Exercises for Function Arithmetic

Question 1 *In Exercises **??** - **??**, use the pair of functions f and g to find each of the following if they exist.*

- $(f+g)(2)$
- $(f-g)(-1)$
- $(g-f)(1)$
- $(fg)\left(\frac{1}{2}\right)$
- $\left(\frac{f}{g}\right)(0)$
- $\left(\frac{g}{f}\right)(-2)$

Problem 1.1 $f(x)=3x+1$ *and* $g(t)=4-t$

- $(f+g)(2)=9$
- $(f-g)(-1)=-7$
- $(g-f)(1)=-1$
- $(fg)\left(\frac{1}{2}\right)=\frac{35}{4}$
- $\left(\frac{f}{g}\right)(0)=\frac{1}{4}$
- $\left(\frac{g}{f}\right)(-2)=-\frac{6}{5}$

Problem 1.2 $f(x)=x^2$ *and* $g(t)=-2t+1$

- $(f+g)(2)=1$
- $(f-g)(-1)=-2$
- $(g-f)(1)=-2$
- $(fg)\left(\frac{1}{2}\right)=0$
- $\left(\frac{f}{g}\right)(0)=0$
- $\left(\frac{g}{f}\right)(-2)=\frac{5}{4}$

Problem 1.3 $f(x)=x^2-x$ *and* $g(t)=12-t^2$

- $(f+g)(2)=10$
- $(f-g)(-1)=-9$
- $(g-f)(1)=11$

- $(fg)\left(\frac{1}{2}\right) = -\frac{47}{16}$
- $\left(\frac{f}{g}\right)(0) = 0$
- $\left(\frac{g}{f}\right)(-2) = \frac{4}{3}$

Problem 1.4 $f(x) = 2x^3$ *and* $g(t) = -t^2 - 2t - 3$

- $(f+g)(2) = 5$
- $(f-g)(-1) = 0$
- $(g-f)(1) = -8$
- $(fg)\left(\frac{1}{2}\right) = -\frac{17}{16}$
- $\left(\frac{f}{g}\right)(0) = 0$
- $\left(\frac{g}{f}\right)(-2) = \frac{3}{16}$

Problem 1.5 $f(x) = \sqrt{x+3}$ *and* $g(t) = 2t - 1$

- $(f+g)(2) = 3 + \sqrt{5}$
- $(f-g)(-1) = 3 + \sqrt{2}$
- $(g-f)(1) = -1$
- $(fg)\left(\frac{1}{2}\right) = 0$
- $\left(\frac{f}{g}\right)(0) = -\sqrt{3}$
- $\left(\frac{g}{f}\right)(-2) = -5$

Problem 1.6 $f(x) = \sqrt{4-x}$ *and* $g(t) = \sqrt{t+2}$

- $(f+g)(2) = 2 + \sqrt{2}$
- $(f-g)(-1) = -1 + \sqrt{5}$
- $(g-f)(1) = 0$
- $(fg)\left(\frac{1}{2}\right) = \frac{\sqrt{35}}{2}$
- $\left(\frac{f}{g}\right)(0) = \sqrt{2}$
- $\left(\frac{g}{f}\right)(-2) = 0$

Problem 1.7 *$f(x) = 2x$ and $g(t) = \dfrac{1}{2t+1}$*

- $(f+g)(2) = \dfrac{21}{5}$
- $(f-g)(-1) = -1$
- $(g-f)(1) = -\dfrac{5}{3}$
- $(fg)\left(\dfrac{1}{2}\right) = \dfrac{1}{2}$
- $\left(\dfrac{f}{g}\right)(0) = 0$
- $\left(\dfrac{g}{f}\right)(-2) = \dfrac{1}{12}$

Problem 1.8 *$f(x) = x^2$ and $g(t) = \dfrac{3}{2t-3}$*

- $(f+g)(2) = 7$
- $(f-g)(-1) = \dfrac{8}{5}$
- $(g-f)(1) = -4$
- $(fg)\left(\dfrac{1}{2}\right) = -\dfrac{3}{8}$
- $\left(\dfrac{f}{g}\right)(0) = 0$
- $\left(\dfrac{g}{f}\right)(-2) = -\dfrac{3}{28}$

