EXERCISES FOR THE OTHER CIRCULAR FUNCTIONS

Question 1 *In Exercises* **??** - **??**, *find the exact value or state that it is undefined.*

Problem 1.1 $\tan\left(\frac{\pi}{4}\right)$

$\tan\left(\frac{\pi}{4}\right) = 1$

Problem 1.2 $\sec\left(\frac{\pi}{6}\right)$

$\sec\left(\frac{\pi}{6}\right) = \frac{2\sqrt{3}}{3}$

Problem 1.3 $\csc\left(\frac{5\pi}{6}\right)$

$\csc\left(\frac{5\pi}{6}\right) = 2$

Problem 1.4 $\cot\left(\frac{4\pi}{3}\right)$

$\cot\left(\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{3}$

Problem 1.5 $\tan\left(-\frac{11\pi}{6}\right)$

$\tan\left(-\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{3}$

Problem 1.6 $\sec\left(-\frac{3\pi}{2}\right)$

$\sec\left(-\frac{3\pi}{2}\right)$ *is undefined*

Problem 1.7 $\csc\left(-\frac{\pi}{3}\right)$

$\csc\left(-\frac{\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$

Problem 1.8 $\cot\left(\frac{13\pi}{2}\right)$

$\cot\left(\frac{13\pi}{2}\right) = 0$

Problem 1.9 $\tan(117\pi)$

$\tan(117\pi) = 0$

Problem 1.10 $\sec\left(-\frac{5\pi}{3}\right)$

$\sec\left(-\frac{5\pi}{3}\right) = 2$

EXERCISES FOR THE OTHER CIRCULAR FUNCTIONS

Problem 1.11 $\csc(3\pi)$

$\csc(3\pi)$ *is undefined*

Problem 1.12 $\cot(-5\pi)$

$\cot(-5\pi)$ *is undefined*

Problem 1.13 $\tan\left(\frac{31\pi}{2}\right)$

$\tan\left(\frac{31\pi}{2}\right)$ *is undefined*

Problem 1.14 $\sec\left(\frac{\pi}{4}\right)$

$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

Problem 1.15 $\csc\left(-\frac{7\pi}{4}\right)$

$\csc\left(-\frac{7\pi}{4}\right) = \sqrt{2}$

Problem 1.16 $\cot\left(\frac{7\pi}{6}\right)$

$\cot\left(\frac{7\pi}{6}\right) = \sqrt{3}$

Problem 1.17 $\tan\left(\frac{2\pi}{3}\right)$

$\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

Problem 1.18 $\sec(-7\pi)$

$\sec(-7\pi) = -1$

Problem 1.19 $\csc\left(\frac{\pi}{2}\right)$

$\csc\left(\frac{\pi}{2}\right) = 1$

Problem 1.20 $\cot\left(\frac{3\pi}{4}\right)$

$\cot\left(\frac{3\pi}{4}\right) = -1$

Question 2 *In Exercises* **??** *-* **??***, use the given the information to determine the quadrant in which the terminal side of the angle lies when plotted in standard position.*

Problem 2.1 $\sin(\theta) > 0$ *but* $\tan(\theta) < 0$.

θ *is in Quadrant II.*

Problem 2.2 $\cot(\alpha) > 0$ *but* $\cos(\alpha) < 0$.

α *is in Quadrant III*

Problem 2.3 $\sin(\beta) > 0$ *and* $\tan(\beta) > 0$.

β *is in Quadrant I.*

Problem 2.4 $\cos(\gamma) > 0$ *but* $\cot(\gamma) < 0$.

γ *is in Quadrant IV*.

Question 3 *In Exercises* **??** *-* **??**, *use the given the information to find the exact values of the circular functions of* θ.

