Exercises for the Inverse Trigonometric Functions

Question 1 *In Exercises* **??** - **??**, *find the exact value. (To enter* π *in an answer box, type "pi".)*

Problem 1.1 $\arcsin(-1)$

$\arcsin(-1) = -\frac{\pi}{2}$

Problem 1.2 $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

Problem 1.3 $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

Problem 1.4 $\arcsin\left(-\frac{1}{2}\right)$

$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Problem 1.5 $\arcsin(0)$

$\arcsin(0) = 0$

Problem 1.6 $\arcsin\left(\frac{1}{2}\right)$

$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Problem 1.7 $\arcsin\left(\frac{\sqrt{2}}{2}\right)$

$\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

Problem 1.8 $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

$\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

Problem 1.9 $\arcsin(1)$

$\arcsin(1) = \frac{\pi}{2}$

Problem 1.10 $\arccos(-1)$

$\arccos(-1) = \pi$

Problem 1.11 $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

Problem 1.12 $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

Problem 1.13 $\arccos\left(-\frac{1}{2}\right)$

$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Problem 1.14 $\arccos(0)$

$\arccos(0) = \frac{\pi}{2}$

Problem 1.15 $\arccos\left(\frac{1}{2}\right)$

$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Problem 1.16 $\arccos\left(\frac{\sqrt{2}}{2}\right)$

$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

Problem 1.17 $\arccos\left(\frac{\sqrt{3}}{2}\right)$

$\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

Problem 1.18 $\arccos(1)$

$\arccos(1) = 0$

Problem 1.19 $\arctan\left(-\sqrt{3}\right)$

$\arctan\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

Problem 1.20 $\arctan(-1)$

$\arctan(-1) = -\frac{\pi}{4}$

Problem 1.21 $\arctan\left(-\frac{\sqrt{3}}{3}\right)$

$$\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$$

Problem 1.22 $\arctan(0)$

$\arctan(0) = 0$

Problem 1.23 $\arctan\left(\frac{\sqrt{3}}{3}\right)$

$$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

Problem 1.24 $\arctan(1)$

$\arctan(1) = \frac{\pi}{4}$

Problem 1.25 $\arctan\left(\sqrt{3}\right)$

$$\arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$$

Problem 1.26 $\mathit{arccot}\left(-\sqrt{3}\right)$

$$\mathit{arccot}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$$

Problem 1.27 $\mathit{arccot}(-1)$

$$\mathit{arccot}(-1) = \frac{3\pi}{4}$$

Problem 1.28 $\mathit{arccot}\left(-\frac{\sqrt{3}}{3}\right)$

$$\mathit{arccot}\left(-\frac{\sqrt{3}}{3}\right) = \frac{2\pi}{3}$$

Problem 1.29 $\mathit{arccot}(0)$

$$\mathit{arccot}(0) = \frac{\pi}{2}$$

Problem 1.30 $\mathit{arccot}\left(\frac{\sqrt{3}}{3}\right)$

$$\mathit{arccot}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{3}$$

Problem 1.31 $\mathit{arccot}(1)$

$$\mathit{arccot}(1) = \frac{\pi}{4}$$

Problem 1.32 $\mathit{arccot}\left(\sqrt{3}\right)$

$$\mathit{arccot}\left(\sqrt{3}\right) = \frac{\pi}{6}$$

Problem 1.33 $arcsec(2)$

$arcsec(2) = \frac{\pi}{3}$

Problem 1.34 $arccsc(2)$

$arccsc(2) = \frac{\pi}{6}$

Problem 1.35 $arcsec\left(\sqrt{2}\right)$

$arcsec\left(\sqrt{2}\right) = \frac{\pi}{4}$

Problem 1.36 $arccsc\left(\sqrt{2}\right)$

$arccsc\left(\sqrt{2}\right) = \frac{\pi}{4}$

Problem 1.37 $arcsec\left(\frac{2\sqrt{3}}{3}\right)$

$arcsec\left(\frac{2\sqrt{3}}{3}\right) = \frac{\pi}{6}$

Problem 1.38 $arccsc\left(\frac{2\sqrt{3}}{3}\right)$

$arccsc\left(\frac{2\sqrt{3}}{3}\right) = \frac{\pi}{3}$

Problem 1.39 $arcsec(1)$

$arcsec(1) = 0$

Problem 1.40 $arccsc(1)$

$arccsc(1) = \frac{\pi}{2}$

Question 2 *In Exercises* **??** *-* **??***, assume that the range of arcsecant is* $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ *and that the range of arccosecant is* $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ *when finding the exact value. (See Section* **??***.)*

Problem 2.1 $arcsec(-2)$

$arcsec(-2) = \frac{2\pi}{3}$

Problem 2.2 $arcsec\left(-\sqrt{2}\right)$

$arcsec\left(-\sqrt{2}\right) = \frac{3\pi}{4}$

Problem 2.3 $arcsec\left(-\frac{2\sqrt{3}}{3}\right)$

$arcsec\left(-\frac{2\sqrt{3}}{3}\right) = \frac{5\pi}{6}$

Problem 2.4 $arcsec(-1)$

$arcsec(-1) = \pi$

Problem 2.5 $arccsc(-2)$

$arccsc(-2) = -\frac{\pi}{6}$

Problem 2.6 $arccsc\left(-\sqrt{2}\right)$

$arccsc\left(-\sqrt{2}\right) = -\frac{\pi}{4}$

Problem 2.7 $arccsc\left(-\frac{2\sqrt{3}}{3}\right)$

$arccsc\left(-\frac{2\sqrt{3}}{3}\right) = -\frac{\pi}{3}$

Problem 2.8 $arccsc(-1)$

$arccsc(-1) = -\frac{\pi}{2}$

Question 3 *In Exercises* **??** *-* **??***, assume that the range of arcsecant is* $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$ *and that the range of arccosecant is* $\left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$ *when finding the exact value. (See Section* **??***.)*

