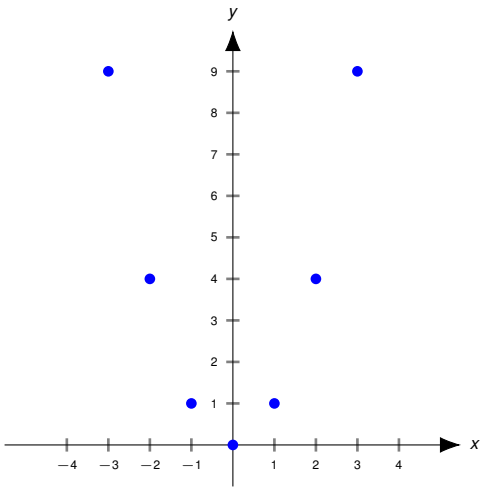
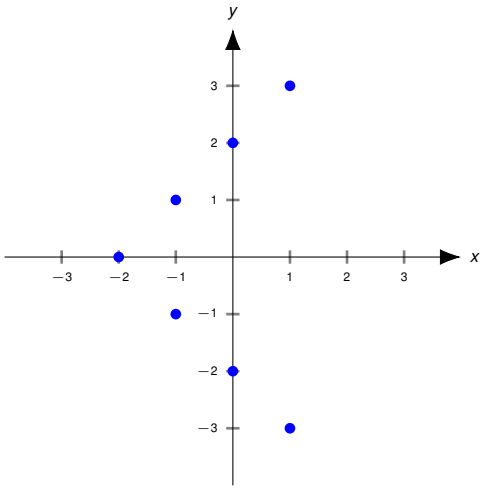
EXERCISES FOR RELATIONS

In Exercises **??** - **??**, graph the given relation in the *xy*-plane.

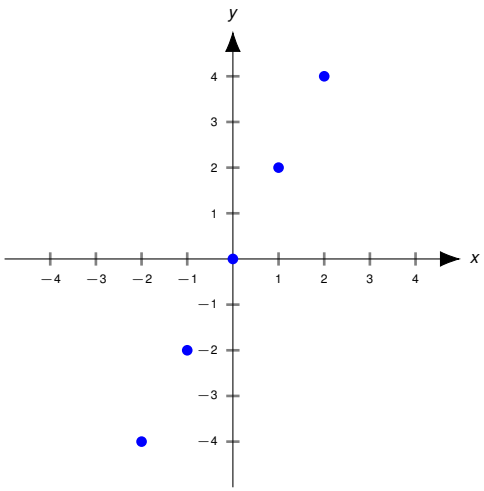
Problem 1 $\{(-3,9),\ (-2,4),\ (-1,1),\ (0,0),\ (1,1),\ (2,4),\ (3,9)\}$



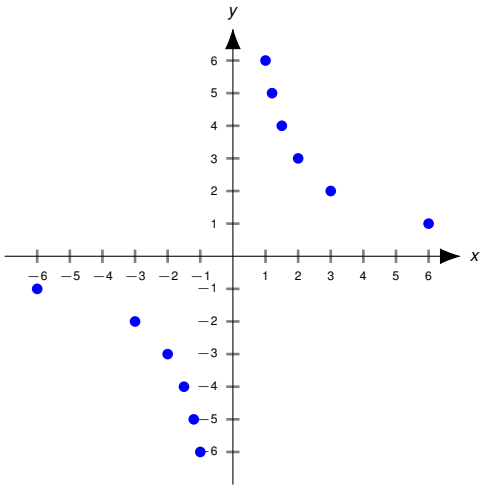
Problem 2 $\{(-2,0),\ (-1,1),\ (-1,-1),\ (0,2),\ (0,-2),\ (1,3),\ (1,-3)\}$



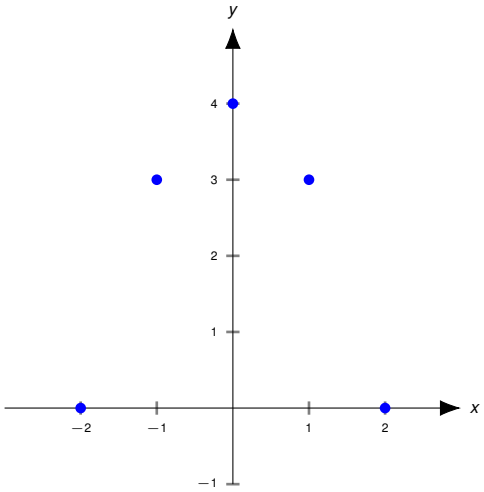
Problem 3 $\{(m,2m) \mid m = 0, \pm 1, \pm 2\}$



