Exercises for Introduction to Derivatives

Question 1 *In Exercises **??** - **??**, find the limit of the following difference quotients.*

1. $\lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$
2. $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Problem 1.1 $f(x) = 2x - 5$

1. $\lim_{h \to 0} 2 = 2$
2. $\lim_{h \to 0} 2 = 2$

Problem 1.2 $f(x) = -3x + 5$

1. $\lim_{h \to 0} -3 = -3$
2. $\lim_{h \to 0} -3 = -3$

Problem 1.3 $f(x) = 6$

1. $\lim_{h \to 0} 0 = 0$
2. $\lim_{h \to 0} 0 = 0$

Problem 1.4 $f(x) = 3x^2 - x$

1. $\lim_{h \to 0} (3h + 11) = 11$
2. $\lim_{h \to 0} (6x + 3h - 1) = 6x - 1$

Problem 1.5 $f(x) = -x^2 + 2x - 1$

1. $\lim_{h \to 0} (-h - 2) = -2$
2. $\lim_{h \to 0} (-2x - h + 2) = -2x + 2$

Problem 1.6 $f(x) = 4x^2$

1. $\lim_{h \to 0} (4h + 16) = 16$
2. $\lim_{h \to 0} (8x + 4h) = 8x$

Question 2 *In Exercises **??** - **??**, find:*

1. $f'(2) = \lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$
2. *The equation of the tangent line at* $(2, f(2))$. *Check your answer graphically.*
3. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

4. *The equation of the tangent line at* $(0, f(0))$*. Check your answer graphically.*

Problem 2.1 $f(x) = x - x^2$

1. $f'(2) = \lim_{h\to 0}(-h-3) = -3$
2. $y = f'(2)(x-2) + f(2) = (-3)(x-2) + (-2)$ *so* $y = -3x + 4$.
3. $f'(x) = \lim_{h\to 0}(-2x - h + 1) = -2x + 1$
4. $y = f'(0)(x-0) + f(0) = (1)(x-0) + 0$ *so* $y = x$.

Problem 2.2 $f(x) = x^3 + 1$

1. $f'(2) = \lim_{h\to 0}\left(h^2 + 6h + 12\right) = 12$
2. $y = f'(2)(x-2) + f(2) = 12(x-2) + 9$ *so* $y = 12x - 15$.
3. $f'(x) = \lim_{h\to 0}\left(3x^2 + 3xh + h^2\right) = 3x^2$
4. $y = f'(0)(x-0) + f(0) = 0(x-0) + 1$ *so* $y = 1$.

Problem 3 *Find* $f'(x) = \lim_{h\to 0}\dfrac{f(x+h) - f(x)}{h}$ *for* $f(x) = mx + b$ *where* $m \neq 0$

$f'(x) = \lim_{h\to 0} m = m$.

Problem 4 *For* $f(x) = ax^2 + bx + c$ *where* $a \neq 0$*:*

1. *find* $f'(x) = \lim_{h\to 0}\dfrac{f(x+h) - f(x)}{h}$.
2. *solve* $f'(x) = 0$ *for* x*. Does this look familiar? Explain.*

1. $f'(x) = \lim_{h\to 0}(2ax + ah + b) = 2ax + b$.
2. *Solving* $f'(x) = 2ax + b = 0$ *for* x *gives* $x = -\dfrac{b}{2a}$ *which is the formula for the* x*-coordinate of the vertex of the parabola* $y = f(x)$*. If we zoom in near the the vertex of a parabola, the graph becomes locally flat so it makes sense the slope of the tangent line,* $f'(x) = 0$ *there.*

Question 5 *In Exercises* **??** - **??**, *find the limit of the following difference quotients:*

1. $\lim_{\Delta x\to 0}\dfrac{f(-1+\Delta x) - f(-1)}{\Delta x}$
2. $\lim_{\Delta x\to 0}\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

Problem 5.1 $f(x) = \dfrac{2}{x}$

1. $\lim_{\Delta x\to 0}\dfrac{2}{\Delta x - 1} = -2$

2. $\lim\limits_{\Delta x \to 0} \dfrac{-2}{x(x+\Delta x)} = -\dfrac{2}{x^2}$

Problem 5.2 $f(x) = \dfrac{3}{1-x}$

1. $\lim\limits_{\Delta x \to 0} \dfrac{-3}{2(\Delta x - 2)} = \dfrac{3}{4}$
2. $\lim\limits_{\Delta x \to 0} \dfrac{3}{(x+\Delta x - 1)(x-1)} = \dfrac{3}{(x-1)^2}$

Problem 5.3 $f(x) = \dfrac{1}{x^2}$

1. $\lim\limits_{\Delta x \to 0} \dfrac{2-\Delta x}{(\Delta x - 1)^2} = 2$
2. $\lim\limits_{\Delta x \to 0} \dfrac{-(2x+\Delta x)}{x^2(x+\Delta x)^2} = -\dfrac{2}{x^3}$

Problem 5.4 $f(x) = \dfrac{2}{x+5}$

1. $\lim\limits_{\Delta x \to 0} \dfrac{-1}{2(\Delta x + 4)} = -\dfrac{1}{8}$
2. $\lim\limits_{\Delta x \to 0} \dfrac{-2}{(x+5)(x+\Delta x+5)} = -\dfrac{2}{(x+5)^2}$

Question 6 *In Exercises* **??** - **??***, find the limit of the following:*

1. $f'(-1) = \lim\limits_{\Delta x \to 0} \dfrac{f(-1+\Delta x) - f(-1)}{\Delta x}$
2. *The equation of the tangent line at* $(-1, f(-1))$.
3. $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$
4. *The equation of the tangent line at* $(0, f(0))$.