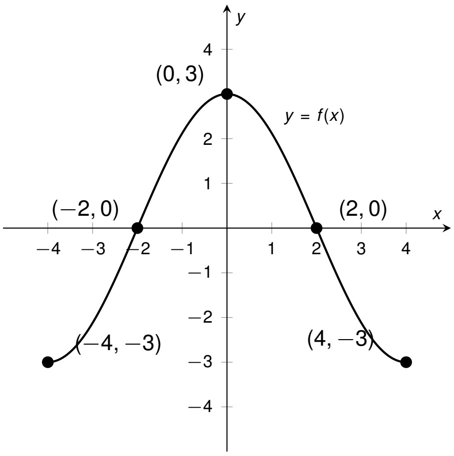
Problem 1.9 *$f(x) = x^2$ and $g(t) = \dfrac{1}{t^2}$*

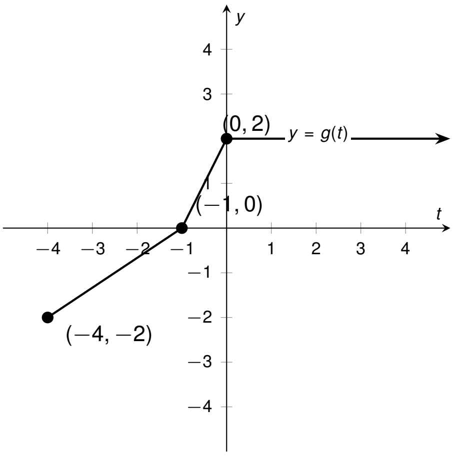
- $(f+g)(2) = \dfrac{17}{4}$
- $(f-g)(-1) = 0$
- $(g-f)(1) = 0$
- $(fg)\left(\dfrac{1}{2}\right) = 1$
- $\left(\dfrac{f}{g}\right)(0)$ *is undefined*
- $\left(\dfrac{g}{f}\right)(-2) = \dfrac{1}{16}$

Problem 1.10 *$f(x) = x^2 + 1$ and $g(t) = \dfrac{1}{t^2+1}$*

- $(f+g)(2) = \frac{26}{5}$
- $(f-g)(-1) = \frac{3}{2}$
- $(g-f)(1) = -\frac{3}{2}$
- $(fg)\left(\frac{1}{2}\right) = 1$
- $\left(\frac{f}{g}\right)(0) = 1$
- $\left(\frac{g}{f}\right)(-2) = \frac{1}{25}$

Question 2 *Exercises **??** - **??** refer to the functions f and g whose graphs are below.*





Problem 2.1 $(f+g)(-4) = -5$

Problem 2.2 $(f+g)(0) = 5$

Problem 2.3 $(f-g)(4) = -5$

Problem 2.4 $(fg)(-4) = 6$

Problem 2.5 $(fg)(-2) = 0$

Problem 2.6 $(fg)(4) = -6$

Problem 2.7 $\left(\frac{f}{g}\right)(0) = \frac{3}{2}$

Problem 2.8 $\left(\frac{f}{g}\right)(2) = 0$

Problem 2.9 $\left(\frac{g}{f}\right)(-1) = 0$

Problem 2.10 *Find the domains of* $f + g$, $f - g$, fg, $\frac{f}{g}$ *and* $\frac{g}{f}$.

The domains of $f + g$, $f - g$ *and* fg *are all* $[-4, 4]$. *The domain of* $\frac{f}{g}$ *is* $[-4, -1) \cup (-1, 4]$ *and the domain of* $\frac{g}{f}$ *is* $[-4, -2) \cup (-2, 2) \cup (2, 4]$.

Question 3 *In Exercises* **??** *-* **??**, *let f be the function defined by*

$$f = \{(-3, 4), (-2, 2), (-1, 0), (0, 1), (1, 3), (2, 4), (3, -1)\}$$

and let g be the function defined by

$$g = \{(-3, -2), (-2, 0), (-1, -4), (0, 0), (1, -3), (2, 1), (3, 2)\}$$

Compute the indicated value if it exists.

Problem 3.1 $(f + g)(-3) = 2$

Problem 3.2 $(f - g)(2) = 3$

Problem 3.3 $(fg)(-1) = 0$

Problem 3.4 $(g + f)(1) = 0$

Problem 3.5 $(g - f)(3) = 3$

Problem 3.6 $(gf)(-3) = -8$

Problem 3.7 $\left(\frac{f}{g}\right)(-2)$

Does not exist.

