Exercises for Function Composition

Question 1 *In Exercises **??** - **??**, use the given pair of functions to find the following values if they exist.*

- $(g \circ f)(0)$
- $(f \circ g)(-1)$
- $(f \circ f)(2)$
- $(g \circ f)(-3)$
- $(f \circ g)\left(\frac{1}{2}\right)$
- $(f \circ f)(-2)$

Problem 1.1 $f(x) = x^2$, $g(t) = 2t + 1$

- $(g \circ f)(0) = 1$
- $(f \circ g)(-1) = 1$
- $(f \circ f)(2) = 16$
- $(g \circ f)(-3) = 19$
- $(f \circ g)\left(\frac{1}{2}\right) = 4$
- $(f \circ f)(-2) = 16$

Problem 1.2 $f(x) = 4 - x$, $g(t) = 1 - t^2$

- $(g \circ f)(0)$
  $(g \circ f)(0) = -15$
- $(f \circ g)(-1)$
  $(f \circ g)(-1) = 4$
- $(f \circ f)(2)$
  $(f \circ f)(2) = 2$
- $(g \circ f)(-3)$
  $(g \circ f)(-3) = -48$
- $(f \circ g)\left(\frac{1}{2}\right)$
  $(f \circ g)\left(\frac{1}{2}\right) = \frac{13}{4}$
- $(f \circ f)(-2)$
  $(f \circ f)(-2) = -2$

Problem 1.3 $f(x) = 4 - 3x$, $g(t) = |t|$

- $(g \circ f)(0) = 4$
- $(f \circ g)(-1) = 1$

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- $(f \circ f)(2) = 10$
- $(g \circ f)(-3) = 13$
- $(f \circ g)\left(\frac{1}{2}\right) = \frac{5}{2}$
- $(f \circ f)(-2) = -26$

Problem 1.4 $f(x) = |x-1|$, $g(t) = t^2 - 5$

- $(g \circ f)(0)$

  $(g \circ f)(0) = -4$
- $(f \circ g)(-1)$

  $(f \circ g)(-1) = 5$
- $(f \circ f)(2)$

  $(f \circ f)(2) = 0$
- $(g \circ f)(-3)$

  $(g \circ f)(-3) = 11$
- $(f \circ g)\left(\frac{1}{2}\right)$

  $(f \circ g)\left(\frac{1}{2}\right) = \frac{23}{4}$
- $(f \circ f)(-2)$

  $(f \circ f)(-2) = 2$

Problem 1.5 $f(x) = 4x + 5$, $g(t) = \sqrt{t}$

- $(g \circ f)(0) = \sqrt{5}$
- $(f \circ g)(-1)$

  $(f \circ g)(-1)$ *is not real*
- $(f \circ f)(2) = 57$
- $(g \circ f)(-3)$

  $(g \circ f)(-3)$ *is not real*
- $(f \circ g)\left(\frac{1}{2}\right) = 5 + 2\sqrt{2}$
- $(f \circ f)(-2) = -7$

Problem 1.6 $f(x) = \sqrt{3-x}$, $g(t) = t^2 + 1$

- $(g \circ f)(0)$

  $(g \circ f)(0) = 4$
- $(f \circ g)(-1)$

  $(f \circ g)(-1) = 1$

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- $(f \circ f)(2)$
  $(f \circ f)(2) = \sqrt{2}$
- $(g \circ f)(-3)$
  $(g \circ f)(-3) = 7$
- $(f \circ g)\left(\frac{1}{2}\right)$
  $(f \circ g)\left(\frac{1}{2}\right) = \frac{\sqrt{7}}{2}$
- $(f \circ f)(-2)$
  $(f \circ f)(-2) = \sqrt{3 - \sqrt{5}}$

Problem 1.7 $f(x) = 6 - x - x^2$, $g(t) = t\sqrt{t + 10}$

- $(g \circ f)(0) = 24$
- $(f \circ g)(-1) = 0$
- $(f \circ f)(2) = 6$
- $(g \circ f)(-3) = 0$
- $(f \circ g)\left(\frac{1}{2}\right) = \frac{27 - 2\sqrt{42}}{8}$
- $(f \circ f)(-2) = -14$