Problem 6.1 $f(x) = \dfrac{1}{4x-3}$

1. $f'(-1) = \lim\limits_{\Delta x \to 0} \dfrac{4}{7(4\Delta x - 7)} = -\dfrac{4}{49}$
2. $y = f'(-1)(x-(-1)) + f(-1) = -\dfrac{4}{49}(x+1) + \left(-\dfrac{1}{7}\right)$ *so* $y = -\dfrac{4}{49}x - \dfrac{11}{49}$.
3. $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{-4}{(4x-3)(4x+4\Delta x-3)} = -\dfrac{4}{(4x-3)^2}$
4. $y = f'(0)(x-0) + f(0) = -\dfrac{4}{9}(x-0) + \left(-\dfrac{1}{3}\right)$ *so* $y = -\dfrac{4}{9}x - \dfrac{1}{3}$

Problem 6.2 $f(x) = \dfrac{3x}{x+2}$

1. $f'(-1) = \lim_{\Delta x \to 0} \dfrac{6}{\Delta x + 1} = 6$
2. $y = f'(-1)(x - (-1)) + f(-1) = 6(x + 1) + (-3)$ *so* $y = 6x + 3$.
3. $f'(x) = \lim_{\Delta x \to 0} \dfrac{6}{(x + 2)(x + \Delta x + 2)} = \dfrac{6}{(x + 2)^2}$
4. $y = f'(0)(x - 0) + f(0) = \dfrac{3}{2}(x - 0) + 0$ *so* $y = \dfrac{3}{2}x$

Problem 6.3 $f(x) = \dfrac{x}{x - 9}$

1. $f'(-1) = \lim_{\Delta x \to 0} \dfrac{9}{10(\Delta x - 10)} = -\dfrac{9}{100}$
2. $y = f'(-1)(x - (-1)) + f(-1) = -\dfrac{9}{100}(x + 1) + \dfrac{1}{10}$ *so* $y = -\dfrac{9}{100}x + \dfrac{1}{100}$.
3. $f'(x) = \lim_{\Delta x \to 0} \dfrac{-9}{(x - 9)(x + \Delta x - 9)} = -\dfrac{9}{(x - 9)^2}$
4. $y = f'(0)(x - 0) + f(0) = -\dfrac{1}{9}(x - 0) + 0$ *so* $y = -\dfrac{1}{9}x$

Problem 6.4 $f(x) = \dfrac{x^2}{2x + 1}$

1. $f'(-1) = \lim_{\Delta x \to 0} \dfrac{\Delta x}{2\Delta x - 1} = 0$
2. $y = f'(-1)(x - (-1)) + f(-1) = (0)(x + 1) + (-1)$ *so* $y = -1$.
3. $f'(x) = \lim_{\Delta x \to 0} \dfrac{2x^2 + 2x\Delta x + 2x + \Delta x}{(2x + 1)(2x + 2\Delta x + 1)} = \dfrac{2x^2 + 2x}{(2x + 1)^2}$
4. $y = f'(0)(x - 0) + f(0) = (0)(x - 0) + 0$ *so* $y = 0$

Question 7 *In Exercises* **??** - **??**, *find the limit of the following difference quotients:*

1. $\lim_{\Delta t \to 0} \dfrac{g(\Delta t) - g(0)}{\Delta t}$
2. $\lim_{\Delta t \to 0} \dfrac{g(t + \Delta t) - g(t)}{\Delta t}$

Problem 7.1 $g(t) = \sqrt{9 - t}$

1. $\lim_{\Delta t \to 0} \dfrac{-1}{\sqrt{9 - \Delta t} + 3} = -\dfrac{1}{6}$
2. $\lim_{\Delta t \to 0} \dfrac{-1}{\sqrt{9 - t - \Delta t} + \sqrt{9 - t}} = -\dfrac{1}{2\sqrt{9 - t}}$

Problem 7.2 $g(t) = \sqrt{2t + 1}$

1. $\lim_{\Delta t \to 0} \dfrac{2}{\sqrt{2\Delta t + 1} + 1} = 1$

2. $\lim\limits_{\Delta t \to 0} \dfrac{2}{\sqrt{2t+2\Delta t+1}+\sqrt{2t+1}} = \dfrac{2}{2\sqrt{2t+1}}$

Question 8 *In Exercises* **??** *-* **??***, find the following:*

1. $g'(0) = \lim\limits_{\Delta t \to 0} \dfrac{g(\Delta t)-g(0)}{\Delta t}$
2. *The equation of the tangent line at* $(0, g(0))$.
3. $g'(t) = \lim\limits_{\Delta t \to 0} \dfrac{g(t+\Delta t)-g(t)}{\Delta t}$
4. *The equation of the tangent line at* $(1, g(1))$.

