$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

Problem 2.19 $\tan^2(\theta) = \frac{3}{2}\sec(\theta)$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

Problem 2.20 $\cos^3(t) = -\cos(t)$

$t = \frac{\pi}{2}, \frac{3\pi}{2}$

Problem 2.21 $\tan(2x) - 2\cos(x) = 0$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Problem 2.22 $\csc^3(\theta) + \csc^2(\theta) = 4\csc(\theta) + 4$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Problem 2.23 $2\tan(t) = 1 - \tan^2(t)$

$t = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

Problem 2.24 $\tan(x) = \sec(x)$

No solution

Question 3 *In Exercises* **??** - **??**, *solve the equation, giving the exact solutions which lie in* $[0, 2\pi)$

Problem 3.1 $\sin(6\theta)\cos(\theta) = -\cos(6\theta)\sin(\theta)$

$\theta = 0, \frac{\pi}{7}, \frac{2\pi}{7}, \frac{3\pi}{7}, \frac{4\pi}{7}, \frac{5\pi}{7}, \frac{6\pi}{7}, \pi, \frac{8\pi}{7}, \frac{9\pi}{7}, \frac{10\pi}{7}, \frac{11\pi}{7}, \frac{12\pi}{7}, \frac{13\pi}{7}$

Problem 3.2 $\sin(3t)\cos(t) = \cos(3t)\sin(t)$

$t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

Problem 3.3 $\cos(2x)\cos(x) + \sin(2x)\sin(x) = 1$

$x = 0$

Problem 3.4 $\cos(5\theta)\cos(3\theta) - \sin(5\theta)\sin(3\theta) = \dfrac{\sqrt{3}}{2}$

$\theta = \dfrac{\pi}{48}, \dfrac{11\pi}{48}, \dfrac{13\pi}{48}, \dfrac{23\pi}{48}, \dfrac{25\pi}{48}, \dfrac{35\pi}{48}, \dfrac{37\pi}{48}, \dfrac{47\pi}{48}, \dfrac{49\pi}{48}, \dfrac{59\pi}{48}, \dfrac{61\pi}{48}, \dfrac{71\pi}{48}, \dfrac{73\pi}{48}, \dfrac{83\pi}{48}, \dfrac{85\pi}{48}, \dfrac{95\pi}{48}$

Problem 3.5 $\sin(t) + \cos(t) = 1$

Select All Correct Answers:

1. $t = 0$ ✓
2. $t = \dfrac{\pi}{6}$
3. $t = \dfrac{\pi}{3}$
4. $t = \dfrac{\pi}{2}$ ✓
5. $t = \dfrac{2\pi}{3}$
6. $t = \dfrac{5\pi}{6}$
7. $t = \pi$

Problem 3.6 $\sin(x) + \sqrt{3}\cos(x) = 1$

$x = \dfrac{\pi}{2}, \dfrac{11\pi}{6}$

Problem 3.7 $\sqrt{2}\cos(\theta) - \sqrt{2}\sin(\theta) = 1$

$\theta = \dfrac{\pi}{12}, \dfrac{17\pi}{12}$

Problem 3.8 $\sqrt{3}\sin(2t) + \cos(2t) = 1$

Select All Correct Answers:

1. $t = 0$ ✓
2. $t = \dfrac{\pi}{3}$ ✓
3. $t = \dfrac{2\pi}{3}$
4. $t = \pi$ ✓
5. $t = \dfrac{4\pi}{3}$ ✓
6. $t = \dfrac{5\pi}{3}$

Problem 3.9 $\cos(2x) - \sqrt{3}\sin(2x) = \sqrt{2}$

$x = \dfrac{17\pi}{24}, \dfrac{41\pi}{24}, \dfrac{23\pi}{24}, \dfrac{47\pi}{24}$

Problem 3.10 $3\sqrt{3}\sin(3\theta) - 3\cos(3\theta) = 3\sqrt{3}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{18}, \frac{5\pi}{6}, \frac{17\pi}{18}, \frac{3\pi}{2}, \frac{29\pi}{18}$

Problem 3.11 $\cos(3t) = \cos(5t)$

$t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

Problem 3.12 $\cos(4x) = \cos(2x)$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

Problem 3.13 $\sin(5\theta) = \sin(3\theta)$

$\theta = 0, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \pi, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