Problem 3.1 $\sin(\theta) = \frac{3}{5}$ *with* θ *in Quadrant II*

$\sin(\theta) = \frac{3}{5}$

$\cos(\theta) = -\frac{4}{5}$

$\tan(\theta) = -\frac{3}{4}$

$\csc(\theta) = \frac{5}{3}$

$\sec(\theta) = -\frac{5}{4}$

$\cot(\theta) = -\frac{4}{3}$

Problem 3.2 $\tan(\theta) = \frac{12}{5}$ *with* θ *in Quadrant III*

$\sin(\theta) = -\frac{12}{13}$

$\cos(\theta) = -\frac{5}{13}$

$\tan(\theta) = \frac{12}{5}$

$\csc(\theta) = -\frac{13}{12}$

$\sec(\theta) = -\frac{13}{5}$

$\cot(\theta) = \frac{5}{12}$

Problem 3.3 $\csc(\theta) = \frac{25}{24}$ *with* θ *in Quadrant I*

$\sin(\theta) = \frac{24}{25}$

$\cos(\theta) = \frac{7}{25}$

$\tan(\theta) = \frac{24}{7}$

$\csc(\theta) = \frac{25}{24}$

$\sec(\theta) = \frac{25}{7}$

$\cot(\theta) = \frac{7}{24}$

Problem 3.4 $\sec(\theta) = 7$ *with* θ *in Quadrant IV*

$\sin(\theta) = \frac{-4\sqrt{3}}{7}$

$\cos(\theta) = \frac{1}{7}$

$\tan(\theta) = -4\sqrt{3}$

$\csc(\theta) = -\frac{7\sqrt{3}}{12}$

$\sec(\theta) = 7$

$\cot(\theta) = -\frac{\sqrt{3}}{12}$

Problem 3.5 $\csc(\theta) = -\frac{10\sqrt{91}}{91}$ *with* θ *in Quadrant III*

$\sin(\theta) = -\frac{\sqrt{91}}{10}$

$\cos(\theta) = -\frac{3}{10}$

$\tan(\theta) = \frac{\sqrt{91}}{3}$

$\csc(\theta) = -\frac{10\sqrt{91}}{91}$

$\sec(\theta) = -\frac{10}{3}$

$\cot(\theta) = \frac{3\sqrt{91}}{91}$

Problem 3.6 $\cot(\theta) = -23$ *with* θ *in Quadrant II*

$\sin(\theta) = \frac{\sqrt{530}}{530}$

$\cos(\theta) = -\frac{23\sqrt{530}}{530}$

$\tan(\theta) = -\frac{1}{23}$

$\csc(\theta) = \sqrt{530}$

$\sec(\theta) = -\frac{\sqrt{530}}{23}$

$\cot(\theta) = -23$

Problem 3.7 $\tan(\theta) = -2$ *with* θ *in Quadrant IV.*

$\sin(\theta) = -\frac{2\sqrt{5}}{5}$

$\cos(\theta) = \frac{\sqrt{5}}{5}$

$\tan(\theta) = -2$

$\csc(\theta) = -\frac{\sqrt{5}}{2}$

$\sec(\theta) = \sqrt{5}$

$\cot(\theta) = -\frac{1}{2}$

Problem 3.8 $\sec(\theta) = -4$ *with* θ *in Quadrant II.*

$\sin(\theta) = \frac{\sqrt{15}}{4}$

$\cos(\theta) = -\frac{1}{4}$

$\tan(\theta) = -\sqrt{15}$

$\csc(\theta) = \frac{4\sqrt{15}}{15}$

$\sec(\theta) = -4$

$\cot(\theta) = -\frac{\sqrt{15}}{15}$

Problem 3.9 $\cot(\theta) = \sqrt{5}$ *with* θ *in Quadrant III.*

$\sin(\theta) = -\frac{\sqrt{6}}{6}$

$\cos(\theta) = -\frac{\sqrt{30}}{6}$

$\tan(\theta) = \frac{\sqrt{5}}{5}$

$\csc(\theta) = -\sqrt{6}$

$\sec(\theta) = -\frac{\sqrt{30}}{5}$

$\cot(\theta) = \sqrt{5}$

Problem 3.10 $\cos(\theta) = \frac{1}{3}$ *with* θ *in Quadrant I.*

$\sin(\theta) = \frac{2\sqrt{2}}{3}$

$\cos(\theta) = \frac{1}{3}$

$\tan(\theta) = 2\sqrt{2}$

$\csc(\theta) = \frac{3\sqrt{2}}{4}$

$\sec(\theta) = 3$

$\cot(\theta) = \frac{\sqrt{2}}{4}$

Problem 3.11 $\cot(\theta) = 2$ *with* $0 < \theta < \frac{\pi}{2}$.

$\sin(\theta) = \frac{\sqrt{5}}{5}$

$\cos(\theta) = \frac{2\sqrt{5}}{5}$

$\tan(\theta) = \frac{1}{2}$

$\csc(\theta) = \sqrt{5}$

$\sec(\theta) = \frac{\sqrt{5}}{2}$

$\cot(\theta) = 2$

Problem 3.12 $\csc(\theta) = 5$ *with* $\frac{\pi}{2} < \theta < \pi$.