Problem 3.1 $arcsec(-2)$

$arcsec(-2) = \frac{4\pi}{3}$

Problem 3.2 $arcsec\left(-\sqrt{2}\right)$

$arcsec\left(-\sqrt{2}\right) = \frac{5\pi}{4}$

Problem 3.3 $arcsec\left(-\frac{2\sqrt{3}}{3}\right)$

$arcsec\left(-\frac{2\sqrt{3}}{3}\right) = \frac{7\pi}{6}$

Problem 3.4 $arcsec(-1)$

$arcsec(-1) = \pi$

Problem 3.5 $arccsc(-2)$

$arccsc(-2) = \frac{7\pi}{6}$

Problem 3.6 $arccsc\left(-\sqrt{2}\right)$

$arccsc\left(-\sqrt{2}\right) = \frac{5\pi}{4}$

Problem 3.7 $\operatorname{arccsc}\left(-\frac{2\sqrt{3}}{3}\right)$

$\operatorname{arccsc}\left(-\frac{2\sqrt{3}}{3}\right) = \frac{4\pi}{3}$

Problem 3.8 $\operatorname{arccsc}(-1)$

$\operatorname{arccsc}(-1) = \frac{3\pi}{2}$

Question 4 *In Exercises* **??** - **??**, *find the exact value or state that it is undefined.*

Problem 4.1 $\sin\left(\arcsin\left(\frac{1}{2}\right)\right)$

$\sin\left(\arcsin\left(\frac{1}{2}\right)\right) = \frac{1}{2}$

Problem 4.2 $\sin\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)$

$\sin\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right) = -\frac{\sqrt{2}}{2}$

Problem 4.3 $\sin\left(\arcsin\left(\frac{3}{5}\right)\right)$

$\sin\left(\arcsin\left(\frac{3}{5}\right)\right) = \frac{3}{5}$

Problem 4.4 $\sin(\arcsin(-0.42))$

$\sin(\arcsin(-0.42)) = -0.42$

Problem 4.5 $\sin\left(\arcsin\left(\frac{5}{4}\right)\right)$

$\sin\left(\arcsin\left(\frac{5}{4}\right)\right)$ *is undefined.*

Problem 4.6 $\cos\left(\arccos\left(\frac{\sqrt{2}}{2}\right)\right)$

$\cos\left(\arccos\left(\frac{\sqrt{2}}{2}\right)\right) = \frac{\sqrt{2}}{2}$

Problem 4.7 $\cos\left(\arccos\left(-\frac{1}{2}\right)\right)$

$\cos\left(\arccos\left(-\frac{1}{2}\right)\right) = -\frac{1}{2}$

Problem 4.8 $\cos\left(\arccos\left(\frac{5}{13}\right)\right)$

$\cos\left(\arccos\left(\frac{5}{13}\right)\right) = \frac{5}{13}$

Problem 4.9 $\cos(\arccos(-0.998))$

$\cos(\arccos(-0.998)) = -0.998$

Problem 4.10 $\cos(\arccos(\pi))$

$\cos(\arccos(\pi))$ *is undefined.*

Problem 4.11 $\tan(\arctan(-1))$

$\tan(\arctan(-1)) = -1$

Problem 4.12 $\tan\left(\arctan\left(\sqrt{3}\right)\right)$

$\tan\left(\arctan\left(\sqrt{3}\right)\right) = \sqrt{3}$

Problem 4.13 $\tan\left(\arctan\left(\frac{5}{12}\right)\right)$

$\tan\left(\arctan\left(\frac{5}{12}\right)\right) = \frac{5}{12}$

Problem 4.14 $\tan(\arctan(0.965))$

$\tan(\arctan(0.965)) = 0.965$

Problem 4.15 $\tan(\arctan(3\pi))$

$\tan(\arctan(3\pi)) = 3\pi$

Problem 4.16 $\cot(\mathit{arccot}(1))$

$\cot(\mathit{arccot}(1)) = 1$

Problem 4.17 $\cot\left(\mathit{arccot}\left(-\sqrt{3}\right)\right)$

$\cot\left(\mathit{arccot}\left(-\sqrt{3}\right)\right) = -\sqrt{3}$

Problem 4.18 $\cot\left(\mathit{arccot}\left(-\frac{7}{24}\right)\right)$

$\cot\left(\mathit{arccot}\left(-\frac{7}{24}\right)\right) = -\frac{7}{24}$

Problem 4.19 $\cot(\mathit{arccot}(-0.001))$

$\cot(\mathit{arccot}(-0.001)) = -0.001$

Problem 4.20 $\cot\left(\mathit{arccot}\left(\frac{17\pi}{4}\right)\right)$

$\cot\left(\mathit{arccot}\left(\frac{17\pi}{4}\right)\right) = \frac{17\pi}{4}$

Problem 4.21 $\sec(\mathit{arcsec}(2))$

$\sec(\mathit{arcsec}(2)) = 2$

Problem 4.22 $\sec(arcsec(-1))$

$\sec(arcsec(-1)) = -1$

Problem 4.23 $\sec\left(arcsec\left(\frac{1}{2}\right)\right)$

$\sec\left(arcsec\left(\frac{1}{2}\right)\right)$ *is undefined.*

Problem 4.24 $\sec(arcsec(0.75))$

$\sec(arcsec(0.75))$ *is undefined.*

Problem 4.25 $\sec(arcsec(117\pi))$

$\sec(arcsec(117\pi)) = 117\pi$

Problem 4.26 $\csc\left(arccsc\left(\sqrt{2}\right)\right)$

$\csc\left(arccsc\left(\sqrt{2}\right)\right) = \sqrt{2}$

Problem 4.27 $\csc\left(arccsc\left(-\frac{2\sqrt{3}}{3}\right)\right)$

$\csc\left(arccsc\left(-\frac{2\sqrt{3}}{3}\right)\right) = -\frac{2\sqrt{3}}{3}$

Problem 4.28 $\csc\left(arccsc\left(\frac{\sqrt{2}}{2}\right)\right)$

$\csc\left(arccsc\left(\frac{\sqrt{2}}{2}\right)\right)$ *is undefined.*

Problem 4.29 $\csc(arccsc(1.0001))$

$\csc(arccsc(1.0001)) = 1.0001$

Problem 4.30 $\csc\left(arccsc\left(\frac{\pi}{4}\right)\right)$

$\csc\left(arccsc\left(\frac{\pi}{4}\right)\right)$ *is undefined.*

Question 5 *In Exercises* **??** - **??**, *find the exact value or state that it is undefined.*

