## Exercises for Graphs of Other Circular Functions

**Question 1** *In Exercises* **??** - **??**, *graph one cycle of the given function. State the period of the function.*

***Problem 1.1*** $y = \tan\left(t - \frac{\pi}{3}\right)$

*Use the Desmos graph below with the settings* $f(t) = \tan(t)$, $h = \frac{\pi}{3}$, $b = 1$, $a = 1$, $k = 0$

*Desmos link:* https://www.desmos.com/calculator/dvsafdw6md

*Period:* $\pi$

***Problem 1.2*** $y = 2\tan\left(\frac{1}{4}t\right) - 3$

*Use the Desmos graph below with the settings* $f(t) = \tan(t)$, $h = 0$, $b = \frac{1}{4}$, $a = 2$, $k = -3$

*Desmos link:* https://www.desmos.com/calculator/dvsafdw6md

*Period:* $4\pi$

***Problem 1.3*** $y = \frac{1}{3}\tan(-2t - \pi) + 1$

*Use the Desmos graph below with the settings* $f(t) = \tan(t)$, $h = \pi$, $b = -2$, $a = \frac{1}{3}$, $k = 1$

*Desmos link:* https://www.desmos.com/calculator/dvsafdw6md

*Period:* $\frac{\pi}{2}$

***Problem 1.4*** $y = \sec\left(t - \frac{\pi}{2}\right)$

*Use the Desmos graph below with the settings* $f(t) = \sec(t)$, $h = \frac{\pi}{2}$, $b = 1$, $a = 1$, $k = 0$

*Desmos link:* https://www.desmos.com/calculator/rtp5lnnwbp

*To graph by hand, start with* $y = \cos\left(t - \frac{\pi}{2}\right)$
*Period:* $2\pi$

***Problem 1.5*** $y = -\csc\left(t + \frac{\pi}{3}\right)$

*Use the Desmos graph below with the settings* $f(t) = \csc(t)$, $h = -\frac{\pi}{3}$, $b = \frac{1}{2}$, $a = \frac{1}{3}$, $k = 0$

*Desmos link:* https://www.desmos.com/calculator/disjovazry

*To graph by hand, start with* $y = -\frac{1}{3}\cos\left(\frac{1}{2}t + \frac{\pi}{3}\right)$
*Period:* $4\pi$

## EXERCISES FOR GRAPHS OF OTHER CIRCULAR FUNCTIONS

***Problem 1.6*** $y = -\frac{1}{3}\sec\left(\frac{1}{2}t + \frac{\pi}{3}\right)$

*Use the Desmos graph below with the settings* $f(t) = \sec(t),\ h = -\frac{\pi}{3},\ b = 1,\ a = -\frac{1}{3},\ k = 0$

*Desmos link:* https://www.desmos.com/calculator/rtp5lnnwbp

*To graph by hand, start with* $y = -\frac{1}{3}\cos\left(\frac{1}{2}t + \frac{\pi}{3}\right)$
*Period:* $4\pi$

***Problem 1.7*** $y = \csc(2t - \pi)$

*Use the Desmos graph below with the settings* $f(t) = \csc(t),\ h = \pi,\ b = 2,\ a = 1,\ k = 0$

*Desmos link:* https://www.desmos.com/calculator/disjovazry

*To graph by hand, start with* $y = \sin(2t - \pi)$
*Period:* $\pi$

***Problem 1.8*** $y = \sec(3t - 2\pi) + 4$

*Use the Desmos graph below with the settings* $f(t) = \sec(t),\ h = 2\pi,\ b = 3,\ a = 1,\ k = 4$

*Desmos link:* https://www.desmos.com/calculator/rtp5lnnwbp

*To graph by hand, start with* $y = \cos(3t - 2\pi) + 4$
*Period:* $\frac{2\pi}{3}$

***Problem 1.9*** $y = \csc\left(-t - \frac{\pi}{4}\right) - 2$

*Use the Desmos graph below with the settings* $f(t) = \csc(t),\ h = \frac{\pi}{4},\ b = -1,\ a = 1,\ k = -2$

*Desmos link:* https://www.desmos.com/calculator/disjovazry

*To graph by hand, start with* $y = \sin\left(-t - \frac{\pi}{4}\right) - 2$
*Period:* $2\pi$

***Problem 1.10*** $y = \cot\left(t + \frac{\pi}{6}\right)$

*Use the Desmos graph below with the settings* $f(t) = \cot(t),\ h = -\frac{\pi}{6},\ b = 1,\ a = 1,\ k = 0$

*Desmos link:* https://www.desmos.com/calculator/kct2qlbajd

*Period:* $\pi$

***Problem 1.11*** $y = -11\cot\left(\frac{1}{5}t\right)$

*Use the Desmos graph below with the settings* $f(t) = \cot(t),\ h = -11,\ b = \frac{1}{5},\ a = 1,\ k = 0$