Problem 3.14 $\cos(5t) = -\cos(2t)$

$t = \frac{\pi}{7}, \frac{\pi}{3}, \frac{3\pi}{7}, \frac{5\pi}{7}, \pi, \frac{9\pi}{7}, \frac{11\pi}{7}, \frac{5\pi}{3}, \frac{13\pi}{7}$

Problem 3.15 $\sin(6x) + \sin(x) = 0$

$x = 0, \frac{2\pi}{7}, \frac{4\pi}{7}, \frac{6\pi}{7}, \frac{8\pi}{7}, \frac{10\pi}{7}, \frac{12\pi}{7}, \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$

Problem 3.16 $\tan(x) = \cos(x)$

$x = \arcsin\left(\frac{-1+\sqrt{5}}{2}\right) \approx 0.6662, \pi - \arcsin\left(\frac{-1+\sqrt{5}}{2}\right) \approx 2.4754$

Question 4 *In Exercises* **??** - **??**, *solve the equation.*

Problem 4.1 $\arccos(2x) = \pi$

$x = -\frac{1}{2}$

Problem 4.2 $\pi - 2\arcsin(t) = 2\pi$

$t = -1$

Problem 4.3 $4\arctan(3x-1) - \pi = 0$

$x = \frac{2}{3}$

Problem 4.4 $6\,arccot(2t) - 5\pi = 0$

$t = -\frac{\sqrt{3}}{2}$

Problem 4.5 $4\,arcsec\left(\frac{x}{2}\right) = \pi$

$x = 2\sqrt{2}$

Problem 4.6 $12\,arccsc\left(\frac{t}{3}\right) = 2\pi$

$t = 6$

Problem 4.7 $9\arcsin^2(x) - \pi^2 = 0$

$x = \pm\dfrac{\sqrt{3}}{2}$

Problem 4.8 $9\arccos^2(t) - \pi^2 = 0$

$t = \dfrac{1}{2}$

Problem 4.9 $8\,arccot^2(x) + 3\pi^2 = 10\pi\,arccot(x)$

$x = -1, 0$

Problem 4.10 $6\arctan(t)^2 = \pi\arctan(x) + \pi^2$

$t = -\sqrt{3}$

Question 5 *In Exercises* **??** *-* **??***, solve the inequality. Express the exact answer in interval notation, restricting your attention to* $0 \leq x \leq 2\pi$*.*

Problem 5.1 $\sin(x) \leq 0$

$[\pi, 2\pi]$

Problem 5.2 $\tan(t) \geq \sqrt{3}$

$\left[\dfrac{\pi}{3}, \dfrac{\pi}{2}\right) \cup \left[\dfrac{4\pi}{3}, \dfrac{3\pi}{2}\right)$

Problem 5.3 $\sec^2(x) \leq 4$

$\left[0, \dfrac{\pi}{3}\right] \cup \left[\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right] \cup \left[\dfrac{5\pi}{3}, 2\pi\right]$

Problem 5.4 $\cos^2(t) > \dfrac{1}{2}$

$\left[0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right) \cup \left(\dfrac{7\pi}{4}, 2\pi\right]$

Problem 5.5 $\cos(2x) \leq 0$

$\left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right] \cup \left[\dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right]$

Problem 5.6 $\sin\left(t + \dfrac{\pi}{3}\right) > \dfrac{1}{2}$

$\left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{11\pi}{6}, 2\pi\right]$

Problem 5.7 $\cot^2(x) \geq \dfrac{1}{3}$

$\left(0, \dfrac{\pi}{3}\right] \cup \left[\dfrac{2\pi}{3}, \pi\right) \cup \left(\pi, \dfrac{4\pi}{3}\right] \cup \left[\dfrac{5\pi}{3}, 2\pi\right)$

Problem 5.8 $2\cos(t) \geq 1$

$\left[0, \dfrac{\pi}{3}\right] \cup \left[\dfrac{5\pi}{3}, 2\pi\right]$

Problem 5.9 $\sin(5x) \geq 5$

No solution

Problem 5.10 $\cos(3t) \leq 1$

$[0, 2\pi]$

Problem 5.11 $\sec(x) \leq \sqrt{2}$

$\left[0, \frac{\pi}{4}\right] \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cup \left[\frac{7\pi}{4}, 2\pi\right]$