Problem 5.1 $\arcsin\left(\sin\left(\frac{\pi}{6}\right)\right)$

$\arcsin\left(\sin\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

Problem 5.2 $\arcsin\left(\sin\left(-\frac{\pi}{3}\right)\right)$

$\arcsin\left(\sin\left(-\frac{\pi}{3}\right)\right) = -\frac{\pi}{3}$

Problem 5.3 $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right)$

$\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right) = \frac{\pi}{4}$

Problem 5.4 $\arcsin\left(\sin\left(\frac{11\pi}{6}\right)\right)$

$\arcsin\left(\sin\left(\frac{11\pi}{6}\right)\right) = -\frac{\pi}{6}$

Problem 5.5 $\arcsin\left(\sin\left(\frac{4\pi}{3}\right)\right)$

$\arcsin\left(\sin\left(\frac{4\pi}{3}\right)\right) = -\frac{\pi}{3}$

Problem 5.6 $\arccos\left(\cos\left(\frac{\pi}{4}\right)\right)$

$\arccos\left(\cos\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$

Problem 5.7 $\arccos\left(\cos\left(\frac{2\pi}{3}\right)\right)$

$\arccos\left(\cos\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

Problem 5.8 $\arccos\left(\cos\left(\frac{3\pi}{2}\right)\right)$

$\arccos\left(\cos\left(\frac{3\pi}{2}\right)\right) = \frac{\pi}{2}$

Problem 5.9 $\arccos\left(\cos\left(-\frac{\pi}{6}\right)\right)$

$\arccos\left(\cos\left(-\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

Problem 5.10 $\arccos\left(\cos\left(\frac{5\pi}{4}\right)\right)$

$\arccos\left(\cos\left(\frac{5\pi}{4}\right)\right) = \frac{3\pi}{4}$

Problem 5.11 $\arctan\left(\tan\left(\frac{\pi}{3}\right)\right)$

$\arctan\left(\tan\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

Problem 5.12 $\arctan\left(\tan\left(-\frac{\pi}{4}\right)\right)$

$\arctan\left(\tan\left(-\frac{\pi}{4}\right)\right) = -\frac{\pi}{4}$

Problem 5.13 $\arctan(\tan(\pi))$

$\arctan(\tan(\pi)) = 0$

Problem 5.14 $\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$

$\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$ *is undefined*

Problem 5.15 $\arctan\left(\tan\left(\frac{2\pi}{3}\right)\right)$

$\arctan\left(\tan\left(\frac{2\pi}{3}\right)\right) = -\frac{\pi}{3}$

Problem 5.16 $\mathit{arccot}\left(\cot\left(\frac{\pi}{3}\right)\right)$

$\mathit{arccot}\left(\cot\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

Problem 5.17 $\mathit{arccot}\left(\cot\left(-\frac{\pi}{4}\right)\right)$

$\mathit{arccot}\left(\cot\left(-\frac{\pi}{4}\right)\right) = \frac{3\pi}{4}$

Problem 5.18 $\mathit{arccot}(\cot(\pi))$

$\mathit{arccot}(\cot(\pi))$ *is undefined*

Problem 5.19 $\mathit{arccot}\left(\cot\left(\frac{3\pi}{2}\right)\right)$

$\mathit{arccot}\left(\cot\left(\frac{3\pi}{2}\right)\right) = \frac{\pi}{2}$

Problem 5.20 $\mathit{arccot}\left(\cot\left(\frac{2\pi}{3}\right)\right)$

$\mathit{arccot}\left(\cot\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

Question 6 *In Exercises* **??** *-* **??***, assume that the range of arcsecant is* $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ *and that the range of arccosecant is* $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ *when finding the exact value. (See Section* **??***.)*

Problem 6.1 $\mathit{arcsec}\left(\sec\left(\frac{\pi}{4}\right)\right)$

$\mathit{arcsec}\left(\sec\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$

Problem 6.2 $\mathit{arcsec}\left(\sec\left(\frac{4\pi}{3}\right)\right)$

$\mathit{arcsec}\left(\sec\left(\frac{4\pi}{3}\right)\right) = \frac{2\pi}{3}$

Problem 6.3 $\mathit{arcsec}\left(\sec\left(\frac{5\pi}{6}\right)\right)$

$\mathit{arcsec}\left(\sec\left(\frac{5\pi}{6}\right)\right) = \frac{5\pi}{6}$

Problem 6.4 *arcsec* $\left(\sec\left(-\frac{\pi}{2}\right)\right)$

arcsec $\left(\sec\left(-\frac{\pi}{2}\right)\right)$ *is undefined.*

Problem 6.5 *arcsec* $\left(\sec\left(\frac{5\pi}{3}\right)\right)$

arcsec $\left(\sec\left(\frac{5\pi}{3}\right)\right) = \frac{\pi}{3}$

Problem 6.6 *arccsc* $\left(\csc\left(\frac{\pi}{6}\right)\right)$

arccsc $\left(\csc\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

Problem 6.7 *arccsc* $\left(\csc\left(\frac{5\pi}{4}\right)\right)$

arccsc $\left(\csc\left(\frac{5\pi}{4}\right)\right) = -\frac{\pi}{4}$

Problem 6.8 *arccsc* $\left(\csc\left(\frac{2\pi}{3}\right)\right)$

arccsc $\left(\csc\left(\frac{2\pi}{3}\right)\right) = \frac{\pi}{3}$

Problem 6.9 *arccsc* $\left(\csc\left(-\frac{\pi}{2}\right)\right)$

arccsc $\left(\csc\left(-\frac{\pi}{2}\right)\right) = -\frac{\pi}{2}$

Problem 6.10 *arccsc* $\left(\csc\left(\frac{11\pi}{6}\right)\right)$

arccsc $\left(\csc\left(\frac{11\pi}{6}\right)\right) = -\frac{\pi}{6}$

Problem 6.11 *arcsec* $\left(\sec\left(\frac{11\pi}{12}\right)\right)$

arcsec $\left(\sec\left(\frac{11\pi}{12}\right)\right) = \frac{11\pi}{12}$

Problem 6.12 *arccsc* $\left(\csc\left(\frac{9\pi}{8}\right)\right)$

arccsc $\left(\csc\left(\frac{9\pi}{8}\right)\right) = -\frac{\pi}{8}$

Question 7 *In Exercises* **??** *-* **??***, assume that the range of arcsecant is* $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$ *and that the range of arccosecant is* $\left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$ *when finding the exact value. (See Section* **??***.)*