Problem 8.1 $g(t) = \sqrt{-4t+5}$

1. $g'(0) = \lim\limits_{\Delta t \to 0} \dfrac{-4}{\sqrt{5-4\Delta t}+\sqrt{5}} = -\dfrac{2}{\sqrt{5}}$
2. $y = g'(0)(x-0)+g(0) = -\dfrac{2}{\sqrt{5}}(x-0)+\sqrt{5}$ *so* $y = -\dfrac{2}{\sqrt{5}}x+\sqrt{5}$.
3. $g'(t) = \lim\limits_{\Delta t \to 0} \dfrac{-4}{\sqrt{-4t-4\Delta t+5}+\sqrt{-4t+5}} = -\dfrac{2}{\sqrt{-4t+5}}$
4. $y = g'(1)(x-1)+g(1) = (-2)(x-1)+1$ *so* $y = -2x+3$

Problem 8.2 $g(t) = \sqrt{4-t}$

1. $g'(0) = \lim\limits_{\Delta t \to 0} \dfrac{-1}{\sqrt{4-\Delta t}+2} = -\dfrac{1}{4}$
2. $y = g'(0)(x-0)+g(0) = -\dfrac{1}{4}(x-0)+2$ *so* $y = -\dfrac{1}{4}x+2$.
3. $g'(t) = \lim\limits_{\Delta t \to 0} \dfrac{-1}{\sqrt{4-t-\Delta t}+\sqrt{4-t}} = -\dfrac{1}{2\sqrt{4-t}}$
4. $y = g'(1)(x-1)+g(1) = -\dfrac{1}{2\sqrt{3}}(x-1)+\sqrt{3}$ *so* $y = -\dfrac{1}{2\sqrt{3}}x+\dfrac{7\sqrt{3}}{6}$.

Problem 9 *For* $g(t) = t\sqrt{t}$*:*

1. *Explain why* $g'(0) = \lim\limits_{\Delta t \to 0} \dfrac{g(\Delta t)-g(0)}{\Delta t}$ *does not exist.*

   If $\Delta t < 0$*, then* $\sqrt{\Delta t}$ *is not a real number. Hence,* $g'(0) = \lim\limits_{\Delta t \to 0} \dfrac{g(\Delta t)-g(0)}{\Delta t}$ *does not exist.*
2. *Find the derivative* ***from the right*** *at* $t=0$*:* $g'_{+}(0) = \lim\limits_{\Delta t \to 0^{+}} \dfrac{g(\Delta t)-g(0)}{\Delta t}$

   $g'_{+}(0) = \lim\limits_{\Delta t \to 0^{+}} (\Delta t)^{\frac{1}{2}} = 0$
3. *Find* $y = g'_{+}(0)(x-0)+g(0)$ *and interpret.*

   $y = g'_{+}(0)(x-0)+g(0) = (0)(x-0)+0$ *so* $y = 0$

   This line is a tangent line to the graph of $y = t\sqrt{t}$ *at* $(0,0)$ *for* $t \geq 0$.

4. *Find* $g'(t) = \lim\limits_{\Delta t \to 0} \dfrac{g(t+\Delta t) - g(t)}{\Delta t}$. *Assume* $t > 0$.

   $g'(t) = \lim\limits_{\Delta t \to 0} \dfrac{3t^2 + 3t\Delta t + (\Delta t)^2}{(t+\Delta t)^{3/2} + t^{3/2}} = \dfrac{3t^2}{2t^{3/2}} = \dfrac{3}{2}\, t^{1/2}$ *provided* $t > 0$.

Problem 10 *Let* $g(t) = \sqrt{at+b}$, $a \neq 0$.

1. *Find* $g'(t) = \lim\limits_{\Delta t \to 0} \dfrac{g(t+\Delta t) - g(t)}{\Delta t}$

   $g'(t) = \lim\limits_{\Delta t \to 0} \dfrac{a}{\sqrt{at + a\Delta t + b} + \sqrt{at+b}} = \dfrac{a}{2\sqrt{at+b}}$.

2. *What restrictions do you place on t so your formula is valid?*

   We are told $a \neq 0$. *In order for the square roots to be happy, we assume* $at + b > 0$. *Hence,* $t > -\dfrac{b}{a}$ *if* $a > 0$ *and* $t < -\dfrac{b}{a}$ *if* $a < 0$.

Problem 11 *Let* $f(x) = |x|$.

1. *Explain why f is continuous at* $x = 0$.

   f *is continuous at* $x = 0$ *since* $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} |x| = 0 = |0| = f(0)$.

2. *Show* $f'(0)$ *does not exist by showing* $\lim\limits_{h \to 0^-} \dfrac{f(h) - f(0)}{h} = -1$ *but* $\lim\limits_{h \to 0^+} \dfrac{f(h) - f(0)}{h} = 1$.

   $\lim\limits_{h \to 0^-} \dfrac{f(h) - f(0)}{h} = \lim\limits_{h \to 0^-} \dfrac{-h}{h} = -1$

   $\lim\limits_{h \to 0^+} \dfrac{f(h) - f(0)}{h} = \lim\limits_{h \to 0^+} \dfrac{h}{h} = 1$

3. *Graph* $y = f(x)$ *near* $(0,0)$. *Interpret your answer to number* **??** *graphically.*

   Near $x = 0$, *the graph of* $y = f(x)$ *looks like a '$\vee$' shape*[1] *consisting of a line of slope* -1 *to the left of* $x = 0$ *and a line with a slope of* $+1$ *to the right of* $x = 0$.