Problem 5.12 $\cot(t) \leq 4$

$[arccot(4), \pi) \cup [\pi + arccot(4), 2\pi)$

Question 6 *In Exercises* **??** *-* **??***, solve the inequality. Express the exact answer in interval notation, restricting your attention to* $-\pi \leq x \leq \pi$*.*

Problem 6.1 $\cos(x) > \frac{\sqrt{3}}{2}$

$\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$

Problem 6.2 $\sin(t) > \frac{1}{3}$

$\left(\arcsin\left(\frac{1}{3}\right), \pi - \arcsin\left(\frac{1}{3}\right)\right)$

Problem 6.3 $\sec(x) \leq 2$

$\left[-\pi, -\frac{\pi}{2}\right) \cup \left[-\frac{\pi}{3}, \frac{\pi}{3}\right] \cup \left(\frac{\pi}{2}, \pi\right]$

Problem 6.4 $\sin^2(t) < \frac{3}{4}$

$\left(-\frac{2\pi}{3}, -\frac{\pi}{3}\right) \cup \left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$

Problem 6.5 $\cot(x) \geq -1$

$\left(-\pi, -\frac{\pi}{4}\right] \cup \left(0, \frac{3\pi}{4}\right]$

Problem 6.6 $\cos(t) \geq \sin(t)$

$\left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$

Question 7 *In Exercises* **??** *-* **??***, solve the inequality. Express the exact answer in interval notation, restricting your attention to* $-2\pi \leq x \leq 2\pi$*.*

Problem 7.1 $\csc(x) > 1$

$\left(-2\pi, -\frac{3\pi}{2}\right) \cup \left(-\frac{3\pi}{2}, -\pi\right) \cup \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

Problem 7.2 $\cos(t) \leq \frac{5}{3}$

$[-2\pi, 2\pi]$

Problem 7.3 $\cot(x) \geq 5$

$(-2\pi, \mathit{arccot}(5) - 2\pi] \cup (-\pi, \mathit{arccot}(5) - \pi] \cup (0, \mathit{arccot}(5)] \cup (\pi, \pi + \mathit{arccot}(5)]$

Problem 7.4 $\tan^2(t) \geq 1$

$$\left[-\frac{7\pi}{4}, -\frac{3\pi}{2}\right) \cup \left(-\frac{3\pi}{2}, -\frac{5\pi}{4}\right] \cup \left[-\frac{3\pi}{4}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right] \cup \left[\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right]$$

Problem 7.5 $\sin(2x) \geq \sin(x)$

$$\left[-2\pi, -\frac{5\pi}{3}\right] \cup \left[-\pi, -\frac{\pi}{3}\right] \cup \left[0, \frac{\pi}{3}\right] \cup \left[\pi, \frac{5\pi}{3}\right]$$

Problem 7.6 $\cos(2t) \leq \sin(x)$

$$\left[-\frac{11\pi}{6}, -\frac{7\pi}{6}\right] \cup \left[\frac{\pi}{6}, \frac{5\pi}{6}\right] \cup, \left\{-\frac{\pi}{2}, \frac{3\pi}{2}\right\}$$

Question 8 *In Exercises* **??** - **??**, *solve the given inequality.*

Problem 8.1 $\arcsin(2x) > 0$

$$\left(0, \frac{1}{2}\right]$$

Problem 8.2 $3\arccos(t) \leq \pi$

$$\left[\frac{1}{2}, 1\right]$$

Problem 8.3 $6\,\mathit{arccot}(7x) \geq \pi$

$$\left(-\infty, \frac{\sqrt{3}}{7}\right]$$

Problem 8.4 $\pi > 2\arctan(t)$

$(-\infty, \infty)$

Problem 8.5 $2\arcsin(x)^2 > \pi \arcsin(x)$

$[-1, 0)$

Problem 8.6 $12\arccos(t)^2 + 2\pi^2 > 11\pi \arccos(t)$

$$\left[-1, -\frac{1}{2}\right) \cup \left(\frac{\sqrt{2}}{2}, 1\right]$$

Question 9 *In Exercises* **??** - **??**, *express the domain of the function using the extended interval notation. (See Example* **??** *and Section* **??** *for details.)*

Problem 9.1 $f(x) = \dfrac{1}{\cos(x) - 1}$

$$\bigcup_{k=-\infty}^{\infty} (2k\pi, (2k+2)\pi)$$

Problem 9.2 $f(t) = \dfrac{\cos(t)}{\sin(t)+1}$

$$\bigcup_{k=-\infty}^{\infty} \left(\frac{(4k-1)\pi}{2}, \frac{(4k+3)\pi}{2} \right)$$