EXERCISES FOR THE INVERSE TRIGONOMETRIC FUNCTIONS

Problem 7.1 $arcsec\left(\sec\left(\frac{\pi}{4}\right)\right)$

$arcsec\left(\sec\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$

Problem 7.2 $arcsec\left(\sec\left(\frac{4\pi}{3}\right)\right)$

$arcsec\left(\sec\left(\frac{4\pi}{3}\right)\right) = \frac{4\pi}{3}$

Problem 7.3 $arcsec\left(\sec\left(\frac{5\pi}{6}\right)\right)$

$arcsec\left(\sec\left(\frac{5\pi}{6}\right)\right) = \frac{7\pi}{6}$

Problem 7.4 $arcsec\left(\sec\left(-\frac{\pi}{2}\right)\right)$

$arcsec\left(\sec\left(-\frac{\pi}{2}\right)\right)$ *is undefined.*

Problem 7.5 $arcsec\left(\sec\left(\frac{5\pi}{3}\right)\right)$

$arcsec\left(\sec\left(\frac{5\pi}{3}\right)\right) = \frac{\pi}{3}$

Problem 7.6 $arccsc\left(\csc\left(\frac{\pi}{6}\right)\right)$

$arccsc\left(\csc\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

Problem 7.7 $arccsc\left(\csc\left(\frac{5\pi}{4}\right)\right)$

$arccsc\left(\csc\left(\frac{5\pi}{4}\right)\right) = \frac{5\pi}{4}$

Problem 7.8 $arccsc\left(\csc\left(\frac{2\pi}{3}\right)\right)$

$arccsc\left(\csc\left(\frac{2\pi}{3}\right)\right) = \frac{\pi}{3}$

Problem 7.9 $arccsc\left(\csc\left(-\frac{\pi}{2}\right)\right)$

$arccsc\left(\csc\left(-\frac{\pi}{2}\right)\right) = \frac{3\pi}{2}$

Problem 7.10 $arccsc\left(\csc\left(\frac{11\pi}{6}\right)\right)$

$arccsc\left(\csc\left(\frac{11\pi}{6}\right)\right) = \frac{7\pi}{6}$

Problem 7.11 $arcsec\left(\sec\left(\frac{11\pi}{12}\right)\right)$

$arcsec\left(\sec\left(\frac{11\pi}{12}\right)\right) = \frac{13\pi}{12}$

Problem 7.12 $arccsc\left(\csc\left(\frac{9\pi}{8}\right)\right)$

$arccsc\left(\csc\left(\frac{9\pi}{8}\right)\right) = \frac{9\pi}{8}$

Question 8 *In Exercises* **??** - **??**, *find the exact value or state that it is undefined.*

Problem 8.1 $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$

$\sin\left(\arccos\left(-\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{2}$

Problem 8.2 $\sin\left(\arccos\left(\frac{3}{5}\right)\right)$

$\sin\left(\arccos\left(\frac{3}{5}\right)\right) = \frac{4}{5}$

Problem 8.3 $\sin(\arctan(-2))$

$\sin(\arctan(-2)) = -\frac{2\sqrt{5}}{5}$

Problem 8.4 $\sin\left(arccot\left(\sqrt{5}\right)\right)$

$\sin\left(arccot\left(\sqrt{5}\right)\right) = \frac{\sqrt{6}}{6}$

Problem 8.5 $\sin(arccsc(-3))$

$\sin(arccsc(-3)) = -\frac{1}{3}$

Problem 8.6 $\cos\left(\arcsin\left(-\frac{5}{13}\right)\right)$

$\cos\left(\arcsin\left(-\frac{5}{13}\right)\right) = \frac{12}{13}$

Problem 8.7 $\cos\left(\arctan\left(\sqrt{7}\right)\right)$

$\cos\left(\arctan\left(\sqrt{7}\right)\right) = \frac{\sqrt{2}}{4}$

Problem 8.8 $\cos(arccot(3))$

$\cos(arccot(3)) = \frac{3\sqrt{10}}{10}$

Problem 8.9 $\cos\left(arcsec\left(5\right)\right)$

$\cos\left(arcsec\left(5\right)\right) = \frac{1}{5}$

Problem 8.10 $\tan\left(\arcsin\left(-\frac{2\sqrt{5}}{5}\right)\right)$

$\tan\left(\arcsin\left(-\frac{2\sqrt{5}}{5}\right)\right) = -2$

Problem 8.11 $\tan\left(\arccos\left(-\frac{1}{2}\right)\right)$

$\tan\left(\arccos\left(-\frac{1}{2}\right)\right) = -\sqrt{3}$

Problem 8.12 $\tan\left(arcsec\left(\frac{5}{3}\right)\right)$

$\tan\left(arcsec\left(\frac{5}{3}\right)\right) = \frac{4}{3}$

Problem 8.13 $\tan\left(arccot\left(12\right)\right)$

$\tan\left(arccot\left(12\right)\right) = \frac{1}{12}$

Problem 8.14 $\cot\left(\arcsin\left(\frac{12}{13}\right)\right)$

$\cot\left(\arcsin\left(\frac{12}{13}\right)\right) = \frac{5}{12}$

Problem 8.15 $\cot\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right)$

$\cot\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right) = \sqrt{3}$

Problem 8.16 $\cot\left(arccsc\left(\sqrt{5}\right)\right)$

$\cot\left(arccsc\left(\sqrt{5}\right)\right) = 2$

Problem 8.17 $\cot\left(\arctan\left(0.25\right)\right)$

$\cot\left(\arctan\left(0.25\right)\right) = 4$

Problem 8.18 $\sec\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right)$

$\sec\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right) = \frac{2\sqrt{3}}{3}$