Problem 1.8 $f(x) = \sqrt[3]{x + 1}$, $g(t) = 4t^2 - t$

- $(g \circ f)(0)$
  $(g \circ f)(0) = 3$
- $(f \circ g)(-1)$
  $(f \circ g)(-1) = \sqrt[3]{6}$
- $(f \circ f)(2)$
  $(f \circ f)(2) = \sqrt[3]{\sqrt[3]{3} + 1}$
- $(g \circ f)(-3)$
  $(g \circ f)(-3) = 4\sqrt[3]{4} + \sqrt[3]{2}$
- $(f \circ g)\left(\frac{1}{2}\right)$
  $(f \circ g)\left(\frac{1}{2}\right) = \frac{\sqrt[3]{12}}{2}$
- $(f \circ f)(-2)$
  $(f \circ f)(-2) = 0$

Problem 1.9 $f(x) = \frac{3}{1 - x}$, $g(t) = \frac{4t}{t^2 + 1}$

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- $(g \circ f)(0) = \frac{6}{5}$
- $(f \circ g)(-1) = 1$
- $(f \circ f)(2) = \frac{3}{4}$
- $(g \circ f)(-3) = \frac{48}{25}$
- $(f \circ g)\left(\frac{1}{2}\right) = -5$
- $(f \circ f)(-2)$
  $(f \circ f)(-2)$ *is undefined*

Problem 1.10 $f(x) = \dfrac{x}{x+5}$, $g(t) = \dfrac{2}{7-t^2}$

- $(g \circ f)(0)$
  $(g \circ f)(0) = \frac{2}{7}$
- $(f \circ g)(-1)$
  $(f \circ g)(-1) = \frac{1}{16}$
- $(f \circ f)(2)$
  $(f \circ f)(2) = \frac{2}{37}$
- $(g \circ f)(-3)$
  $(g \circ f)(-3) = \frac{8}{19}$
- $(f \circ g)\left(\frac{1}{2}\right)$
  $(f \circ g)\left(\frac{1}{2}\right) = \frac{8}{143}$
- $(f \circ f)(-2)$
  $(f \circ f)(-2) = -\frac{2}{13}$

Problem 1.11 $f(x) = \dfrac{2x}{5-x^2}$, $g(t) = \sqrt{4t+1}$

- $(g \circ f)(0) = 1$
- $(f \circ g)(-1)$
  $(f \circ g)(-1)$ *is not real*
- $(f \circ f)(2) = -\frac{8}{11}$
- $(g \circ f)(-3) = \sqrt{7}$
- $(f \circ g)\left(\frac{1}{2}\right) = \sqrt{3}$

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- $(f \circ f)(-2) = \frac{8}{11}$

Problem 1.12 $f(x) = \sqrt{2x+5}$, $g(t) = \frac{10t}{t^2+1}$

- $(g \circ f)(0)$
  $(g \circ f)(0) = \frac{5\sqrt{5}}{3}$
- $(f \circ g)(-1)$
  $(f \circ g)(-1)$ *is not real*
- $(f \circ f)(2)$
  $(f \circ f)(2) = \sqrt{11}$
- $(g \circ f)(-3)$
  $(g \circ f)(-3)$ *is not real*
- $(f \circ g)\left(\frac{1}{2}\right)$
  $(f \circ g)\left(\frac{1}{2}\right) = \sqrt{13}$
- $(f \circ f)(-2)$
  $(f \circ f)(-2) = \sqrt{7}$

Question 2 *In Exercises* **??** - **??**, *use the given pair of functions to find and simplify expressions for the following functions and state the domain of each using interval notation.*

- $(g \circ f)(x)$
- $(f \circ g)(t)$
- $(f \circ f)(x)$

Problem 2.1 $f(x) = 2x+3$, $g(t) = t^2 - 9$

- $(g \circ f)(x) = 4x^2 + 12x$
  Domain: $(-\infty, \infty)$
- $(f \circ g)(t) = 2t^2 - 15$
  Domain: $(-\infty, \infty)$
- $(f \circ f)(x) = 4x + 9$
  Domain: $(-\infty, \infty)$

Problem 2.2 $f(x) = x^2 - x + 1$, $g(t) = 3t - 5$

- $(g \circ f)(x)$
  $(g \circ f)(x) = 3x^2 - 3x - 2$
  Domain: $(-\infty, \infty)$
- $(f \circ g)(t)$
  $(f \circ g)(t) = 9t^2 - 33t + 31$
  Domain: $(-\infty, \infty)$