*Desmos link:* https://www.desmos.com/calculator/kct2qlbajd

*Period:* $5\pi$

***Problem 1.12*** $y = \frac{1}{3}\cot\left(2t + \frac{3\pi}{2}\right) + 1$

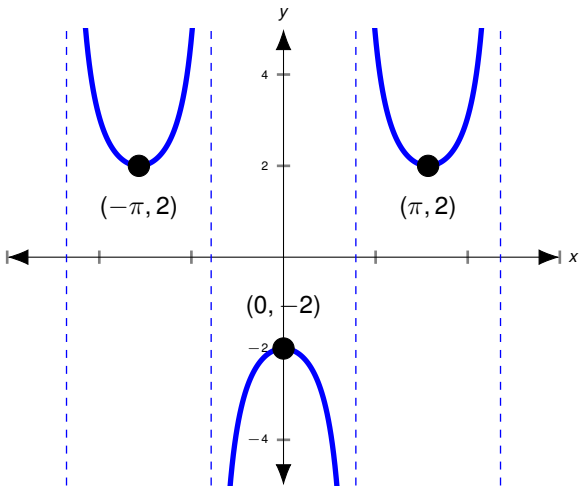
*Use the Desmos graph below with the settings* $f(t) = \cot(t)$, $h = -\frac{3\pi}{2}$, $b = 2$, $a = \frac{1}{3}$, $k = 1$

*Desmos link:* https://www.desmos.com/calculator/kct2qlbajd

*Period:* $\frac{\pi}{2}$

**Question 2** *In Exercises* **??** *-* **??***, the graph of a (co)secant function is given. Find a formula for the function in the form* $F(t) = A\sec(\omega t + \phi) + B$ *and* $G(t) = A\csc(\omega t + \phi) + B$*. Select* $\omega$ *so* $\omega > 0$*. Check your answer by graphing.*

***Problem 2.1*** *Asymptotes:* $t = \pm\frac{\pi}{2}$, $t = \pm\frac{3\pi}{2}$, ...

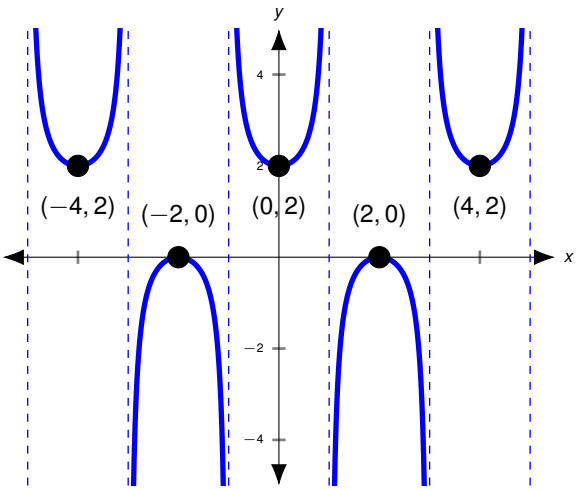



$F(t) = 2\sec(t - \pi)$, $G(t) = 2\csc\left(t - \frac{\pi}{2}\right)$

*Use the Desmos graph below to check that both of your formulas agree.*

*Desmos link:* https://www.desmos.com/calculator/i0yo9vhhbp

***Problem 2.2*** *Asymptotes:* $t = \pm 1$, $t = \pm 3$, $t = \pm 5$, ...



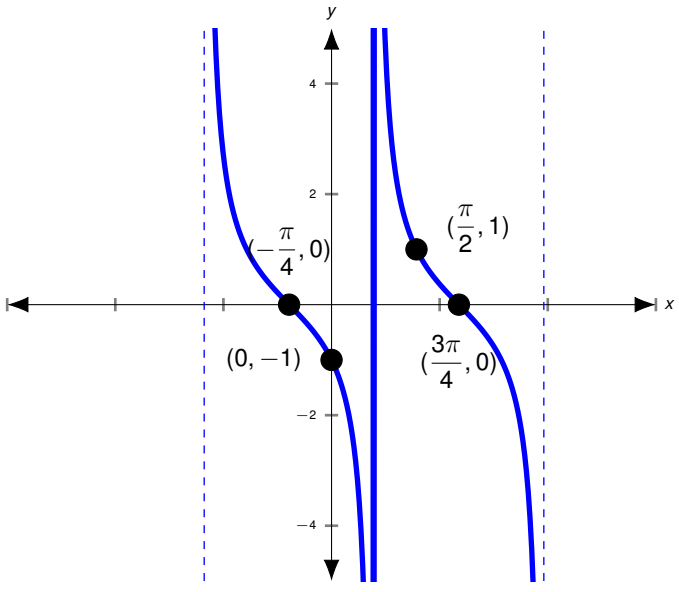

$F(t) = \sec\left(\frac{\pi}{2}t\right) + 1$, $G(t) = \csc\left(\frac{\pi}{2}t + \frac{\pi}{2}\right) + 1$