Problem 8.19 $\sec\left(\arcsin\left(-\frac{12}{13}\right)\right)$

$\sec\left(\arcsin\left(-\frac{12}{13}\right)\right) = \frac{13}{5}$

Problem 8.20 $\sec(\arctan(10))$

$\sec(\arctan(10)) = \sqrt{101}$

Problem 8.21 $\sec\left(\mathit{arccot}\left(-\frac{\sqrt{10}}{10}\right)\right)$

$\sec\left(\mathit{arccot}\left(-\frac{\sqrt{10}}{10}\right)\right) = -\sqrt{11}$

Problem 8.22 $\csc(\mathit{arccot}(9))$

$\csc(\mathit{arccot}(9)) = \sqrt{82}$

Problem 8.23 $\csc\left(\arcsin\left(\frac{3}{5}\right)\right)$

$\csc\left(\arcsin\left(\frac{3}{5}\right)\right) = \frac{5}{3}$

Problem 8.24 $\csc\left(\arctan\left(-\frac{2}{3}\right)\right)$

$\csc\left(\arctan\left(-\frac{2}{3}\right)\right) = -\frac{\sqrt{13}}{2}$

Question 9 *In Exercises* **??** - **??**, *find the exact value or state that it is undefined.*

Problem 9.1 $\sin\left(\arcsin\left(\frac{5}{13}\right) + \frac{\pi}{4}\right)$

$\sin\left(\arcsin\left(\frac{5}{13}\right) + \frac{\pi}{4}\right) = \frac{17\sqrt{2}}{26}$

Problem 9.2 $\cos(\mathit{arcsec}(3) + \arctan(2))$

$\cos(\mathit{arcsec}(3) + \arctan(2)) = \frac{\sqrt{5} - 4\sqrt{10}}{15}$

Problem 9.3 $\tan\left(\arctan(3) + \arccos\left(-\frac{3}{5}\right)\right)$

$\tan\left(\arctan(3) + \arccos\left(-\frac{3}{5}\right)\right) = \frac{1}{3}$

Problem 9.4 $\sin\left(2\arcsin\left(-\frac{4}{5}\right)\right)$

$\sin\left(2\arcsin\left(-\frac{4}{5}\right)\right) = -\frac{24}{25}$

Problem 9.5 $\sin\left(2\mathit{arccsc}\left(\frac{13}{5}\right)\right)$

$\sin\left(2\mathit{arccsc}\left(\frac{13}{5}\right)\right) = \frac{120}{169}$

Problem 9.6 $\sin(2\arctan(2))$

$$\sin(2\arctan(2)) = \frac{4}{5}$$

Problem 9.7 $\cos\left(2\arcsin\left(\frac{3}{5}\right)\right)$

$$\cos\left(2\arcsin\left(\frac{3}{5}\right)\right) = \frac{7}{25}$$

Problem 9.8 $\cos\left(2arcsec\left(\frac{25}{7}\right)\right)$

$$\cos\left(2arcsec\left(\frac{25}{7}\right)\right) = -\frac{527}{625}$$

Problem 9.9 $\cos\left(2arccot\left(-\sqrt{5}\right)\right)$

$$\cos\left(2arccot\left(-\sqrt{5}\right)\right) = \frac{2}{3}$$

Problem 9.10 $\sin\left(\frac{\arctan(2)}{2}\right)$

$$\sin\left(\frac{\arctan(2)}{2}\right) = \sqrt{\frac{5-\sqrt{5}}{10}}$$

Question 10 *In Exercises* **??** *-* **??**, *rewrite each of the following composite functions as algebraic functions of x and state the domain.*

Problem 10.1 $f(x) = \sin(\arccos(x))$

$f(x) = \sin(\arccos(x)) = \sqrt{1-x^2}$ *for* $-1 \leq x \leq 1$

Problem 10.2 $f(x) = \cos(\arctan(x))$

$f(x) = \cos(\arctan(x)) = \frac{1}{\sqrt{1+x^2}}$ *for all* x

Problem 10.3 $f(x) = \tan(\arcsin(x))$

$f(x) = \tan(\arcsin(x)) = \frac{x}{\sqrt{1-x^2}}$ *for* $-1 < x < 1$

Problem 10.4 $f(x) = \sec(\arctan(x))$

$f(x) = \sec(\arctan(x)) = \sqrt{1+x^2}$ *for all* x

Problem 10.5 $f(x) = \csc(\arccos(x))$

$f(x) = \csc(\arccos(x)) = \frac{1}{\sqrt{1-x^2}}$ *for* $-1 < x < 1$

Problem 10.6 $f(x) = \sin(2\arctan(x))$

$f(x) = \sin(2\arctan(x)) = \frac{2x}{x^2+1}$ *for all* x

Problem 10.7 $f(x) = \sin(2\arccos(x))$

$f(x) = \sin(2\arccos(x)) = 2x\sqrt{1-x^2}$ *for* $-1 \leq x \leq 1$

Problem 10.8 $f(x) = \cos(2\arctan(x))$

$f(x) = \cos(2\arctan(x)) = \dfrac{1-x^2}{1+x^2}$ *for all* x

Problem 10.9 $f(x) = \sin(\arccos(2x))$

$f(x) = \sin(\arccos(2x)) = \sqrt{1-4x^2}$ *for* $-\dfrac{1}{2} \leq x \leq \dfrac{1}{2}$

Problem 10.10 $f(x) = \sin\left(\arccos\left(\dfrac{x}{5}\right)\right)$

$f(x) = \sin\left(\arccos\left(\dfrac{x}{5}\right)\right) = \dfrac{\sqrt{25-x^2}}{5}$ *for* $-5 \leq x \leq 5$

Problem 10.11 $f(x) = \cos\left(\arcsin\left(\dfrac{x}{2}\right)\right)$

$f(x) = \cos\left(\arcsin\left(\dfrac{x}{2}\right)\right) = \dfrac{\sqrt{4-x^2}}{2}$ *for* $-2 \leq x \leq 2$

