Exercises for The Circular Functions Sine and Cosine

Question 1 *In Exercises - , find the exact value of the cosine and sine of the given angle.*

Problem 1.1 $\theta = 0$

$\cos(0) = 1$

$\sin(0) = 0$

Problem 1.2 $\theta = \frac{\pi}{4}$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Problem 1.3 $\theta = \frac{\pi}{3}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Problem 1.4 $\theta = \frac{\pi}{2}$

$\cos\left(\frac{\pi}{2}\right) = 0$

$\sin\left(\frac{\pi}{2}\right) = 1$

Problem 1.5 $\theta = \frac{2\pi}{3}$

$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Problem 1.6 $\theta = \frac{3\pi}{4}$

$\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Problem 1.7 $\theta = \pi$

$\cos(\pi) = -1$

$\sin(\pi) = 0$

Problem 1.8 $\theta = \frac{7\pi}{6}$

$\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

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Problem 1.9 $\theta = \frac{5\pi}{4}$

$$\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

Problem 1.10 $\theta = \frac{4\pi}{3}$

$$\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$$

$$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Problem 1.11 $\theta = \frac{3\pi}{2}$

$$\cos\left(\frac{3\pi}{2}\right) = 0$$

$$\sin\left(\frac{3\pi}{2}\right) = -1$$

Problem 1.12 $\theta = \frac{5\pi}{3}$

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$$

$$\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Problem 1.13 $\theta = \frac{7\pi}{4}$

$$\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

Problem 1.14 $\theta = \frac{23\pi}{6}$

$$\cos\left(\frac{23\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{23\pi}{6}\right) = -\frac{1}{2}$$

Problem 1.15 $\theta = -\frac{13\pi}{2}$

$$\cos\left(-\frac{13\pi}{2}\right) = 0$$

$$\sin\left(-\frac{13\pi}{2}\right) = -1$$

Problem 1.16 $\theta = -\dfrac{43\pi}{6}$

$\cos\left(-\dfrac{43\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

$\sin\left(-\dfrac{43\pi}{6}\right) = \dfrac{1}{2}$

Problem 1.17 $\theta = -\dfrac{3\pi}{4}$

$\cos\left(-\dfrac{3\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

$\sin\left(-\dfrac{3\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

Problem 1.18 $\theta = -\dfrac{\pi}{6}$

$\cos\left(-\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

$\sin\left(-\dfrac{\pi}{6}\right) = -\dfrac{1}{2}$

Problem 1.19 $\theta = \dfrac{10\pi}{3}$

$\cos\left(\dfrac{10\pi}{3}\right) = -\dfrac{1}{2}$

$\sin\left(\dfrac{10\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$

Problem 1.20 $\theta = 117\pi$

$\cos(117\pi) = -1$

$\sin(117\pi) = 0$

Question 2 *In Exercises - , find all of the angles which satisfy the given equation.*

For the problems where you are prompted to type in an answer, type a response in the range $0 \leq \theta < 2\pi$.

Problem 2.1 $\sin(\theta) = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{6} + 2\pi k$ *or* $\theta = \dfrac{5\pi}{6} + 2\pi k$ *for any integer* k.

Problem 2.2 $\cos(\theta) = -\dfrac{\sqrt{3}}{2}$

$\theta = \dfrac{5\pi}{6} + 2\pi k$ *(Quadrant II) or* $\theta = \dfrac{7\pi}{6} + 2\pi k$ *(Quadrant III) for any integer* k.

Problem 2.3 $\sin(\theta) = 0$

$\theta = \pi k$ *for any integer* k.

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Problem 2.4 $\cos(\theta) = \dfrac{\sqrt{2}}{2}$

$\theta = \dfrac{\pi}{4} + 2\pi k$ or $\theta = \dfrac{7\pi}{4} + 2\pi k$ for any integer k.

Problem 2.5 $\sin(\theta) = \dfrac{\sqrt{3}}{2}$

$\theta = \dfrac{\pi}{3} + 2\pi k$ (Quadrant I) or $\theta = \dfrac{2\pi}{3} + 2\pi k$ (Quadrant II) for any integer k.

Problem 2.6 $\cos(\theta) = -1$

$\theta = (2k + 1)\pi$ for any integer k.