*Use the Desmos graph below to check that both of your formulas agree.*

*Desmos link:* https://www.desmos.com/calculator/i0yo9vhhbp

**Question 3** *In Exercises* **??** *-* **??***, the graph of a (co)tangent function given. Find a formula the function in the form* $J(t) = A\tan(\omega t + \phi) + B$ *and* $K(t) = A\cot(\omega t + \phi) + B$*. Select* $\omega$ *so* $\omega > 0$*. Check your answer by graphing.*

***Problem 3.1*** *Asymptotes:* $t = -\frac{3\pi}{4}$, $t = \frac{\pi}{4}$, $t = \frac{5\pi}{4}$, ...



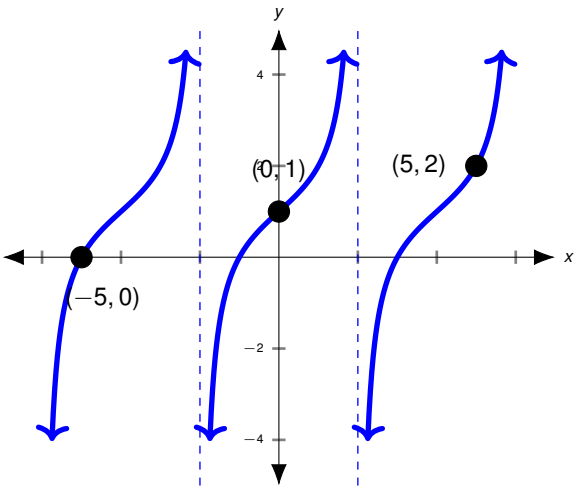

$J(t) = -\tan\left(t + \frac{\pi}{4}\right)$, $K(t) = \cot\left(t - \frac{\pi}{4}\right)$

*Use the Desmos graph below to check that both of your formulas agree.*

*Desmos link:* https://www.desmos.com/calculator/c5y26k1oax

***Problem 3.2*** *Asymptotes:* $t = \pm 2$, $t = \pm 6$, $t = \pm 10$, ...

## EXERCISES FOR GRAPHS OF OTHER CIRCULAR FUNCTIONS




$J(t) = \tan\left(\frac{\pi}{4}t\right) + 1$, $K(t) = -\cot\left(\frac{\pi}{4}t + \frac{\pi}{2}\right) + 1$

*Use the Desmos graph below to check that both of your formulas agree.*

*Desmos link:* `https://www.desmos.com/calculator/c5y26k1oax`

**Question 4** *In Section* **??**, *we observed*[1] *that the cosine and sine functions are continuous. As such, the other four circular functions are continuous on their domains.*[2] *In Exercises* **??** - **??**, *determine the given limit. Use the symbols '$-\infty$' and '$\infty$'as appropriate. Check your answers graphically.*

***Problem 4.1*** $\lim_{t\to 0} \tan(t)$.

$\lim_{t\to 0} \tan(t) = \tan(0) = 0.$

***Problem 4.2*** $\lim_{t\to \pi} \sec(t)$

$\lim_{t\to \pi} \sec(t) = \sec(\pi) = -1$

***Problem 4.3*** $\lim_{t\to 3\pi} \cot\left(\frac{t}{2}\right)$

$\lim_{t\to 3\pi} \cot\left(\frac{t}{2}\right) = \cot\left(\frac{3\pi}{2}\right) = 0$

***Problem 4.4*** $\lim_{\theta\to 0} \csc\left(2\theta + \frac{\pi}{4}\right)$.

$\lim_{\theta\to 0} \csc\left(2\theta + \frac{\pi}{4}\right) = \csc\left(\frac{\pi}{4}\right) = \sqrt{2}.$

***Problem 4.5*** $\lim_{\theta\to \pi} (\cos(\theta) - \sec(\theta))$

$\lim_{\theta\to \pi} (\cos(\theta) - \sec(\theta)) = \cos(\pi) - \sec(\pi) = -1 - (-1) = 0$

***Problem 4.6*** $\lim_{\theta\to \frac{\pi}{4}} \sec(\theta)\tan(\theta)$

$\lim_{\theta\to \frac{\pi}{4}} (\sec(\theta)\tan(\theta)) = \sec\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right) = \sqrt{2}$

[1] insisted?

[2] See the remarks following Definition **??** in Section **??**.