Problem 10.12 $f(x) = \cos(\arctan(3x))$

$f(x) = \cos(\arctan(3x)) = \dfrac{1}{\sqrt{1+9x^2}}$ *for all* x

Problem 10.13 $f(x) = \sin(2\arcsin(7x))$

$f(x) = \sin(2\arcsin(7x)) = 14x\sqrt{1-49x^2}$ *for* $-\dfrac{1}{7} \leq x \leq \dfrac{1}{7}$

Problem 10.14 $f(x) = \sin\left(2\arcsin\left(\dfrac{x\sqrt{3}}{3}\right)\right)$

$f(x) = \sin\left(2\arcsin\left(\dfrac{x\sqrt{3}}{3}\right)\right) = \dfrac{2x\sqrt{3-x^2}}{3}$ *for* $-\sqrt{3} \leq x \leq \sqrt{3}$

Problem 10.15 $f(x) = \cos(2\arcsin(4x))$

$f(x) = \cos(2\arcsin(4x)) = 1 - 32x^2$ *for* $-\dfrac{1}{4} \leq x \leq \dfrac{1}{4}$

Problem 10.16 $f(x) = \sec(\arctan(2x))\tan(\arctan(2x))$

$f(x) = \sec(\arctan(2x))\tan(\arctan(2x)) = 2x\sqrt{1+4x^2}$ *for all* x

Problem 10.17 $f(x) = \sin(\arcsin(x) + \arccos(x))$

$f(x) = \sin(\arcsin(x) + \arccos(x)) = 1$ *for* $-1 \leq x \leq 1$

Problem 10.18 $f(x) = \cos(\arcsin(x) + \arctan(x))$

$f(x) = \cos(\arcsin(x) + \arctan(x)) = \dfrac{\sqrt{1-x^2} - x^2}{\sqrt{1+x^2}}$ *for* $-1 \leq x \leq 1$

Problem 10.19 $f(x) = \tan(2\arcsin(x))$

$f(x) = \tan(2\arcsin(x)) = \dfrac{2x\sqrt{1-x^2}}{1-2x^2}$ *for*[1] x *in* $\left(-1, -\dfrac{\sqrt{2}}{2}\right) \cup \left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right) \cup \left(\dfrac{\sqrt{2}}{2}, 1\right)$

Problem 10.20 $f(x) = \sin\left(\dfrac{1}{2}\arctan(x)\right)$

$$f(x) = \sin\left(\frac{1}{2}\arctan(x)\right) = \begin{cases} \sqrt{\dfrac{\sqrt{x^2+1}-1}{2\sqrt{x^2+1}}} & \text{for } x \geq 0 \\ -\sqrt{\dfrac{\sqrt{x^2+1}-1}{2\sqrt{x^2+1}}} & \text{for } x < 0 \end{cases}$$

Problem 11 *If* $\theta = \arcsin\left(\dfrac{x}{2}\right)$*, find an expression for* $\theta + \sin(2\theta)$ *in terms of* x*.*

$\theta + \sin(2\theta) = \arcsin\left(\dfrac{x}{2}\right) + \dfrac{x\sqrt{4-x^2}}{2}$

Problem 12 *If* $\theta = \arctan\left(\dfrac{x}{7}\right)$*, find an expression for* $\dfrac{1}{2}\theta - \dfrac{1}{2}\sin(2\theta)$ *in terms of* x*.*

$\dfrac{1}{2}\theta - \dfrac{1}{2}\sin(2\theta) = \dfrac{1}{2}\arctan\left(\dfrac{x}{7}\right) - \dfrac{7x}{x^2+49}$

Problem 13 *If* $\theta = arcsec\left(\dfrac{x}{4}\right)$*, find an expression for* $4\tan(\theta) - 4\theta$ *in terms of* x *assuming* $x \geq 4$*.*

$4\tan(\theta) - 4\theta = \sqrt{x^2-16} - 4arcsec\left(\dfrac{x}{4}\right)$

Question 14 *In Exercises* **??** *-* **??***, solve the equation using the techniques discussed in Example* **??** *then approximate the solutions which lie in the interval* $[0, 2\pi)$ *to four decimal places.*

Problem 14.1 $\sin(\theta) = \dfrac{7}{11}$

$\theta = \arcsin\left(\dfrac{7}{11}\right) + 2\pi k$ *or* $\theta = \pi - \arcsin\left(\dfrac{7}{11}\right) + 2\pi k$*, in* $[0, 2\pi)$*,* $\theta \approx 0.6898,\ 2.4518$

Problem 14.2 $\cos(\theta) = -\dfrac{2}{9}$

$\theta = \arccos\left(-\dfrac{2}{9}\right) + 2\pi k$ *or* $\theta = -\arccos\left(-\dfrac{2}{9}\right) + 2\pi k$*, in* $[0, 2\pi)$*,* $\theta \approx 1.7949,\ 4.4883$

Problem 14.3 $\sin(\theta) = -0.569$

$\theta = \pi + \arcsin(0.569) + 2\pi k$ *or* $\theta = 2\pi - \arcsin(0.569) + 2\pi k$*, in* $[0, 2\pi)$*,* $\theta \approx 3.7469,\ 5.6779$

Problem 14.4 $\cos(\theta) = 0.117$

$\theta = \arccos(0.117) + 2\pi k$ *or* $\theta = 2\pi - \arccos(0.117) + 2\pi k$*, in* $[0, 2\pi)$*,* $\theta \approx 1.4535,\ 4.8297$

Problem 14.5 $\sin(\theta) = 0.008$

$\theta = \arcsin(0.008) + 2\pi k$ *or* $\theta = \pi - \arcsin(0.008) + 2\pi k$*, in* $[0, 2\pi)$*,* $\theta \approx 0.0080,\ 3.1336$

[1] The equivalence for $x = \pm 1$ can be verified independently of the derivation of the formula, but Calculus is required to fully understand what is happening at those x values. You'll see what we mean when you work through the details of the identity for $\tan(2t)$. For now, we exclude $x = \pm 1$ from our answer.