$\sin(\theta) = \frac{1}{5}$

$\cos(\theta) = -\frac{2\sqrt{6}}{5}$

$\tan(\theta) = -\frac{\sqrt{6}}{12}$

$\csc(\theta) = 5$

$\sec(\theta) = -\frac{5\sqrt{6}}{12}$

$\cot(\theta) = -2\sqrt{6}$

Problem 3.13 $\tan(\theta) = \sqrt{10}$ *with* $\pi < \theta < \frac{3\pi}{2}$.

$\sin(\theta) = -\frac{\sqrt{110}}{11}$

$\cos(\theta) = -\frac{\sqrt{11}}{11}$

$\tan(\theta) = \sqrt{10}$

$\csc(\theta) = -\frac{\sqrt{110}}{10}$

$\sec(\theta) = -\sqrt{11}$

$\cot(\theta) = \frac{\sqrt{10}}{10}$

Problem 3.14 $\sec(\theta) = 2\sqrt{5}$ *with* $\frac{3\pi}{2} < \theta < 2\pi$.

$\sin(\theta) = -\frac{\sqrt{95}}{10}$

$\cos(\theta) = \frac{\sqrt{5}}{10}$

$\tan(\theta) = -\sqrt{19}$

$\csc(\theta) = -\frac{2\sqrt{95}}{19}$

$\sec(\theta) = 2\sqrt{5}$

$\cot(\theta) = -\frac{\sqrt{19}}{19}$

Question 4 *In Exercises* **??** - **??**, *use your calculator to approximate the given value to three decimal places. Make sure your calculator is in the proper angle measurement mode!*

Problem 4.1 $\csc(78.95^\circ)$

$\csc(78.95^\circ) \approx 1.019$

Problem 4.2 $\tan(-2.01)$

$\tan(-2.01) \approx 2.129$

Problem 4.3 $\cot(392.994)$

$\cot(392.994) \approx 3.292$

Problem 4.4 $\sec(207^\circ)$

$\sec(207^\circ) \approx -1.122$

Problem 4.5 $\csc(5.902)$

$\csc(5.902) \approx -2.688$

Problem 4.6 $\tan(39.672^\circ)$

$\tan(39.672^\circ) \approx 0.829$

Problem 4.7 $\cot(3^\circ)$

$\cot(3^\circ) \approx 19.081$

Problem 4.8 $\sec(0.45)$

$\sec(0.45) \approx 1.111$

Question 5 *In Exercises* **??** - **??**, *find all of the angles which satisfy the equation.*

For the problems where you are prompted to type in an answer, type a response in the range $0 \leq \theta < 2\pi$.

Problem 5.1 $\tan(\theta) = \sqrt{3}$

$\theta = \frac{\pi}{3} + \pi k$ *for any integer k*

Problem 5.2 $\sec(\theta) = 2$

$\theta = \frac{\pi}{3} + 2\pi k$ *(Quadrant I) or* $\theta = \frac{5\pi}{3} + 2\pi k$ *(Quadrant III) for any integer k*

Problem 5.3 $\csc(\theta) = -1$

$\theta = \frac{3\pi}{2} + 2\pi k$ *for any integer k.*

Problem 5.4 $\cot(\theta) = \frac{\sqrt{3}}{3}$

$\theta = \frac{\pi}{3} + \pi k$ *(Quadrant I) for any integer k*

EXERCISES FOR THE OTHER CIRCULAR FUNCTIONS

Problem 5.5 $\tan(\theta) = 0$

$\theta = \pi k$ *for any integer* k

Problem 5.6 $\sec(\theta) = 1$

$\theta = 2\pi k$ *for any integer* k

Problem 5.7 $\csc(\theta) = 2$

$\theta = \frac{\pi}{6} + 2\pi k$ *(Quadrant I) or* $\theta = \frac{5\pi}{6} + 2\pi k$ *(Quadrant II) for any integer* k.