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- $(f \circ f)(x)$
  $(f \circ f)(x) = x^4 - 2x^3 + 2x^2 - x + 1$
  Domain: $(-\infty, \infty)$

Problem 2.3 $f(x) = x^2 - 4$, $g(t) = |t|$

- $(g \circ f)(x) = |x^2 - 4|$
  Domain: $(-\infty, \infty)$
- $(f \circ g)(t) = t^2 - 4$
  Domain: $(-\infty, \infty)$
- $(f \circ f)(x) = x^4 - 8x^2 + 12$
  Domain: $(-\infty, \infty)$

Problem 2.4 $f(x) = 3x - 5$, $g(t) = \sqrt{t}$

- $(g \circ f)(x)$
  $(g \circ f)(x) = \sqrt{3x - 5}$
  Domain: $\left[\frac{5}{3}, \infty\right)$
- $(f \circ g)(t)$
  $(f \circ g)(t) = 3\sqrt{t} - 5$
  Domain: $[0, \infty)$
- $(f \circ f)(x)$
  $(f \circ f)(x) = 9x - 20$
  Domain: $(-\infty, \infty)$

Problem 2.5 $f(x) = |x + 1|$, $g(t) = \sqrt{t}$

- $(g \circ f)(x) = \sqrt{|x + 1|}$
  Domain: $(-\infty, \infty)$
- $(f \circ g)(t) = \sqrt{t} + 1$
  Domain: $[0, \infty)$
- $(f \circ f)(x) = |x + 1| + 1$
  Domain: $(-\infty, \infty)$

Problem 2.6 $f(x) = 3 - x^2$, $g(t) = \sqrt{t + 1}$

- $(g \circ f)(x)$
  $(g \circ f)(x) = \sqrt{4 - x^2}$
  Domain: $[-2, 2]$
- $(f \circ g)(t)$
  $(f \circ g)(t) = 2 - t$
  Domain: $[-1, \infty)$

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- $(f \circ f)(x)$

  $(f \circ f)(x) = -x^4 + 6x^2 - 6$

  Domain: $(-\infty, \infty)$

Problem 2.7 $f(x) = |x|$, $g(t) = \sqrt{4-t}$

- $(g \circ f)(x) = \sqrt{4-|x|}$

  Domain: $[-4, 4]$

- $(f \circ g)(t) = \sqrt{4-t}$

  Domain: $(-\infty, 4]$

- $(f \circ f)(x) = |x|$

  Domain: $(-\infty, \infty)$

Problem 2.8 $f(x) = x^2 - x - 1$, $g(t) = \sqrt{t-5}$

- $(g \circ f)(x)$

  $(g \circ f)(x) = \sqrt{x^2 - x - 6}$

  Domain: $(-\infty, -2] \cup [3, \infty)$

- $(f \circ g)(t)$

  $(f \circ g)(t) = t - 6 - \sqrt{t-5}$

  Domain: $[5, \infty)$

- $(f \circ f)(x)$

  $(f \circ f)(x) = x^4 - 2x^3 - 2x^2 + 3x + 1$

  Domain: $(-\infty, \infty)$

Problem 2.9 $f(x) = 3x - 1$, $g(t) = \dfrac{1}{t+3}$

- $(g \circ f)(x) = \dfrac{1}{3x+2}$

  Domain: $\left(-\infty, -\dfrac{2}{3}\right) \cup \left(-\dfrac{2}{3}, \infty\right)$

- $(f \circ g)(t) = -\dfrac{t}{t+3}$

  Domain: $(-\infty, -3) \cup (-3, \infty)$

- $(f \circ f)(x) = 9x - 4$

  Domain: $(-\infty, \infty)$

Problem 2.10 $f(x) = \dfrac{3x}{x-1}$, $g(t) = \dfrac{t}{t-3}$

- $(g \circ f)(x)$

  $(g \circ f)(x) = x$

  Domain: $(-\infty, 1) \cup (1, \infty)$

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- $(f \circ g)(t)$

  $(f \circ g)(t) = t$

  Domain: $(-\infty, 3) \cup (3, \infty)$

- $(f \circ f)(x)$

  $(f \circ f)(x) = \dfrac{9x}{2x+1}$

  Domain: $\left(-\infty, -\dfrac{1}{2}\right) \cup \left(-\dfrac{1}{2}, 1\right) \cup (1, \infty)$