4. *Find and simplify* $f'(x) = \lim\limits_{h \to 0} \dfrac{|x+h| - |x|}{h}$ *assuming* $x \neq 0$.

   HINT: *Consider the two cases* $x > 0$ *and* $x < 0 \ldots$

   If $x < 0$, $f'(x) = lim_{h \to 0} \dfrac{|x+h| - |x|}{h} = -1$.

   If $x > 0$, $f'(x) = \lim\limits_{h \to 0} \dfrac{|x+h| - |x|}{h} = 1$.

Problem 12 *Let* $g(t) = \sqrt[3]{t}$.

1. *Explain why g is continuous at* $t = 0$.

   g is continuous at $t = 0$ *since* $\lim\limits_{t \to 0} g(t) = \lim\limits_{t \to 0} \sqrt[3]{t} = 0 = \sqrt[3]{0} = g(0)$.

2. *Show* $g'(0)$ *does not exist by showing* $\lim\limits_{\Delta t \to 0} \dfrac{g(\Delta t) - g(0)}{\Delta t} = \infty$.

   $\lim\limits_{\Delta t \to 0} \dfrac{g(\Delta t) - g(0)}{\Delta t} = \lim\limits_{\Delta t \to 0} \dfrac{1}{(\Delta t)^{\frac{2}{3}}} = \infty$

[1] We say the graph of f has a **corner** at $(0,0)$ since we have two different, but finite slopes meeting at a point.

3. *Graph $y = g(t)$ near $(0,0)$. Interpret your answer to number **??** graphically.*

   The graph of $y = g(t)$ near $(0,0)$ is a vertical line.[2] *Since g is increasing through $(0,0)$, the 'slope' of this vertical line could be seen as $+\infty$.*

4. *Find and simplify $g'(t) = \lim\limits_{\Delta t \to 0} \dfrac{g(t+\Delta t) - g(t)}{\Delta t}$ assuming $t \neq 0$.*

   HINT: $(a-b)\left(a^2+ab+b^2\right) = a^3 - b^3$

   $$g'(t) = \lim_{\Delta t \to 0} \frac{1}{(t+\Delta t)^{\frac{2}{3}} + (t+\Delta t)^{\frac{1}{3}} t^{\frac{1}{3}} + t^{\frac{2}{3}}} = \frac{1}{3t^{\frac{2}{3}}},\ t \neq 0.$$

Problem 13 *Let $h(x) = x^{\frac{2}{3}}$.*

1. *Explain why h is continuous at $x = 0$.*

   $\lim\limits_{x\to 0} h(x) = \lim\limits_{x \to 0} x^{\frac{2}{3}} = 0 = 0^{\frac{2}{3}} = h(0)$.

2. *Show $h'(0)$ does not exist by showing $\lim\limits_{\Delta x \to 0^-} \dfrac{h(\Delta x) - h(0)}{\Delta x} = -\infty$ and $\lim\limits_{\Delta x \to 0^+} \dfrac{h(\Delta x) - h(0)}{\Delta x} = \infty$*

   $$\lim_{\Delta x \to 0^-} \frac{h(\Delta x) - h(0)}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{1}{(\Delta x)^{\frac{1}{3}}} = -\infty$$
   $$\lim_{\Delta x \to 0^+} \frac{h(\Delta x) - h(0)}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{1}{(\Delta x)^{\frac{1}{3}}} = \infty$$

3. *Graph $y = h(x)$ near $(0,0)$. Interpret your answer to number **??** graphically.*

   The graph $y = h(x)$ near $(0,0)$ shows a steeply decreasing graph as we approach $(0,0)$ from the left followed by a steeply increasing graph as we approach $(0,0)$ from the right.[3]

4. *Find and simplify $h'(x) = \lim\limits_{\Delta x \to 0} \dfrac{h(x+\Delta x) - h(x)}{\Delta x}$ assuming $x \neq 0$.*

   HINT: $(a-b)\left(a^2+ab+b^2\right) = a^3 - b^3$ *and* $a^2 - b^2 = (a+b)(a-b)$.

   $$h'(x) = \lim_{\Delta x \to 0} \frac{h(x+\Delta x) - h(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x + \Delta x}{(x+\Delta x)^{\frac{4}{3}} + (x+\Delta x)^{\frac{2}{3}} x^{\frac{2}{3}} + x^{\frac{4}{3}}} = \frac{2}{3x^{\frac{1}{3}}},\ x \neq 0.$$

Problem 14 *Recall from Exercise **??** in Section **??** that Carl's friend Jason participates in the Highland Games. In one event, the hammer throw, the height $h(t)$ in feet of the hammer above the ground t seconds after Jason lets it go is modeled by the function $h(t) = -16t^2 + 22.08t + 6$.*

1. *Find and simplify a formula for the velocity of the hammer, $v(t) = h'(t) = \lim\limits_{\Delta t \to 0} \dfrac{h(t+\Delta t) - h(t)}{\Delta t}$.*

   $$v(t) = \lim_{\Delta t \to 0} \frac{h(t+\Delta t) - h(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{-16(\Delta t)^2 - 32t\,\Delta t + 22.08\Delta t}{\Delta t} = -32t + 22.08.$$