Problem 5.8 $\cot(\theta) = 0$

$\theta = \frac{\pi}{2} + \pi k$ *for any integer* k

Problem 5.9 $\tan(\theta) = -1$

$\theta = \frac{3\pi}{4} + \pi k$ *(Quadrant II) for any integer* k

Problem 5.10 $\sec(\theta) = 0$

$\sec(\theta) = 0$ *has no solution.*

Problem 5.11 $\csc(\theta) = -\frac{1}{2}$

$\csc(\theta) = -\frac{1}{2}$ *has no (real) solution.*

Problem 5.12 $\sec(\theta) = -1$

$\theta = \pi + 2\pi k = (2k+1)\pi$ *for any integer* k

Problem 5.13 $\tan(\theta) = -\sqrt{3}$

$\theta = \frac{2\pi}{3} + \pi k$ *(Quadrant II) for any integer* k

Problem 5.14 $\csc(\theta) = -2$

$\theta = \frac{7\pi}{6} + 2\pi k$ *or* $\theta = \frac{11\pi}{6} + 2\pi k$ *for any integer* k

Problem 5.15 $\cot(\theta) = -1$

$\theta = \frac{3\pi}{4} + \pi k$ *(Quadrant II) for any integer* k

Question 6 *In Exercises* **??** - **??**, *solve the equation for* t. *Give exact values.*

Problem 6.1 $\cot(t) = 1$

$t = \frac{\pi}{4} + \pi k$ *for any integer* k

Problem 6.2 $\tan(t) = \frac{\sqrt{3}}{3}$

$t = \frac{\pi}{6} + \pi k$ *for any integer* k

Problem 6.3 $\sec(t) = -\frac{2\sqrt{3}}{3}$

$t = \frac{5\pi}{6} + 2\pi k$ *or* $t = \frac{7\pi}{6} + 2\pi k$ *for any integer* k

Problem 6.4 $\csc(t) = 0$

$\csc(t) = 0$ *has no solution.*

Problem 6.5 $\cot(t) = -\sqrt{3}$

$t = \frac{5\pi}{6} + \pi k$ *for any integer* k

Problem 6.6 $\tan(t) = -\frac{\sqrt{3}}{3}$

$t = \frac{5\pi}{6} + \pi k$ *for any integer* k

Problem 6.7 $\sec(t) = \frac{2\sqrt{3}}{3}$

$t = \frac{\pi}{6} + 2\pi k$ *or* $t = \frac{11\pi}{6} + 2\pi k$ *for any integer* k

Problem 6.8 $\csc(t) = \frac{2\sqrt{3}}{3}$

$t = \frac{\pi}{3} + 2\pi k$ *or* $t = \frac{2\pi}{3} + 2\pi k$ *for any integer* k

Question 7 *In Exercises* **??** *-* **??***, write the given function as a nontrivial decomposition of functions as directed.*

Problem 7.1 *For* $f(t) = 3t^2 + 2\tan(3t)$*, find functions* g *and* h *so that* $f = g + h$*.*

One solution is $g(t) = 3t^2$ *and* $h(t) = 2\tan(3t)$*.*

Problem 7.2 *For* $f(\theta) = \sec(\theta) - \tan(\theta)$*, find functions* g *and* h *so that* $f = g - h$*.*

One solution is $g(\theta) = \sec(\theta)$ *and* $h(\theta) = \tan(\theta)$*.*

Problem 7.3 *For* $f(t) = -\csc(t)\cot(t)$*, find functions* g *and* h *so that* $f = gh$*.*

One solution is $g(t) = -\csc(t)$ *and* $h(t) = \cot(t)$*.*

Problem 7.4 *For* $r(t) = \frac{\tan(3t)}{t}$*, find functions* f *and* g *so* $r = \frac{f}{g}$*.*

One solution is $f(t) = \tan(3t)$ *and* $g(t) = t$*.*

Problem 7.5 *For* $T(\theta) = \tan(4\theta)$*, find functions* f *and* g *so* $T = g \circ f$*.*

One solution is $f(\theta) = 4\theta$ *and* $g(\theta) = \tan(\theta)$*.*

Problem 7.6 *For* $s(\theta) = \sec^2(\theta)$*, find functions* f *and* g *so* $s = g \circ f$*.*

Since $\sec^2(\theta) = (\sec(\theta))^2$*, one solution is* $f(\theta) = \sec(\theta)$ *and* $g(\theta) = \theta^2$*.*

Problem 7.7 *For $L(x) = \ln(\sin(x))$, find functions f and g so $L = g \circ f$.*

One solution is $f(x) = \sin(x)$ and $g(x) = \ln(x)$.