Problem 2.7 $\sin(\theta) = -1$

$\theta = \dfrac{3\pi}{2} + 2\pi k$ for any integer k.

Problem 2.8 $\cos(\theta) = \dfrac{\sqrt{3}}{2}$

$\theta = \dfrac{\pi}{6} + 2\pi k$ (Quadrant I) or $\theta = \dfrac{11\pi}{6} + 2\pi k$ (Quadrant IV) for any integer k.

Problem 2.9 $\cos(\theta) = -1.001$

Since $-1 \leq \cos(\theta) \leq 1$, $\cos(\theta) = -1.001$ has no (real) solution.

Question 3 *In Exercises - , solve the equation for t. (See the remarks on page* **??***.)*

Problem 3.1 $\cos(t) = 0$

$t = \dfrac{\pi}{2} + \pi k$ for any integer k.

Problem 3.2 $\sin(t) = -\dfrac{\sqrt{2}}{2}$

$t = \dfrac{5\pi}{4} + 2\pi k$ or $t = \dfrac{7\pi}{4} + 2\pi k$ for any integer k.

Problem 3.3 $\cos(t) = 3$

Since $-1 \leq \cos(t) \leq 1$, $\cos(t) = 3$ has no (real) solution.

Problem 3.4 $\sin(t) = -\dfrac{1}{2}$

$t = \dfrac{7\pi}{6} + 2\pi k$ or $t = \dfrac{11\pi}{6} + 2\pi k$ for any integer k.

Problem 3.5 $\cos(t) = \dfrac{1}{2}$

$t = \dfrac{\pi}{3} + 2\pi k$ or $t = \dfrac{5\pi}{3} + 2\pi k$ for any integer k.

Problem 3.6 $\sin(t) = -2$

Since $-1 \leq \sin(t) \leq 1$, $\sin(t) = -2$ has no (real) solution.

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Problem 3.7 $\cos(t) = 1$

$t = 2\pi k$ *for any integer* k.

Problem 3.8 $\sin(t) = 1$

$t = \frac{\pi}{2} + 2\pi k$ *for any integer* k.

Problem 3.9 $\cos(t) = -\frac{\sqrt{2}}{2}$

$t = \frac{3\pi}{4} + 2\pi k$ *or* $t = \frac{5\pi}{4} + 2\pi k$ *for any integer* k.

Question 4 *In Exercises - , let θ be the angle in standard position whose terminal side contains the given point then compute* $\cos(\theta)$ *and* $\sin(\theta)$.

Problem 4.1 $P(-7, 24)$

$\cos(\theta) = -\frac{7}{25}$

$\sin(\theta) = \frac{24}{25}$

Problem 4.2 $Q(3, 4)$

$\cos(\theta) = \frac{3}{5}$

$\sin(\theta) = \frac{4}{5}$

Problem 4.3 $R(5, -9)$

$\cos(\theta) = \frac{5\sqrt{106}}{106}$

$\sin(\theta) = -\frac{9\sqrt{106}}{106}$

Problem 4.4 $T(-2, -11)$

$\cos(\theta) = -\frac{2\sqrt{5}}{25}$

$\sin(\theta) = -\frac{11\sqrt{5}}{25}$

Question 5 *In Exercises - , use the results developed throughout the section to find the requested value.*

Problem 5.1 *If* $\sin(\theta) = -\frac{7}{25}$ *with θ in Quadrant IV, what is* $\cos(\theta)$*?*

$\cos(\theta) = \frac{24}{25}$.

Problem 5.2 *If* $\cos(\theta) = \frac{4}{9}$ *with θ in Quadrant I, what is* $\sin(\theta)$*?*

$\sin(\theta) = \frac{\sqrt{65}}{9}$.

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Problem 5.3 *If* $\sin(\theta) = \frac{5}{13}$ *with* θ *in Quadrant II, what is* $\cos(\theta)$*?*

$\cos(\theta) = -\frac{12}{13}$.

Problem 5.4 *If* $\cos(\theta) = -\frac{2}{11}$ *with* θ *in Quadrant III, what is* $\sin(\theta)$*?*

$\sin(\theta) = -\frac{\sqrt{117}}{11}$.