Problem 2.11 $f(x) = \dfrac{x}{2x+1}$, $g(t) = \dfrac{2t+1}{t}$

- $(g \circ f)(x) = \dfrac{4x+1}{x}$

  Domain: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 0) \cup (0, \infty)$

- $(f \circ g)(t) = \dfrac{2t+1}{5t+2}$

  Domain: $(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, 0) \cup (0, \infty)$

- $(f \circ f)(x) = \dfrac{x}{4x+1}$

  Domain: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, -\frac{1}{4}) \cup (-\frac{1}{4}, \infty)$

Problem 2.12 $f(x) = \dfrac{2x}{x^2-4}$, $g(t) = \sqrt{1-t}$

- $(g \circ f)(x)$

  $(g \circ f)(x) = \sqrt{\dfrac{x^2-2x-4}{x^2-4}}$

  Domain: $(-\infty, -2) \cup \left[1-\sqrt{5}, 2\right) \cup \left[1+\sqrt{5}, \infty\right)$

- $(f \circ g)(t)$

  $(f \circ g)(t) = -\dfrac{2\sqrt{1-t}}{t+3}$

  Domain: $(-\infty, -3) \cup (-3, 1]$

- $(f \circ f)(x)$

  $(f \circ f)(x) = \dfrac{4x-x^3}{x^4-9x^2+16}$

  Domain: $\left(-\infty, -\dfrac{1+\sqrt{17}}{2}\right) \cup \left(-\dfrac{1+\sqrt{17}}{2}, -2\right) \cup \left(-2, \dfrac{1-\sqrt{17}}{2}\right) \cup \left(\dfrac{1-\sqrt{17}}{2}, \dfrac{-1+\sqrt{17}}{2}\right) \cup \left(\dfrac{-1+\sqrt{17}}{2}, 2\right) \cup \left(2, \dfrac{1+\sqrt{17}}{2}\right) \cup \left(\dfrac{1+\sqrt{17}}{2}, \infty\right)$

Question 3 *In Exercises* **??** - **??**, *use* $f(x) = -2x$, $g(t) = \sqrt{t}$ *and* $h(s) = |s|$ *to find and simplify expressions for the following functions and state the domain of each using interval notation.*

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Problem 3.1 $(h \circ g \circ f)(x)$

$(h \circ g \circ f)(x) = |\sqrt{-2x}| = \sqrt{-2x}$, *domain:* $(-\infty, 0]$

Problem 3.2 $(h \circ f \circ g)(t)$

$(h \circ f \circ g)(t) = |-2\sqrt{t}| = 2\sqrt{t}$, *domain:* $[0, \infty)$

Problem 3.3 $(g \circ f \circ h)(s)$

$(g \circ f \circ h)(s) = \sqrt{-2|s|}$, *domain:* $\{0\}$

Problem 3.4 $(g \circ h \circ f)(x)$

$(g \circ h \circ f)(x) = \sqrt{|-2x|} = \sqrt{2|x|}$, *domain:* $(-\infty, \infty)$

Problem 3.5 $(f \circ h \circ g)(t)$

$(f \circ h \circ g)(t) = -2|\sqrt{t}| = -2\sqrt{t}$, *domain:* $[0, \infty)$

Problem 3.6 $(f \circ g \circ h)(s)$

$(f \circ g \circ h)(s) = -2\sqrt{|s|}$, , *domain:* $(-\infty, \infty)$

Question 4 *In Exercises* **??** - **??**, *let* f *be the function defined by*

$$f = \{(-3,4),(-2,2),(-1,0),(0,1),(1,3),(2,4),(3,-1)\}$$

and let g *be the function defined by*

$$g = \{(-3,-2),(-2,0),(-1,-4),(0,0),(1,-3),(2,1),(3,2)\}.$$

Find the following, if it exists.