Problem 3.8 $\left(\frac{f}{g}\right)(-1) = 0$

Problem 3.9 $\left(\frac{f}{g}\right)(2) = 4$

Problem 3.10 $\left(\frac{g}{f}\right)(-1)$

Does not exist.

Problem 3.11 $\left(\frac{g}{f}\right)(3) = -2$

Problem 3.12 $\left(\frac{g}{f}\right)(-3) = -\frac{1}{2}$

Question 4 *In Exercises* **??** *-* **??**, *use the pair of functions f and g to find the domain of the indicated function then find and simplify an expression for it.*

- $(f + g)(x)$

- $(f-g)(x)$
- $(fg)(x)$
- $\left(\frac{f}{g}\right)(x)$

Problem 4.1 $f(x) = 2x + 1$ *and* $g(x) = x - 2$

- $(f+g)(x) = 3x - 1$ *Domain:* $(-\infty, \infty)$
- $(f-g)(x) = x + 3$ *Domain:* $(-\infty, \infty)$
- $(fg)(x) = 2x^2 - 3x - 2$ *Domain:* $(-\infty, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{2x+1}{x-2}$ *Domain:* $(-\infty, 2) \cup (2, \infty)$

Problem 4.2 $f(x) = 1 - 4x$ *and* $g(x) = 2x - 1$

- $(f+g)(x) = -2x$ *Domain:* $(-\infty, \infty)$
- $(f-g)(x) = 2 - 6x$ *Domain:* $(-\infty, \infty)$
- $(fg)(x) = -8x^2 + 6x - 1$ *Domain:* $(-\infty, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{1-4x}{2x-1}$ *Domain:* $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

Problem 4.3 $f(x) = x^2$ *and* $g(x) = 3x - 1$

- $(f+g)(x) = x^2 + 3x - 1$ *Domain:* $(-\infty, \infty)$
- $(f-g)(x) = x^2 - 3x + 1$ *Domain:* $(-\infty, \infty)$
- $(fg)(x) = 3x^3 - x^2$ *Domain:* $(-\infty, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{x^2}{3x-1}$ *Domain:* $\left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$

Problem 4.4 $f(x) = x^2 - x$ *and* $g(x) = 7x$

- $(f+g)(x) = x^2 + 6x$ *Domain:* $(-\infty, \infty)$
- $(f-g)(x) = x^2 - 8x$ *Domain:* $(-\infty, \infty)$
- $(fg)(x) = 7x^3 - 7x^2$ *Domain:* $(-\infty, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{x-1}{7}$ *Domain:* $(-\infty, 0) \cup (0, \infty)$

Problem 4.5 $f(x) = x^2 - 4$ *and* $g(x) = 3x + 6$

- $(f+g)(x) = x^2 + 3x +$ *Domain:* $(-\infty, \infty)$
- $(f-g)(x) = x^2 - 3x - 10$ *Domain:* $(-\infty, \infty)$
- $(fg)(x) = 3x^3 + 6x^2 - 12x - 24$ *Domain:* $(-\infty, \infty)$

- $\left(\frac{f}{g}\right)(x) = \frac{x-2}{3}$ *Domain:* $(-\infty, -2) \cup (-2, \infty)$

Problem 4.6 *$f(x) = -x^2 + x + 6$ and $g(x) = x^2 - 9$*

- $(f+g)(x) = x - 3$ *Domain:* $(-\infty, \infty)$
- $(f-g)(x) = -2x^2 + x + 15$ *Domain:* $(-\infty, \infty)$
- $(fg)(x) = -x^4 + x^3 + 15x^2 - 9x - 54$ *Domain:* $(-\infty, \infty)$
- $\left(\frac{f}{g}\right)(x) = -\frac{x+2}{x+3}$ *Domain:* $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Problem 4.7 *$f(x) = \frac{x}{2}$ and $g(x) = \frac{2}{x}$*

- $(f+g)(x) = \frac{x^2+4}{2x}$ *Domain:* $(-\infty, 0) \cup (0, \infty)$
- $(f-g)(x) = \frac{x^2-4}{2x}$ *Domain:* $(-\infty, 0) \cup (0, \infty)$
- $(fg)(x) = 1$ *Domain:* $(-\infty, 0) \cup (0, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{x^2}{4}$ *Domain:* $(-\infty, 0) \cup (0, \infty)$