Problem 5.5 *If* $\sin(\theta) = -\frac{2}{3}$ *with* θ *in Quadrant III, what is* $\cos(\theta)$*?*

$\cos(\theta) = -\frac{\sqrt{5}}{3}$.

Problem 5.6 *If* $\cos(\theta) = \frac{28}{53}$ *with* θ *in Quadrant IV, what is* $\sin(\theta)$*?*

$\sin(\theta) = -\frac{45}{53}$.

Problem 5.7 *If* $\sin(\theta) = \frac{2\sqrt{5}}{5}$ *and* $\frac{\pi}{2} < \theta < \pi$*, what is* $\cos(\theta)$*?*

$\cos(\theta) = -\frac{\sqrt{5}}{5}$.

Problem 5.8 *If* $\cos(\theta) = \frac{\sqrt{10}}{10}$ *and* $2\pi < \theta < \frac{5\pi}{2}$*, what is* $\sin(\theta)$*?*

$\sin(\theta) = \frac{3\sqrt{10}}{10}$.

Problem 5.9 *If* $\sin(\theta) = -0.42$ *and* $\pi < \theta < \frac{3\pi}{2}$*, what is* $\cos(\theta)$*? (Round your answer to four decimal places.)*

$\cos(\theta) = -\sqrt{0.8236} \approx -0.9075$.

Problem 5.10 *If* $\cos(\theta) = -0.98$ *and* $\frac{\pi}{2} < \theta < \pi$*, what is* $\sin(\theta)$*?*

$\sin(\theta) = \sqrt{0.0396} \approx 0.1990$.

Question 6 *In Exercises - , use your calculator to approximate the given value to three decimal places. Make sure your calculator is in the proper angle measurement mode!*

Problem 6.1 $\sin(78.95^\circ)$

$\sin(78.95^\circ) \approx 0.981$

Problem 6.2 $\cos(-2.01)$

$\cos(-2.01) \approx -0.425$

Problem 6.3 $\sin(392.994)$

$\sin(392.994) \approx -0.291$

Problem 6.4 $\cos(207^\circ)$

$\cos(207^\circ) \approx -0.891$

Problem 6.5 $\sin\left(\pi^\circ\right)$

$\sin\left(\pi^\circ\right) \approx 0.055$

Problem 6.6 $\cos(e)$

$\cos(e) \approx -0.912$

Question 7 *In Exercises - , write the given function as a nontrivial decomposition of functions as directed.*

Problem 7.1 *For* $f(t) = 3t + \sin(2t)$*, find functions* g *and* h *so that* $f = g + h$*.*

One solution is $g(t) = 3t$ *and* $h(t) = \sin(2t)$*.*

Problem 7.2 *For* $f(\theta) = 3\cos(\theta) - \sin(4\theta)$*, find functions* g *and* h *so that* $f = g - h$*.*

One solution is $g(\theta) = 3\cos(\theta)$ *and* $h(\theta) = \sin(4\theta)$*.*

Problem 7.3 *For* $f(t) = e^{-0.1t}\sin(3t)$*, find functions* g *and* h *so that* $f = gh$*.*

One solution is $g(t) = e^{-0.1t}$ *and* $h(t) = \sin(3t)$*.*

Problem 7.4 *For* $r(t) = \dfrac{\sin(t)}{t}$*, find functions* f *and* g *so* $r = \dfrac{f}{g}$*.*

One solution is $f(t) = \sin(t)$ *and* $g(t) = t$*.*

Problem 7.5 *For* $r(\theta) = \sqrt{3\cos(\theta)}$*, find functions* f *and* g *so* $r = g \circ f$*.*

One solution is $f(\theta) = 3\cos(\theta)$ *and* $g(\theta) = \sqrt{\theta}$*.*

Problem 8 *For each function* $S(t)$ *listed below, compute the average rate of change over the indicated interval.*[1] *What trends do you notice? Be sure your calculator is in radian mode!*

$S(t)$	$[-0.1, 0.1]$	$[-0.01, 0.01]$	$[-0.001, 0.001]$
$\sin(t)$			
$\sin(2t)$			
$\sin(3t)$			
$\sin(4t)$			

As we zoom in towards 0*, the average rate of change of* $\sin(kt)$ *approaches* k*.*