Problem 4.1 $(f \circ g)(3)$

$(f \circ g)(3) = f(g(3)) = f(2) = 4$

Problem 4.2 $f(g(-1))$

$f(g(-1)) = f(-4)$ *which is undefined*

Problem 4.3 $(f \circ f)(0)$

$(f \circ f)(0) = f(f(0)) = f(1) = 3$

Problem 4.4 $(f \circ g)(-3)$

$(f \circ g)(-3) = f(g(-3)) = f(-2) = 2$

Problem 4.5 $(g \circ f)(3)$

$(g \circ f)(3) = g(f(3)) = g(-1) = -4$

Problem 4.6 $g(f(-3))$

$g(f(-3)) = g(4)$ *which is undefined*

Problem 4.7 $(g \circ g)(-2)$

$(g \circ g)(-2) = g(g(-2)) = g(0) = 0$

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Problem 4.8 $(g \circ f)(-2)$

$(g \circ f)(-2) = g(f(-2)) = g(2) = 1$

Problem 4.9 $g(f(g(0)))$

$g(f(g(0))) = g(f(0)) = g(1) = -3$

Problem 4.10 $f(f(f(-1)))$

$f(f(f(-1))) = f(f(0)) = f(1) = 3$

Problem 4.11 $f(f(f(f(f(1)))))$

$f(f(f(f(f(1))))) = f(f(f(f(3)))) =$
$f(f(f(-1))) = f(f(0)) = f(1) = 3$

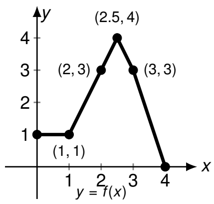
Problem 4.12 $\underbrace{(g \circ g \circ \cdots \circ g)}_{n \text{ times}}(0)$

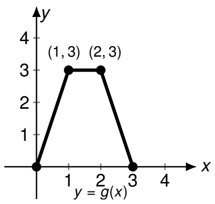
$\underbrace{(g \circ g \circ \cdots \circ g)}_{n \text{ times}}(0) = 0$

Problem 4.13 *Find the domain and range of $f \circ g$ and $g \circ f$.*

- *The domain of $f \circ g$ is $\{-3, -2, 0, 1, 2, 3\}$ and the range of $f \circ g$ is $\{1, 2, 3, 4\}$.*
- *The domain of $g \circ f$ is $\{-2, -1, 0, 1, 3\}$ and the range of $g \circ f$ is $\{-4, -3, 0, 1, 2\}$.*

Question 5 *In Exercises **??** - **??**, use the graphs of $y = f(x)$ and $y = g(x)$ below to find the following if it exists.*





Problem 5.1 $(g \circ f)(1)$

$(g \circ f)(1) = 3$

Problem 5.2 $(f \circ g)(3)$

$(f \circ g)(3) = 1$

Problem 5.3 $(g \circ f)(2)$

$(g \circ f)(2) = 0$

Problem 5.4 $(f \circ g)(0)$

$(f \circ g)(0) = 1$

Problem 5.5 $(f \circ f)(4)$

$(f \circ f)(4) = 1$

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Problem 5.6 $(g \circ g)(1)$

$(g \circ g)(1) = 0$

Problem 5.7 *Find the domain and range of $f \circ g$ and $g \circ f$.*

- *The domain of $f \circ g$ is $[0, 3]$ and the range of $f \circ g$ is $[1, 4.5]$.*
- *The domain of $g \circ f$ is $[0, 2] \cup [3, 4]$ and the range is $[0, 3]$.*

Question 6 *In Exercises **??** - **??**, write the given function as a composition of two or more non-identity functions. (There are several correct answers, so check your answer using function composition.)*

Problem 6.1 $p(x) = (2x + 3)^3$

Let $f(x) = 2x + 3$ and $g(x) = x^3$, then $p(x) = (g \circ f)(x)$.

Problem 6.2 $P(x) = \left(x^2 - x + 1\right)^5$

Let $f(x) = x^2 - x + 1$ and $g(x) = x^5$, $P(x) = (g \circ f)(x)$.

Problem 6.3 $h(t) = \sqrt{2t - 1}$

Let $f(t) = 2t - 1$ and $g(t) = \sqrt{t}$, then $h(t) = (g \circ f)(t)$.

Problem 6.4 $H(t) = |7 - 3t|$

Let $f(t) = 7 - 3t$ and $g(t) = |t|$, then $H(t) = (g \circ f)(t)$.

Problem 6.5 $r(s) = \dfrac{2}{5s + 1}$

Let $f(s) = 5s + 1$ and $g(s) = \dfrac{2}{s}$, then $r(s) = (g \circ f)(s)$.