Problem 9.3 $f(x) = \sqrt{\tan^2(x) - 1}$

$$\bigcup_{k=-\infty}^{\infty} \left\{ \left[\frac{(4k+1)\pi}{4}, \frac{(2k+1)\pi}{2} \right) \cup \left(\frac{(2k+1)\pi}{2}, \frac{(4k+3)\pi}{4} \right] \right\}$$

Problem 9.4 $f(t) = \sqrt{2 - \sec(t)}$

$$\bigcup_{k=-\infty}^{\infty} \left\{ \left[\frac{(6k-1)\pi}{3}, \frac{(6k+1)\pi}{3} \right] \cup \left(\frac{(4k+1)\pi}{2}, \frac{(4k+3)\pi}{2} \right) \right\}$$

Problem 9.5 $f(x) = \csc(2x)$

$$\bigcup_{k=-\infty}^{\infty} \left(\frac{k\pi}{2}, \frac{(k+1)\pi}{2} \right)$$

Problem 9.6 $f(t) = \dfrac{\sin(t)}{2+\cos(t)}$

$(-\infty, \infty)$

Problem 9.7 $f(x) = 3\csc(x) + 4\sec(x)$

$$\bigcup_{k=-\infty}^{\infty} \left(\frac{k\pi}{2}, \frac{(k+1)\pi}{2} \right)$$

Problem 9.8 $f(t) = \ln\left(|\cos(t)|\right)$

$$\bigcup_{k=-\infty}^{\infty} \left(\frac{(2k-1)\pi}{2}, \frac{(2k+1)\pi}{2} \right)$$

Problem 9.9 $f(x) = \arcsin(\tan(x))$

$$\bigcup_{k=-\infty}^{\infty} \left[\frac{(4k-1)\pi}{4}, \frac{(4k+1)\pi}{4} \right]$$

Problem 10

1. *With the help of your classmates, determine the number of solutions to* $\sin(x) = \dfrac{1}{2}$ *in* $[0, 2\pi)$. *Then find the number of solutions to* $\sin(2x) = \dfrac{1}{2}$, $\sin(3x) = \dfrac{1}{2}$ *and* $\sin(4x) = \dfrac{1}{2}$ *in* $[0, 2\pi)$. *What pattern emerges? Explain how this pattern would help you solve equations like* $\sin(11x) = \dfrac{1}{2}$.
2. *Repeat the above exercise focusing on* $\sin\left(\dfrac{x}{2}\right) = \dfrac{1}{2}$, $\sin\left(\dfrac{3x}{2}\right) = \dfrac{1}{2}$ *and* $\sin\left(\dfrac{5x}{2}\right) = \dfrac{1}{2}$. *What pattern emerges here?*
3. *Replace sine with tangent and* $\dfrac{1}{2}$ *with* 1 *and repeat the whole exploration.*

Problem 11 *Suppose an object weighing* 10 *pounds is suspended from the ceiling by a spring which stretches* 2 *feet to its equilibrium position when the object is attached.*

1. *Find the spring constant* k *in* $\frac{lbs.}{ft.}$ *and the mass of the object in slugs.*

   $k = 5\ \frac{lbs.}{ft.}$ *and* $m = \frac{5}{16}$ *slugs*

2. *Find the equation of motion of the object if it is released from* 1 *foot below the equilibrium position from rest. When is the first time the object passes through the equilibrium position? In which direction is it heading?*

   $x(t) = \sin\left(4t + \frac{\pi}{2}\right)$. *The object first passes through the equilibrium point when* $t = \frac{\pi}{8} \approx 0.39$ *seconds after the motion starts. At this time, the object is heading upwards.*

3. *Find the equation of motion of the object if it is released from* 6 *inches above the equilibrium position with a downward velocity of* 2 *feet per second. Find when the object passes through the equilibrium position heading downwards for the third time.*

   $x(t) = \frac{\sqrt{2}}{2}\sin\left(4t + \frac{7\pi}{4}\right)$. *The object passes through the equilibrium point heading downwards for the third time when* $t = \frac{17\pi}{16} \approx 3.34$ *seconds.*

Problem 12 *In Example* **??**, $f(x) = 2\sin(x) - \sin(2x)$ *restricted to* $0 \leq x \leq 2\pi$. *If* $f''(x) = 4\sin(2x) - 2\sin(x)$, *find the inflection points of the graph of* $y = f(x)$.