***Problem 4.7*** $\lim_{x\to 0^+} \csc(3x)$

$\lim_{x\to 0^+} \csc(3x) = \infty$

***Problem 4.8*** $\lim_{x\to \pi^-} (\sin(2x) - \tan(x))$

$\lim_{x\to \pi^-} (\sin(2x) - \tan(x)) = \sin(2\pi) - \tan(\pi) = 0$

***Problem 4.9*** $\lim_{x\to \pi^+} (\cos(x) + \cot(x))$

$\lim_{x\to \pi^+} (\cos(2x) + \cot(x)) = \infty$

**Problem 5** *In Exercise* **??** *in Section* **??**, *we proved* $\lim_{\theta\to 0} \dfrac{\sin(\theta)}{\theta} = 1$.

1. *Use Theorem* **??** *from Section* **??** *to show* $\lim_{\theta\to 0} \dfrac{\theta}{\sin(\theta)} = 1$.

   $\lim_{\theta\to 0} \dfrac{\theta}{\sin(\theta)} = \lim_{\theta\to 0} \left[\dfrac{\sin(\theta)}{\theta}\right]^{-1} = 1^{-1} = 1$

2. *Which indeterminate form is present in the limit* $\lim_{\theta\to 0^+} 2\theta \csc(\theta)$*?*

   *As* $\theta \to 0^+$, $2\theta \csc(\theta) \to 0 \cdot \infty$.

3. *Graph* $f(\theta) = 2\theta \csc(\theta)$ *near* $\theta = 0$. *What appears to be the limit?*

   *The graph of* $f(\theta) = 2\theta \csc(\theta)$ *approaches* $(0, 2)$ *so* $\lim_{\theta\to 0^+} 2\theta \csc(\theta)$ *appears to be* 2.

4. *Rewrite* $2\theta \csc(\theta)$ *in terms of* $\theta$ *and* $\sin(\theta)$ *and use part* **??** *to analytically find* $\lim_{\theta\to 0^+} 2\theta \csc(\theta)$.

   $\lim_{\theta\to 0^+} 2\theta \csc(\theta) = \lim_{\theta\to 0^+} 2\,\theta \dfrac{1}{\sin(\theta)} = \lim_{\theta\to 0^+} 2\,\dfrac{\theta}{\sin(\theta)} = 2(1) = 2.$

5. *Use the same methodology as above to help you analytically determine* $\lim_{\theta\to 0^+} 2\theta \cot(\theta)$

   $\lim_{\theta\to 0^+} 2\theta \cot(\theta) = \lim_{\theta\to 0^+} 2\,\theta \dfrac{\cos(\theta)}{\sin(\theta)} = \lim_{\theta\to 0^+} 2\cos(\theta)\,\dfrac{\theta}{\sin(\theta)} = 2\cos(0)(1) = 2(1)(1) = 2$

**Problem 6** *Use the conversion formulas listed in Theorem* **??** *to create conversion formulas between secant and cosecant functions.*

$\csc\left(t + \dfrac{\pi}{2}\right) = \sec(t)$ *and* $\sec\left(t - \dfrac{\pi}{2}\right) = \csc(t)$.

$f(t) = \sec\left(2t - \dfrac{7\pi}{6}\right) - 1 = \csc\left(\left[2t - \dfrac{7\pi}{6}\right] + \dfrac{\pi}{2}\right) - 1 = \csc\left(2t - \dfrac{2\pi}{3}\right) - 1$, *in terms of cosecants.*

**Problem 7** *Use a conversion formula to rewrite our first answer to Example* **??**, $f(t) = \sec\left(2t - \dfrac{7\pi}{6}\right) - 1$, *in terms of cosecants.*

**Problem 8** *Rework Example* **??** *and find answers with* $A < 0$.

**Problem 9** *Prove Theorem* **??** *using Theorem* **??**.

**Problem 10** *Prove Theorem* **??** *using Theorem* **??**.

## EXERCISES FOR GRAPHS OF OTHER CIRCULAR FUNCTIONS

**Problem 11** *In this Exercise, we argue the range of the tangent function is* $(-\infty, \infty)$*. Let M be a fixed, but arbitrary positive real number.*

1. *Show there is an acute angle* $\theta$ *with* $\tan(\theta) = M$*. (Hint: think right triangles.)*
2. *Using the symmetry of the Unit Circle, explain why there are angles* $\theta$ *with* $\tan(\theta) = -M$*.*
3. *Find angles with* $\tan(\theta) = 0$*.*
4. *Combine the three parts above to conclude the range of the tangent function is* $(-\infty, \infty)$*.*

**Problem 12** *Prove* $\cot(t)$ *is odd. (Hint: mimic the proof given in the text that* $\tan(t)$ *is odd.)*