Problem 6.6 $R(s) = \dfrac{7}{s^2 - 1}$

Let $f(s) = s^2 - 1$ and $g(s) = \dfrac{7}{s}$, then $R(s) = (g \circ f)(s)$.

Problem 6.7 $q(z) = \dfrac{|z| + 1}{|z| - 1}$

Let $f(z) = |z|$ and $g(z) = \dfrac{z + 1}{z - 1}$, then $q(z) = (g \circ f)(z)$.

Problem 6.8 $Q(z) = \dfrac{2z^3 + 1}{z^3 - 1}$

Let $f(z) = z^3$ and $g(z) = \dfrac{2z + 1}{z - 1}$, then $Q(z) = (g \circ f)(z)$.

Problem 6.9 $v(x) = \dfrac{2x + 1}{3 - 4x}$

Let $f(x) = 2x$ and $g(x) = \dfrac{x + 1}{3 - 2x}$, then $v(x) = (g \circ f)(x)$.

Problem 6.10 $w(x) = \dfrac{x^2}{x^4+1}$

Let $f(x) = x^2$ *and* $g(x) = \dfrac{x}{x^2+1}$, *then* $w(x) = (g \circ f)(x)$.

Problem 7 *Write the function* $F(x) = \sqrt{\dfrac{x^3+6}{x^3-9}}$ *as a composition of three or more non-identity functions.*

$F(x) = \sqrt{\dfrac{x^3+6}{x^3-9}} = (h(g(f(x)))$ *where* $f(x) = x^3$, $g(x) = \dfrac{x+6}{x-9}$ *and* $h(x) = \sqrt{x}$.

Problem 8 *Let* $g(x) = -x$, $h(x) = x+2$, $j(x) = 3x$ *and* $k(x) = x-4$. *In what order must these functions be composed with* $f(x) = \sqrt{x}$ *to create* $F(x) = 3\sqrt{-x+2} - 4$?

$F(x) = 3\sqrt{-x+2} - 4 = k(j(f(h(g(x)))))$

Problem 9 *What linear functions could be used to transform* $f(x) = x^3$ *into* $F(x) = -\frac{1}{2}(2x-7)^3 + 1$? *What is the proper order of composition?*

One solution is $F(x) = -\frac{1}{2}(2x-7)^3 + 1 = k(j(f(h(g(x)))))$ *where* $g(x) = 2x$, $h(x) = x - 7$, $j(x) = -\frac{1}{2}x$ *and* $k(x) = x + 1$. *You could also have* $F(x) = H(f(G(x)))$ *where* $G(x) = 2x - 7$ *and* $H(x) = -\frac{1}{2}x + 1$.

Problem 10 *Let* $f(x) = 3x+1$ *and let* $g(x) = \begin{cases} 2x-1 & \text{if } x \leq 3 \\ 4-x & \text{if } x > 3 \end{cases}$. *Find expressions for* $(f \circ g)(x)$ *and* $(g \circ f)(x)$.

$(f \circ g)(x) = \begin{cases} 6x-2 & \text{if } x \leq 3 \\ 13-3x & \text{if } x > 3 \end{cases}$ *and* $(g \circ f)(x) = \begin{cases} 6x+1 & \text{if } x \leq \frac{2}{3} \\ 3-3x & \text{if } x > \frac{2}{3} \end{cases}$

Problem 11 *The volume* V *of a cube is a function of its side length* x. *Let's assume that* $x = t + 1$ *is also a function of time* t, *where* x *is measured in inches and* t *is measured in minutes. Find a formula for* V *as a function of* t.

$V(x) = x^3$ *so* $V(x(t)) = (t+1)^3$

Problem 12 *Suppose a local vendor charges* \$2 *per hot dog and that the number of hot dogs sold per hour* x *is given by* $x(t) = -4t^2 + 20t + 92$, *where* t *is the number of hours since* 10 *AM,* $0 \leq t \leq 4$.

1. *Find an expression for the revenue per hour* R *as a function of* x.

   $R(x) = 2x$

2. *Find and simplify* $(R \circ x)(t)$. *What does this represent?*

   $(R \circ x)(t) = -8t^2 + 40t + 184$, $0 \leq t \leq 4$. *This gives the revenue per hour as a function of time.*

3. *What is the revenue per hour at noon?*

   Noon corresponds to $t = 2$, *so* $(R \circ x)(2) = 232$. *The hourly revenue at noon is* \$232 *per hour.*

4. *Using Example* **??** *as a guide, verify* $\dfrac{\Delta[R(x)]}{\Delta x} \cdot \dfrac{\Delta[x(t)]}{\Delta t} = \dfrac{\Delta[R(t)]}{\Delta t}$.