The inflection points are: $\left(\arccos\left(\frac{1}{4}\right), \frac{3\sqrt{15}}{8}\right)$, $(\pi, 0)$, *and* $\left(2\pi - \arccos\left(\frac{1}{4}\right), -\frac{3\sqrt{15}}{8}\right)$

Problem 13 *In Example* **??**, $f(x) = x - 2\cos(x)$ *restricted to* $0 \leq x \leq 2\pi$. *If* $f'(x) = 2\sin(x) + 1$, *list the open intervals over which* f *is increasing and decreasing. Find the local extrema.*

f *is increasing on* $\left(0, \frac{7\pi}{6}\right)$ *and again on* $\left(\frac{11\pi}{6}, 2\pi\right)$; f *is decreasing on* $\left(\frac{7\pi}{6}, \frac{11\pi}{6}\right)$; *local (absolute) max:* $\left(\frac{7\pi}{6}, \frac{7\pi}{6} + \sqrt{3}\right)$; *local min:* $\left(\frac{11\pi}{6}, \frac{11\pi}{6} - \sqrt{3}\right)$

Problem 14 *Let* $f(x) = e^{-x}\sin(x)$ *for* $x > 0$.

1. *Use the Squeeze Theorem, Theorem* **??**, *to find* $\lim_{x\to\infty} f(x)$. *Interpret your answer graphically.*

   Hint: *Since* $-1 \leq \sin(x) \leq 1$, $-e^{-x} \leq e^{-x}\sin(x) \leq e^{-x}$ ...

   $\lim_{x\to\infty} f(x) = 0$. *We have a horizontal asymptote* $y = 0$.

2. *Use the fact that* $f'(x) = e^{-x}\cos(x) - e^{-x}\sin(x)$ *to help you find the intervals over which* f *is increasing and decreasing.*

   - f *is increasing on* $\left(0, \frac{\pi}{4}\right)$, $\left(\frac{5\pi}{4}, \frac{9\pi}{4}\right)$, $\left(\frac{13\pi}{4}, \frac{17\pi}{4}\right)$, ...:

     In other words: on $\left(0, \frac{\pi}{4}\right)$ *along with* $\left(\frac{(8k+5)\pi}{4}, \frac{(8k+9)\pi}{4}\right)$ *for* $k = 0, 1, 2, 3, ...$
   - f *is decreasing on* $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$, $\left(\frac{9\pi}{4}, \frac{13\pi}{4}\right)$, $\left(\frac{17\pi}{4}, \frac{21\pi}{4}\right)$, ...:

     In other words: on $\left(\frac{(8k+1)\pi}{4}, \frac{(8k+5)\pi}{4}\right)$ *for* $k = 0, 1, 2, 3$ ...

3. *Use the fact that* $f''(x) = -2e^{-x}\cos(x)$ *to help you find the intervals over which the graph of* f *is concave up and concave down.*

   - *the graph of* f *is concave up on* $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right), \left(\frac{5\pi}{2}, \frac{7\pi}{2}\right), \left(\frac{9\pi}{2}, \frac{11\pi}{2}\right), \ldots$:

     In other words: on $\left(\frac{(4k+1)\pi}{2}, \frac{(4k+3)\pi}{2}\right)$ *for* $k = 0, 1, 2, 3, \ldots$

   - *the graph of* f *is concave down on* $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right), \left(\frac{7\pi}{2}, \frac{9\pi}{2}\right), \ldots$:

     In other words: on $\left(0, \frac{\pi}{2}\right)$ *along with* $\left(\frac{(4k+3)\pi}{2}, \frac{(4k+5)\pi}{2}\right)$ *for* $k = 0, 1, 2, 3 \ldots$

Problem 15 *Recall* $x(t) = 10e^{-t/5}\sin\left(t + \frac{\pi}{3}\right)$ *from Example* **??** *number* **??** *models underdamped motion. Use the Squeeze Theorem, Theorem* **??**, *to prove* $\lim_{t\to\infty} x(t) = 0$.

HINT: *Since* $-1 \leq \sin\left(t + \frac{\pi}{3}\right) \leq 1$, $-10e^{-t/5} \leq 10e^{-t/5}\sin\left(t + \frac{\pi}{3}\right) \leq 10e^{-t/5}$ ...