Problem 14.6 $\cos(\theta) = \frac{359}{360}$

$\theta = \arccos\left(\frac{359}{360}\right) + 2\pi k$ *or* $\theta = 2\pi - \arccos\left(\frac{359}{360}\right) + 2\pi k$, *in* $[0, 2\pi)$, $\theta \approx 0.0746,\ 6.2086$

Problem 14.7 $\tan(t) = 117$

$t = \arctan(117) + \pi k$, *in* $[0, 2\pi)$, $t \approx 1.56225,\ 4.70384$ $t = \arctan\left(-\frac{1}{12}\right) + \pi k$, *in* $[0, 2\pi)$, $t \approx 3.0585,\ 6.2000$

Problem 14.8 $\cot(t) = -12$

$t = \arctan\left(-\frac{1}{12}\right) + \pi k$, *in* $[0, 2\pi)$, $t \approx 3.0585,\ 6.2000$

Problem 14.9 $\sec(t) = \frac{3}{2}$

$t = \arccos\left(\frac{2}{3}\right) + 2\pi k$ *or* $t = 2\pi - \arccos\left(\frac{2}{3}\right) + 2\pi k$, *in* $[0, 2\pi)$, $t \approx 0.8411,\ 5.4422$

Problem 14.10 $\csc(t) = -\frac{90}{17}$

$t = \pi + \arcsin\left(\frac{17}{90}\right) + 2\pi k$ *or* $t = 2\pi - \arcsin\left(\frac{17}{90}\right) + 2\pi k$, *in* $[0, 2\pi)$, $t \approx 3.3316,\ 6.0932$

Problem 14.11 $\tan(t) = -\sqrt{10}$

$t = \arctan\left(-\sqrt{10}\right) + \pi k$, *in* $[0, 2\pi)$, $t \approx 1.8771,\ 5.0187$

Problem 14.12 $\sin(t) = \frac{3}{8}$

$t = \arcsin\left(\frac{3}{8}\right) + 2\pi k$ *or* $t = \pi - \arcsin\left(\frac{3}{8}\right) + 2\pi k$, *in* $[0, 2\pi)$, $t \approx 0.3844,\ 2.7572$

Problem 14.13 $\cos(x) = -\frac{7}{16}$

$x = \arccos\left(-\frac{7}{16}\right) + 2\pi k$ *or* $x = -\arccos\left(-\frac{7}{16}\right) + 2\pi k$, *in* $[0, 2\pi)$, $x \approx 2.0236,\ 4.2596$

Problem 14.14 $\tan(x) = 0.03$

$x = \arctan(0.03) + \pi k$, *in* $[0, 2\pi)$, $x \approx 0.0300,\ 3.1716$

Problem 14.15 $\sin(x) = 0.3502$

$x = \arcsin(0.3502) + 2\pi k$ *or* $x = \pi - \arcsin(0.3502) + 2\pi k$, *in* $[0, 2\pi)$, $x \approx 0.3578,\ 2.784$

Problem 14.16 $\sin(x) = -0.721$

$x = \pi + \arcsin(0.721) + 2\pi k$ *or* $x = 2\pi - \arcsin(0.721) + 2\pi k$, *in* $[0, 2\pi)$, $x \approx 3.9468,\ 5.4780$

Problem 14.17 $\cos(x) = 0.9824$

$x = \arccos(0.9824) + 2\pi k$ *or* $x = 2\pi - \arccos(0.9824) + 2\pi k$, *in* $[0, 2\pi)$, $x \approx 0.1879,\ 6.0953$

Problem 14.18 $\cos(x) = -0.5637$

$x = \arccos(-0.5637) + 2\pi k$ *or* $x = -\arccos(-0.5637) + 2\pi k$, *in* $[0, 2\pi)$, $x \approx 2.1697, 4.1135$

Problem 14.19 $\cot(x) = \dfrac{1}{117}$

$x = \arctan(117) + \pi k$, *in* $[0, 2\pi)$, $x \approx 1.5622, 4.7038$

Problem 14.20 $\tan(x) = -0.6109$

$x = \arctan(-0.6109) + \pi k$, *in* $[0, 2\pi)$, $x \approx 2.5932, 5.7348$

Question 15 *In Exercises* **??** - **??**, *rewrite the given function as a sinusoid of the form* $C(t) = A\cos(\omega t + \phi)$ *and* $S(t) = A\sin(\omega t + \phi)$ *(See Example* **??***.) Approximate the value of* ϕ *(which is in radians, of course) to four decimal places.*

Problem 15.1 $f(t) = 5\sin(3t) + 12\cos(3t)$

$$f(t) = 5\sin(3t) + 12\cos(3t) = 13\sin\left(3t + \arcsin\left(\frac{12}{13}\right)\right) \approx 13\sin(3t + 1.1760)$$

$$f(t) = 5\sin(3t) + 12\cos(3t) = 13\cos\left(3t + \arcsin\left(-\frac{5}{13}\right)\right) \approx 13\cos(3t - 0.3948)$$

Problem 15.2 $f(t) = 3\cos(2t) + 4\sin(2t)$

$$f(t) = 3\cos(2t) + 4\sin(2t) = 5\sin\left(2t + \arcsin\left(\frac{3}{5}\right)\right) \approx 5\sin(2t + 0.6435)$$

$$f(t) = 3\cos(2t) + 4\sin(2t) = 5\cos\left(2t + \arcsin\left(-\frac{4}{5}\right)\right) \approx 5\cos(2t - 0.9273)$$

Problem 15.3 $f(t) = \cos(t) - 3\sin(t)$

$$f(t) = \cos(t) - 3\sin(t) = \sqrt{10}\sin\left(t + \arccos\left(-\frac{3\sqrt{10}}{10}\right)\right) \approx \sqrt{10}\sin(t + 2.8198)$$

$$f(t) = \cos(t) - 3\sin(t) = \sqrt{10}\cos\left(t + \arcsin\left(\frac{3\sqrt{10}}{10}\right)\right) \approx \sqrt{10}\cos(t + 1.2490)$$

Problem 15.4 $f(t) = 7\sin(10t) - 24\cos(10t)$

$$f(t) = 7\sin(10t) - 24\cos(10t) = 25\sin\left(10t + \arcsin\left(-\frac{24}{25}\right)\right) \approx 25\sin(10t - 1.2870)$$

$$f(t) = 7\sin(10t) - 24\cos(10t) = 25\cos\left(10t + \pi + \arcsin\left(\frac{7}{25}\right)\right) \approx 25\cos(10t + 3.4254)$$