2. *Find and interpret $v(0)$.*

   $v(0) = -32(0) + 22.08 = 22.08$. *This means initially (when Jason lets go of the hammer), the hammer is traveling upwards at* 22.08 *feet per second.*

3. *Solve $v(t) = 0$ and interpret.*

   $v(t) = 0$ *when* $t = \dfrac{22.08}{32} = 0.69$. *This means the (vertical) velocity zeros out* 0.69 *seconds after Jason lets go of the hammer. In this scenario, this corresponds to when the hammer reaches its peak height.*

[2] We say the graph of g has a **vertical tangent line** at $(0,0)$. This is the more formal way to describe all of the 'unusual steepness' we saw back in Chapter **??**.

[3] We say the graph of f has a **cusp** at $(0,0)$ since we have two different, infinite slopes meeting at a point.

4. *Find the velocity of the hammer when it hits the ground, rounded to three decimal places.*

   We first find when the hammer hits the ground by solving $h(t) = 0$*. The positive answer here is* $t \approx 1.612$ *seconds. The velocity of the hammer is:* $v(1.612) = -32(1.612) + 22.08 = -29.504$*. The hammer hits the ground going (approximately)* 29.504 *feet per second.*[4]

Problem 15 *In Exercise **??** in Section **??**, the average fuel economy* $F(t)$ *in miles per gallon (mpg) for passenger cars in the US t years after 1980 is modeled by* $F(t) = -0.0076t^2 + 0.45t + 16$, $0 \leq t \leq 28$.

1. *Find and simplify a formula for* $F'(t) = \lim_{h\to 0} \dfrac{F(t+h) - F(t)}{h}$.

   $F'(t) = \lim_{h\to 0} \dfrac{F(t+h) - F(t)}{h} = \lim_{h\to 0} \dfrac{-0.0076h^2 - 0.0152th + 0.45h}{h} = -0.0152t + 0.45.$

2. *Find and interpret* $F'(0)$, $F'(5)$ *and* $F'(10)$.

   $F'(0) = 0.45$*, so fuel economy was increasing at a rate of* 0.45 *mpg per year in 1980.*[5]

   $F'(5) = 0.374$*, so fuel economy was increasing at a rate of* 0.374 *mpg per year in 1985.*

   $F'(10) = 0.298$*, so fuel economy was increasing at a rate of* 0.298 *mpg per year in 1990.*

3. *Interpret the trend you observe in your answers to part **??**.*

   Based on the model, we have that during the years 1980 - 1990, fuel economy was increasing, but less so as the decade wore on. Technical and cost limitations could be at work here.

Problem 16 *Let us return to Example **??** where* $C(x) = .03x^3 - 4.5x^2 + 225x + 250$ *denotes the cost, in dollars, of producing x PortaBoy game systems.*

1. *Find and interpret* $C'(75) = \lim_{h\to 0} \dfrac{C(75+h) - C(75)}{h}$.

   $C'(75) = \lim_{h\to 0} \dfrac{C(75+h) - C(75)}{h} = \lim_{h\to 0} \dfrac{0.03h^3 + 2.25h^2 + 56.25h}{h} = 56.25$*. This means when producing 75 systems, the cost is increasing at a rate of* \$56.25 *per system.*

2. *Recall in Exercise **??** in Section **??**, we found the marginal cost,* $MC(75) = 58.53$*, which means it will cost an additional* \$58.53 *to produce the* 76*th item. Compare* $C'(75)$ *and* $MC(75)$.

   We see that $C'(75)$ *is numerically close to* $MC(75)$ *but the former is a rate of change (measured in dollars per system) where the latter is a change (measured in dollars). Note that if we set* $h = 1$ *in the difference quotient:*

   $$\frac{C(75+h) - C(75)}{h} = \frac{C(75+1) - C(75)}{1} = C(76) - C(75)$$

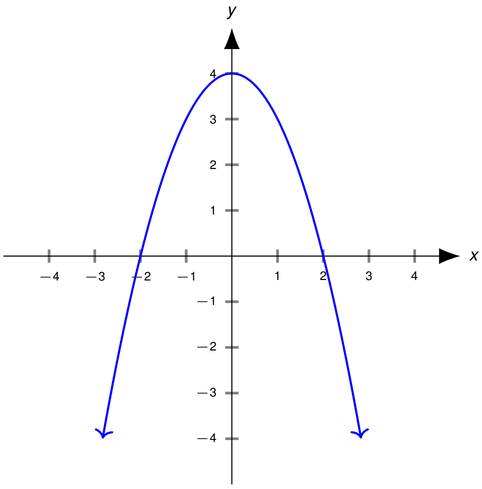
   we see these two quantities can be used to approximate each other.