Problem 7.8 *For $\ell(\theta) = \ln|\sec(\theta) - \tan(\theta)|$, find find functions f, g, and h so $\ell = h \circ (f - g)$.*

One solution is $f(\theta) = \sec(\theta)$, $g(\theta) = \tan(\theta)$, and $h(\theta) = \ln|\theta|$.

Problem 8 *Let $S(t) = \sin(t)$ and $C(t) = \cos(t)$, $F(t) = \tan(t)$, and $G(t) = \cot(t)$. Explain why $F = \dfrac{S}{C}$ but $F \neq \dfrac{1}{G}$.*

HINT: Think about domains . . .

Problem 9 *For each function $T(t)$ listed below, compute the average rate of change over the indicated interval.*[1] *What trends do you notice? Compare your answer with what you discovered in Section* **??** *number* **??**. *Be sure your calculator is in radian mode!*

$T(t)$	$[-0.1, 0.1]$	$[-0.01, 0.01]$	$[-0.001, 0.001]$
$\tan(t)$			
$\tan(2t)$			
$\tan(3t)$			
$\tan(4t)$			

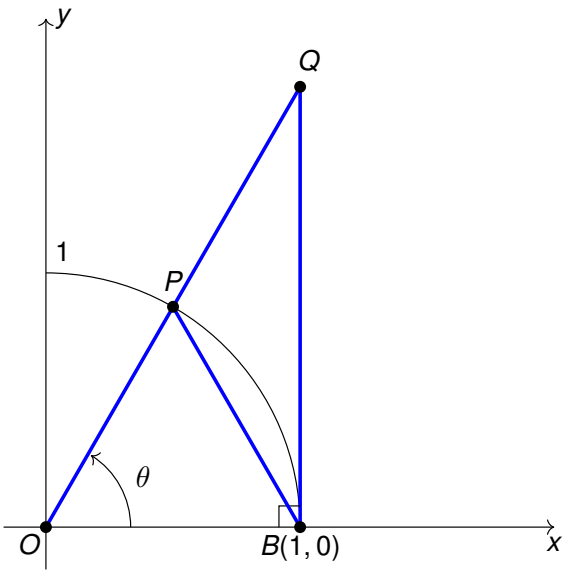
As we zoom in towards 0, the average rate of change of $\tan(kt)$ approaches k. This is the same trend we observed for $\sin(kt)$ in Section **??** *number* **??**.

$T(t)$	$[-0.1, 0.1]$	$[-0.01, 0.01]$	$[-0.001, 0.001]$
$\tan(t)$	≈ 1.0033	≈ 1	≈ 1
$\tan(2t)$	≈ 2.0271	≈ 2.0003	≈ 2
$\tan(3t)$	≈ 3.0933	≈ 3.0009	≈ 3
$\tan(4t)$	≈ 4.2279	≈ 4.0021	≈ 4

Problem 10 *We wish to establish the inequality $\cos(\theta) < \dfrac{\sin(\theta)}{\theta} < 1$ for $0 < \theta < \dfrac{\pi}{2}$. Use the diagram from the beginning of the section, partially reproduced below, to answer the following.*

[1] See Definition **??** in Section **??** for a review of this concept, as needed.

EXERCISES FOR THE OTHER CIRCULAR FUNCTIONS



1. *Show that triangle OPB has area* $\frac{1}{2}\sin(\theta)$ *and triangle OQB has area* $\frac{1}{2}\tan(\theta)$.
2. *Show that the circular sector OPB with central angle* θ *has area* $\frac{1}{2}\theta$.
3. *Comparing areas, show that* $\sin(\theta) < \theta < \tan(\theta)$ *for* $0 < \theta < \frac{\pi}{2}$.
4. *Use the inequality* $\sin(\theta) < \theta$ *to show that* $\frac{\sin(\theta)}{\theta} < 1$ *for* $0 < \theta < \frac{\pi}{2}$.
5. *Use the inequality* $\theta < \tan(\theta)$ *to show that* $\cos(\theta) < \frac{\sin(\theta)}{\theta}$ *for* $0 < \theta < \frac{\pi}{2}$. *Combine this with the previous part to complete the proof.*

Problem 11 *Show that* $\cos(\theta) < \frac{\sin(\theta)}{\theta} < 1$ *also holds for* $-\frac{\pi}{2} < \theta < 0$.

Problem 12 *Use the results from Exercises* **??** *and* **??** *along with the Squeeze Theorem,*[2] *to prove* $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$.

[2] Theorem **??**