Problem 15.5 $f(t) = -\cos(t) - 2\sqrt{2}\sin(t)$

$$f(t) = -\cos(t) - 2\sqrt{2}\sin(t) = 3\sin\left(t + \pi + \arcsin\left(\frac{1}{3}\right)\right) \approx 3\sin(t + 3.4814)$$

$$f(t) = -\cos(t) - 2\sqrt{2}\sin(t) = 3\cos\left(t + \arccos\left(-\frac{1}{3}\right)\right) \approx 3\sin(t + 1.9106)$$

Problem 15.6 $f(t) = 2\sin(t) - \cos(t)$

$$f(t) = 2\sin(t) - \cos(t) = \sqrt{5}\sin\left(t + \arcsin\left(-\frac{\sqrt{5}}{5}\right)\right) \approx \sqrt{5}\sin(t - 0.4636)$$

$f(t) = 2\sin(t) - \cos(t) = \sqrt{5}\cos\left(t + \pi + \arcsin\left(\frac{2\sqrt{5}}{5}\right)\right) \approx \sqrt{5}\cos(t + 4.2487)$

Question 16 *In Exercises* **??** *-* **??***, find the domain of the given function. Write your answers in interval notation.*

Problem 16.1 $f(x) = \arcsin(5x)$

$\left[-\frac{1}{5}, \frac{1}{5}\right]$

Problem 16.2 $f(x) = \arccos\left(\frac{3x-1}{2}\right)$

$\left[-\frac{1}{3}, 1\right]$

Problem 16.3 $f(x) = \arcsin\left(2x^2\right)$

$\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

Problem 16.4 $f(x) = \arccos\left(\frac{1}{x^2-4}\right)$

$(-\infty, -\sqrt{5}] \cup [-\sqrt{3}, \sqrt{3}] \cup [\sqrt{5}, \infty)$

Problem 16.5 $f(x) = \arctan(4x)$

$(-\infty, \infty)$

Problem 16.6 $f(x) = \mathit{arccot}\left(\frac{2x}{x^2-9}\right)$

$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Problem 16.7 $f(x) = \arctan(\ln(2x-1))$

$\left(\frac{1}{2}, \infty\right)$

Problem 16.8 $f(x) = \mathit{arccot}(\sqrt{2x-1})$

$\left[\frac{1}{2}, \infty\right)$

Problem 16.9 $f(x) = \mathit{arcsec}(12x)$

$\left(-\infty, -\frac{1}{12}\right] \cup \left[\frac{1}{12}, \infty\right)$

Problem 16.10 $f(x) = \mathit{arccsc}(x+5)$

$(-\infty, -6] \cup [-4, \infty)$

Problem 16.11 $f(x) = \mathit{arcsec}\left(\frac{x^3}{8}\right)$

$(-\infty, -2] \cup [2, \infty)$

Problem 16.12 $f(x) = \operatorname{arccsc}\left(e^{2x}\right)$

$[0, \infty)$

Problem 17 *Find the following limits.*

1. $\lim_{x \to 1^-} \arcsin(x)$

   $\lim_{x \to 1^-} \arcsin(x) = \frac{\pi}{2}$

2. $\lim_{x \to -\infty} \arctan(3x)$

   $\lim_{x \to -\infty} \arctan(3x) = -\infty$

3. $\lim_{x \to 1} \mathit{arcsec}(2x)$

   $\lim_{x \to 1} \mathit{arcsec}(2x) = \frac{\pi}{3}$

Problem 18 *Find a nonzero number x where* $\mathit{arccot}(x) \neq \arctan\left(\frac{1}{x}\right)$.

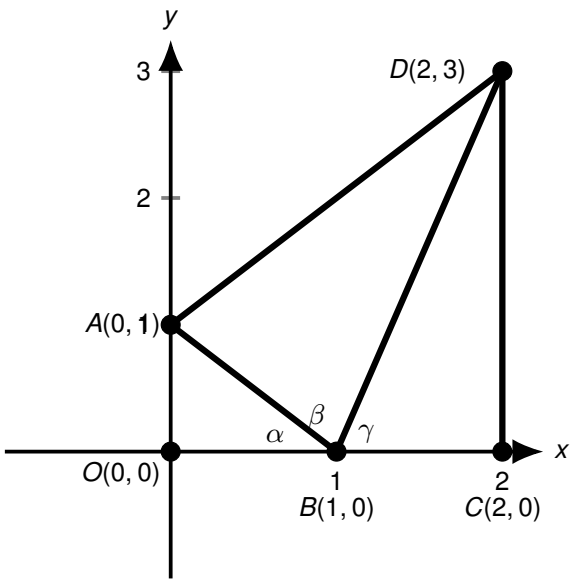
Problem 19 *Find an example where* $\mathit{arcsec}(x) \neq \arccos\left(\frac{1}{x}\right)$ *if we use* $\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$ *as the range of* $f(x) = \mathit{arcsec}(x)$.

Problem 20 *Show that* $\arcsin(x) + \arccos(x) = \frac{\pi}{2}$ *for* $-1 \leq x \leq 1$.

Problem 21 *Discuss with your classmates why* $\arcsin\left(\frac{1}{2}\right) \neq 30$.

Problem 22 *Use the diagram below along with the accompanying questions to show:*

$$\arctan(1) + \arctan(2) + \arctan(3) = \pi$$



EXERCISES FOR THE INVERSE TRIGONOMETRIC FUNCTIONS

1. *Clearly $\triangle AOB$ and $\triangle BCD$ are right triangles because the line through O and A and the line through C and D are perpendicular to the x-axis. Use the distance formula to show that $\triangle BAD$ is also a right triangle (with $\angle BAD$ being the right angle) by showing that the sides of the triangle satisfy the Pythagorean Theorem.*
2. *Use $\triangle AOB$ to show that α* = arctan(1)
3. *Use $\triangle BAD$ to show that β* = arctan(2)
4. *Use $\triangle BCD$ to show that γ* = arctan(3)
5. *Use the fact that O, B and C all lie on the x-axis to conclude that $\alpha + \beta + \gamma = \pi$. Thus* arctan(1) + arctan(2) + arctan(3) = π.