   $\dfrac{\Delta[R(x)]}{\Delta x} = 2$, $\dfrac{\Delta[x(t)]}{\Delta t} = -8t + 4\Delta t + 20$.

   $\dfrac{\Delta[R(x)]}{\Delta x} \cdot \dfrac{\Delta[x(t)]}{\Delta t} = (2)(-8t + 4\Delta t + 20) = -16t + 8\Delta t + 40 = \dfrac{\Delta[R(t)]}{\Delta t}$ ✓

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EXERCISES FOR FUNCTION COMPOSITION

Problem 13 *The book in Example **??** plunges into a lake and generates a circular wave pattern. If the waves are tracked as traveling at a constant* 0.5 *meters per second* $\left(\frac{m}{s}\right)$*, use Theorem **??** to find the rate at which the area of the disturbance is changing with respect to time as the radius changes from* $r = 1$ *to* $r = 1.1$ *meters (m). Be sure to include units on your answer.*

Hint: *Recall the area, A, enclosed by a circle of radius r is given by* $A = \pi r^2$*. Here,* $\frac{\Delta r}{\Delta t} = 0.5\,\frac{m}{s}$*.*

$\frac{\Delta A}{\Delta t} = \frac{\Delta A}{\Delta r} \cdot \frac{\Delta r}{\Delta t}$. $\frac{\Delta A}{\Delta r} = \frac{A(1.1) - A(1)}{1.1 - 1} = \frac{\pi(1.1)^2 - \pi(1)^2}{0.1} = 2.1\,\frac{m^2}{m}$, $\frac{\Delta r}{\Delta t} = 0.5\,\frac{m}{s}$.
Hence, $\frac{\Delta A}{\Delta t} = \left(2.1\,\frac{m^2}{m}\right)\left(0.5\,\frac{m}{s}\right) = 1.05\,\frac{m^2}{s}$

Problem 14 *Perfectly fine precalculus textbooks which have no Calculus content are being fed into a shredder at a rate of 2 books per minute in order to make room for precalculus textbooks with Calculus content. The shredder creates a pile of debris which is in the shape of a right circular cone whose height is twice its width.*

1. *Assume the volume of the conical pile, V, is given by* $V = \frac{1}{3}\pi r^2 h$ *where r is the radius of the base of the pile and h is the height of the pile. Given the pile is twice as tall as it is wide, show we can write* $V = \frac{4}{3}\pi r^3$*.*
   The 'width' of the pile is the diameter of the circular base of the pile. Since the diameter of a circle is twice the radius, $h = 2(2r) = 4r$*. Hence,* $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(4r) = \frac{4}{3}\pi r^3$*.*

2. *Assuming a typical precalculus textbook is* 0.10 *cubic feet* $\left(ft^3\right)$*, use Theorem **??** to find the rate of change of the radius of the pile with respect to time as the radius changes from* 2 *to* 2.1 *feet. Be sure to include units on your answer.*
   We have $\frac{\Delta V}{\Delta t} = \frac{\Delta V}{\Delta r} \cdot \frac{\Delta r}{\Delta t}$*. Two textbooks per minute into the shredder amounts to the volume of the cone increasing at a rate of* $2(0.1) = 0.2\,\frac{ft^3}{min}$. $\frac{\Delta V}{\Delta r} = \frac{V(2.1) - V(2)}{2.1 - 2} = 16.81\overline{3}\,\pi\,\frac{ft^3}{ft}$.
   Hence, $0.2\,\frac{ft^3}{min} = 16.81\overline{3}\,\pi\,\frac{ft^3}{ft}\,\frac{\Delta r}{\Delta t}$ *so* $\frac{\Delta r}{\Delta t} = \frac{0.2}{16.81\overline{3}\,\pi} \approx 0.0038\,\frac{ft}{min}$.

Problem 15 *Discuss with your classmates how 'real-world' processes such as filling out federal income tax forms or computing your final course grade could be viewed as a use of function composition. Find a process for which composition with itself (iteration) makes sense.*

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