Question 17 *In Exercises **??** - **??**, match the graph of the function with a plausible graph of its derivative. Choose from Graph A, B, or C.*

Problem 17.1 $y = f(x)$*:*

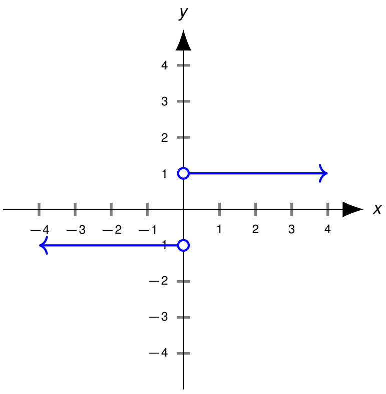
[4]The negative '$-$' here on $v(1.612)$ indicates the hammer is heading **downwards** when it strikes the ground.

[5]Since the domain of F is $0 \leq t \leq 28$, $F'(0)$ is actually $F'_{+}(0)$.

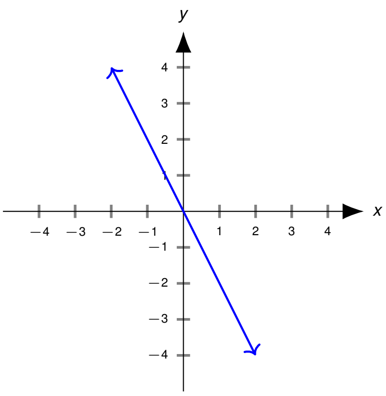


Multiple Choice:

1. *Graph A:*



2. *Graph B.:*



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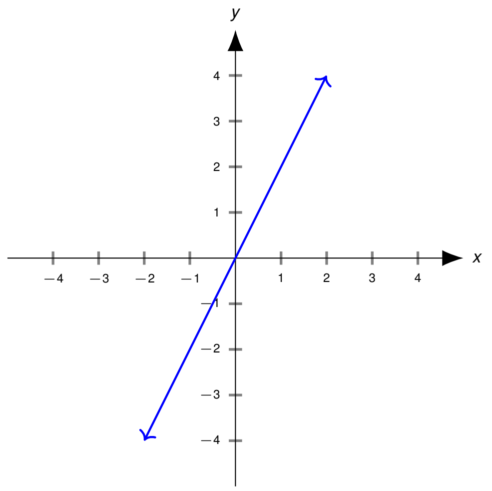
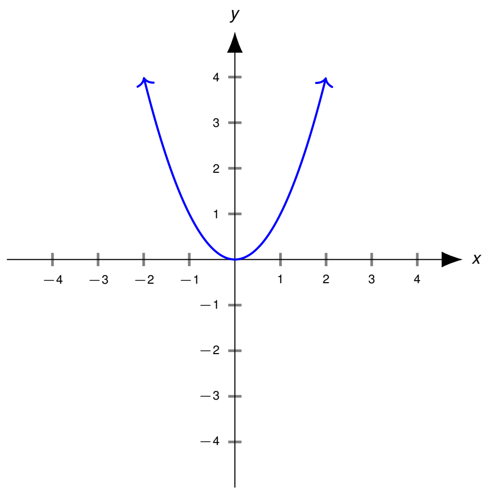
3. *Graph C.:*

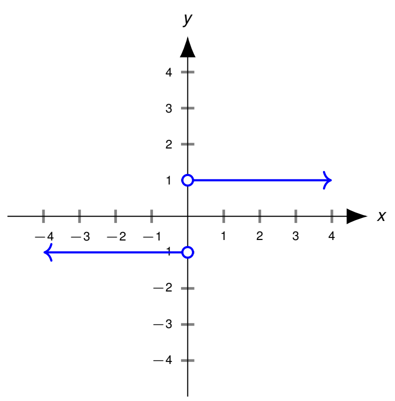


Problem 17.2 *$y = g(x)$:*

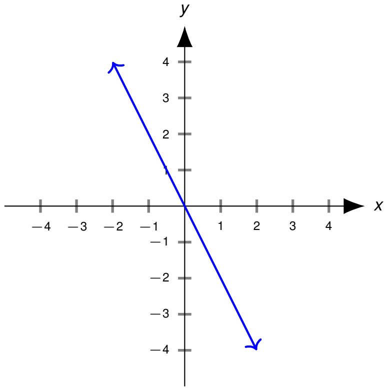


Multiple Choice:

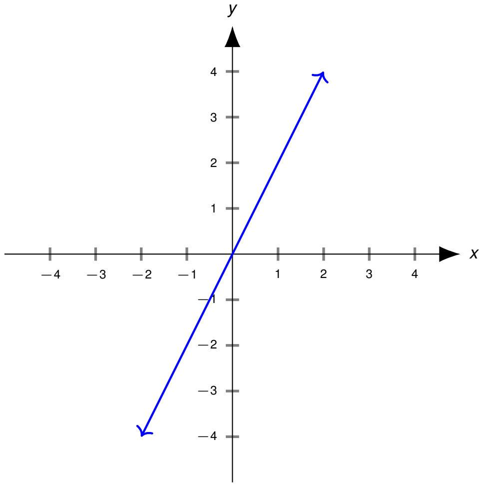
1. *Graph A:*



2. *Graph B.:*

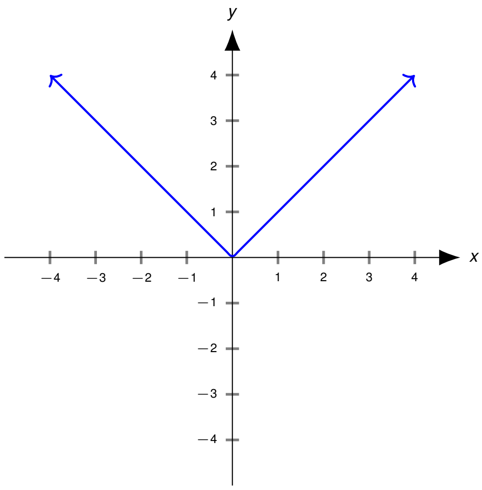


3. *Graph C.:*



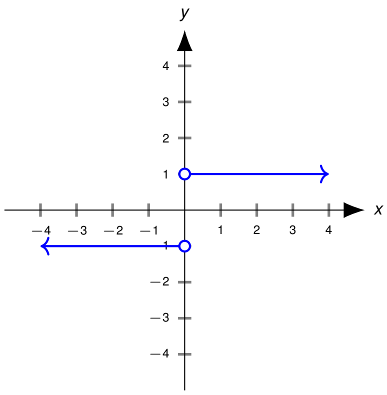
✓

Problem 17.3 $y = h(x)$:



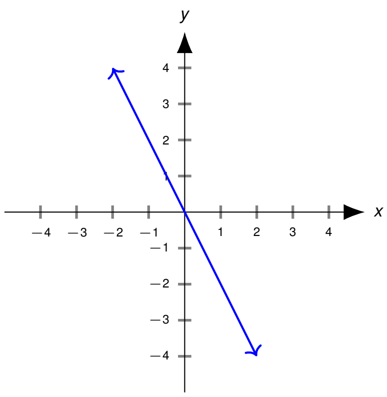
Multiple Choice:

1. *Graph A:*

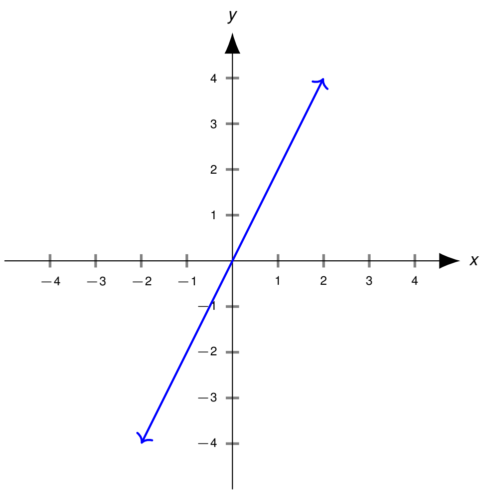


✓

2. *Graph B.:*

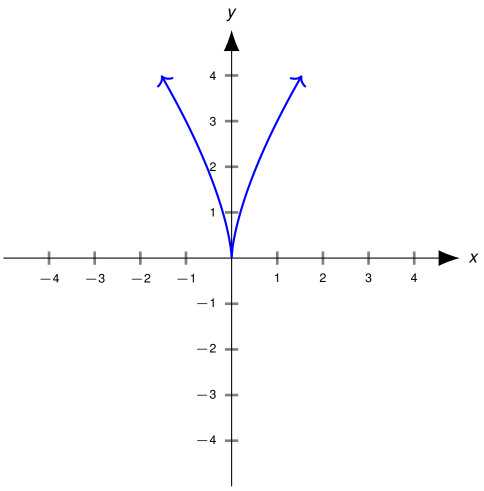


3. *Graph C.:*



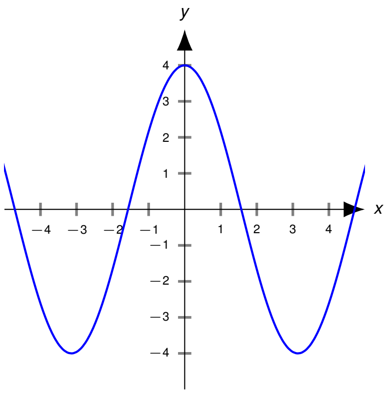
Question 18 *In Exercises* **??** - **??**, *match the graph of the function with a plausible graph of its derivative. Choose from Graph A, B, or C below.*

Problem 18.1 $y = f(x)$:

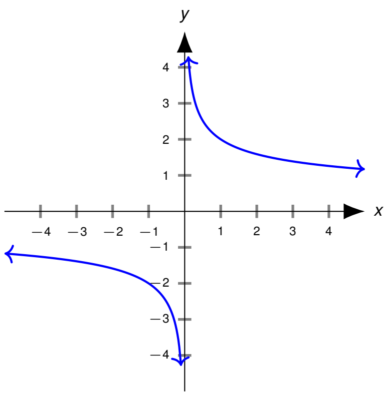


Multiple Choice:

1. *Graph A:*

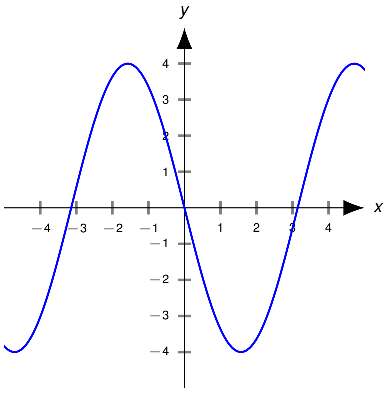


2. *Graph B:*

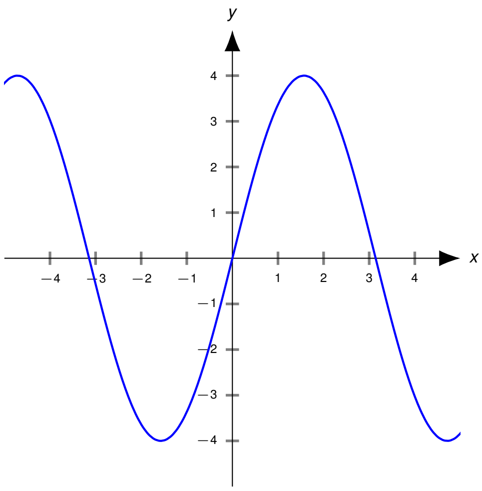


✓

3. *Graph C:*

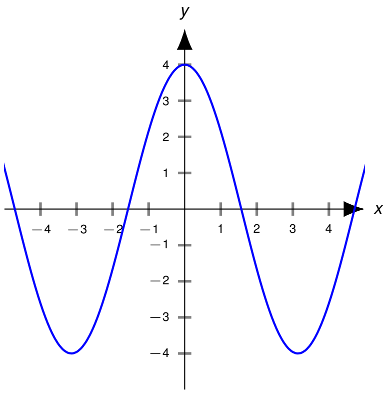


Problem 18.2 *$y = g(x)$:*



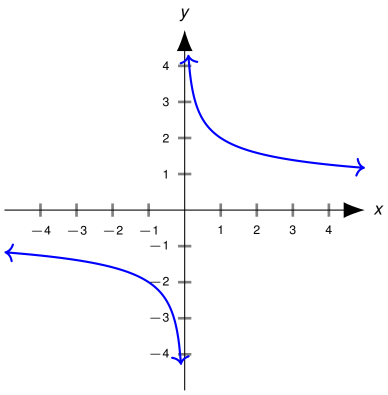
Multiple Choice:

1. *Graph A:*

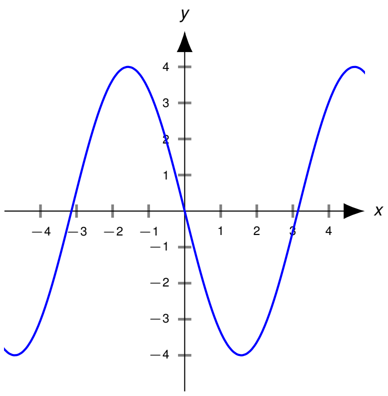


✓

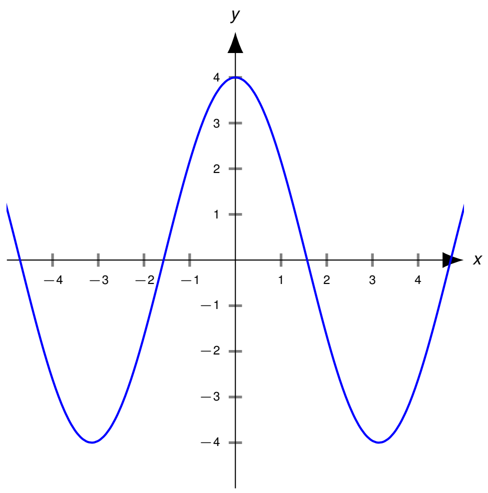
2. *Graph B:*



3. *Graph C:*

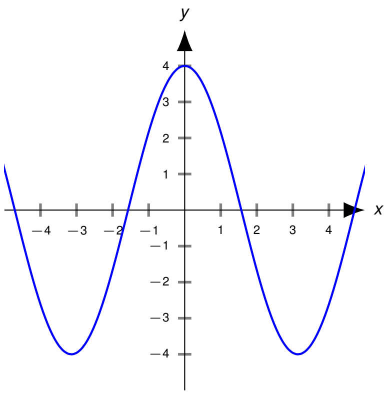


Problem 18.3 *$y = h(x)$:*

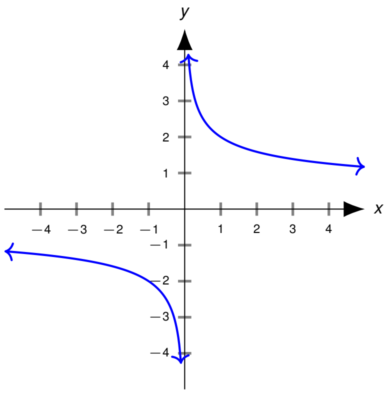


Multiple Choice:

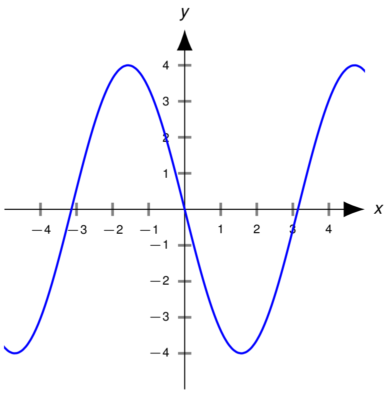
1. *Graph A:*



2. *Graph B:*



3. *Graph C:*



✓