Problem 4.8 *$f(x) = x - 1$ and $g(x) = \frac{1}{x-1}$*

- $(f+g)(x) = \frac{x^2-2x+2}{x-1}$ *Domain:* $(-\infty, 1) \cup (1, \infty)$
- $(f-g)(x) = \frac{x^2-2x}{x-1}$ *Domain:* $(-\infty, 1) \cup (1, \infty)$
- $(fg)(x) = 1$ *Domain:* $(-\infty, 1) \cup (1, \infty)$
- $\left(\frac{f}{g}\right)(x) = x^2 - 2x + 1$ *Domain:* $(-\infty, 1) \cup (1, \infty)$

Problem 4.9 *$f(x) = x$ and $g(x) = \sqrt{x+1}$*

- $(f+g)(x) = x + \sqrt{x+1}$ *Domain:* $[-1, \infty)$
- $(f-g)(x) = x - \sqrt{x+1}$ *Domain:* $[-1, \infty)$
- $(fg)(x) = x\sqrt{x+1}$ *Domain:* $[-1, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{x}{\sqrt{x+1}}$ *Domain:* $(-1, \infty)$

Problem 4.10 *$f(x) = \sqrt{x-5}$ and $g(x) = f(x) = \sqrt{x-5}$*

- $(f+g)(x) = 2\sqrt{x-5}$ *Domain:* $[5, \infty)$
- $(f-g)(x) = 0$ *Domain:* $[5, \infty)$

- $(fg)(x) = x - 5$ *Domain:* $[5, \infty)$
- $\left(\dfrac{f}{g}\right)(x) = 1$ *Domain:* $(5, \infty)$

Question 5 *In Exercises* **??** - **??**, *write the given function as a nontrivial decomposition of functions as directed.*

Problem 5.1 *For* $p(z) = 4z - z^3$*, find functions* f *and* g *so that* $p = f - g$*. One solution is* $f(z) = 4z$ *and* $g(z) = z^3$*.*

Problem 5.2 *For* $p(z) = 4z - z^3$*, find functions* f *and* g *so that* $p = f + g$*. One solution is* $f(z) = 4z$ *and* $g(z) = -z^3$*.*

Problem 5.3 *For* $g(t) = 3t|2t - 1|$*, find functions* f *and* h *so that* $g = fh$*. One solution is* $f(t) = 3t$ *and* $h(t) = |2t - 1|$

Problem 5.4 *For* $r(x) = \dfrac{3 - x}{x + 1}$*, find functions* f *and* g *so* $r = \dfrac{f}{g}$*. One solution is* $f(x) = 3 - x$ *and* $g(x) = x + 1$*.*

Problem 5.5 *For* $r(x) = \dfrac{3 - x}{x + 1}$*, find functions* f *and* g *so* $r = fg$*. One solution is* $f(x) = 3 - x$ *and* $g(x) = (x + 1)^{-1}$*.*

Problem 6 *Can* $f(x) = x$ *be decomposed as* $f = g - h$ *where* $g(x) = x + \dfrac{1}{x}$ *and* $h(x) = \dfrac{1}{x}$*? No. The equivalence does not hold when* $x = 0$*.*

Problem 7 *Discuss with your classmates how to phrase the quantities revenue and profit in Definition* **??** *terms of function arithmetic as defined in Definition* **??**.

Problem 8 *In this exercise, we explore decomposing a function into its positive and negative parts. Given a function* f*, we define the* ***positive part*** *of* f*, denoted* f_+ *and* ***negative part*** *of* f*, denoted* f_- *by:*

$$f_+(x) = \frac{f(x) + |f(x)|}{2}, \qquad \text{and} \qquad f_-(x) = \frac{f(x) - |f(x)|}{2}.$$

1. *Using a graphing utility, graph each of the functions* f *below along with* f_+ *and* f_-*.*
   - $f(x) = x - 3$
   - $f(x) = x^2 - x - 6$
   - $f(x) = 4x - x^3$

   Why is f_+ *called the 'positive part' of* f *and* f_- *called the 'negative part' of* f*?*
2. *Show that* $f = f_+ + f_-$*.*
3. *Use Definition* **??** *to rewrite the expressions for* $f_+(x)$ *and* $f_-(x)$ *as piecewise defined functions.*