$S(t)$	$[-0.1, 0.1]$	$[-0.01, 0.01]$	$[-0.001, 0.001]$
$\sin(t)$	≈ 0.9983	≈ 1	≈ 1
$\sin(2t)$	≈ 1.9867	≈ 1.9999	≈ 2
$\sin(3t)$	≈ 2.9552	≈ 2.9995	≈ 3
$\sin(4t)$	≈ 3.8942	≈ 3.9989	≈ 4

Question 9 *In Exercises - , find the equations of motion for the given scenario. Assume that the center of the motion is the origin, the motion is counter-clockwise and that* $t = 0$ *corresponds to a position along the positive* x*-axis. (See Equation* **??** *and Example* **??***.)*

[1] See Definition **??** in Section **??** for a review of this concept, as needed.

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EXERCISES FOR THE CIRCULAR FUNCTIONS SINE AND COSINE

Problem 9.1 *A point on the edge of the spinning yo-yo in Exercise* **??** *from Section* **??**.

Recall: The diameter of the yo-yo is 2.25 inches and it spins at 4500 revolutions per minute.

$r = 1.125$ *inches,* $\omega = 9000\pi \, \frac{\text{radians}}{\text{minute}}$, $x = 1.125\cos(9000\pi\, t)$, $y = 1.125\sin(9000\pi\, t)$. *Here x and y are measured in inches and t is measured in minutes.*

Problem 9.2 *The yo-yo in exercise* **??** *from Section* **??**.

Recall: The radius of the circle is 28 inches and it completes one revolution in 3 seconds.

$r = 28$ *inches,* $\omega = \frac{2\pi}{3} \, \frac{\text{radians}}{\text{second}}$, $x = 28\cos\left(\frac{2\pi}{3}t\right)$, $y = 28\sin\left(\frac{2\pi}{3}t\right)$. *Here x and y are measured in inches and t is measured in seconds.*

Problem 9.3 *A point on the edge of the hard drive in Exercise* **??** *from Section* **??**.

Recall: The diameter of the hard disk is 2.5 inches and it spins at 7200 revolutions per minute.

$r = 1.25$ *inches,* $\omega = 14400\pi \, \frac{\text{radians}}{\text{minute}}$, $x = 1.25\cos(14400\pi\, t)$, $y = 1.25\sin(14400\pi\, t)$. *Here x and y are measured in inches and t is measured in minutes.*

Problem 9.4 *A passenger on the Big Wheel in Exercise* **??** *from Section* **??**.

Recall: The diameter is 128 feet and completes 2 revolutions in 2 minutes, 7 seconds.

$r = 64$ *feet,* $\omega = \frac{4\pi}{127} \, \frac{\text{radians}}{\text{second}}$, $x = 64\cos\left(\frac{4\pi}{127}t\right)$, $y = 64\sin\left(\frac{4\pi}{127}t\right)$. *Here x and y are measured in feet and t is measured in seconds*

Problem 10 *Consider the numbers:* $0, 1, 2, 3, 4$. *Take the square root of each of these numbers, then divide each by* 2. *The resulting numbers should look hauntingly familiar.*

Problem 11 *On page* **??**, *we see that the sine and cosine functions of angles can be considered functions of real numbers. With help from your classmates, discuss the domains and ranges of* $f(t) = \sin(t)$ *and* $g(t) = \cos(t)$. *Write your answers using interval notation.*

Problem 12 *Another way to establish Theorem* **??** *is to use transformations. Re-read the discussion following Theorem* **??** *in Chapter* **??** *and transform the Unit Circle,* $x^2 + y^2 = 1$, *to* $x^2 + y^2 = r^2$ *using horizontal and vertical stretches. Show if the coordinates on the Unit Circle are* $(\cos(\theta), \sin(\theta))$, *then the corresponding coordinates on* $x^2 + y^2 = r^2$ *are* $(r\cos(\theta), r\sin(\theta))$.

Problem 13 *In the scenario of Equation* **??**, *we assumed that at* $t = 0$, *the object was at the point* $(r, 0)$. *If this is not the case, we can adjust the equations of motion by introducing a 'time delay.' If* $t_0 > 0$ *is the first time the object passes through the point* $(r, 0)$, *show, with the help of your classmates, the equations of motion are* $x = r\cos(\omega(t - t_0))$ *and* $y = r\sin(\omega(t - t_0))$.

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