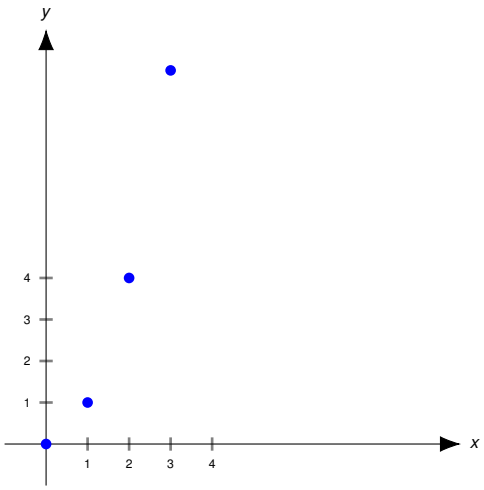
Problem 4 $\left\{\left(\frac{6}{k}, k\right) \mid k = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6\right\}$



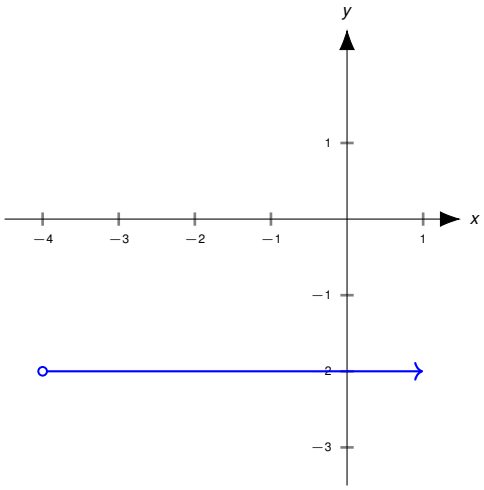
Problem 5 $\left\{\left(n, 4 - n^2\right) \mid n = 0, \pm 1, \pm 2\right\}$



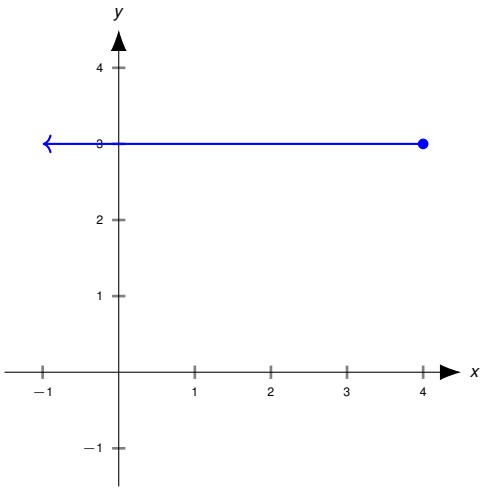
Problem 6 $\left\{\left(\sqrt{j}, j\right) \mid j = 0, 1, 4, 9\right\}$



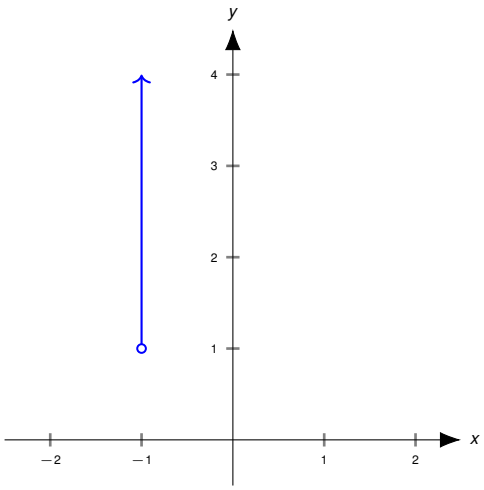
Problem 7 $\{(x, -2) \mid x > -4\}$



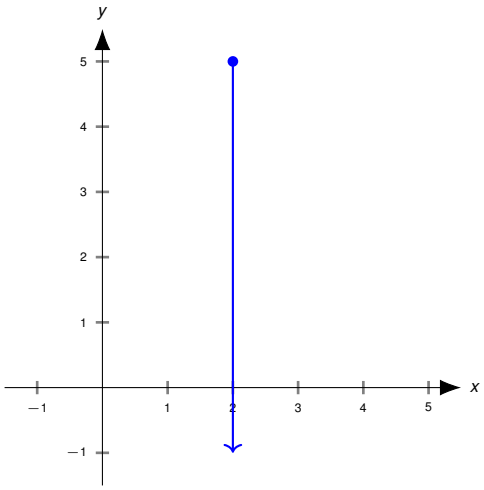
Problem 8 $\{(x, 3) \mid x \leq 4\}$



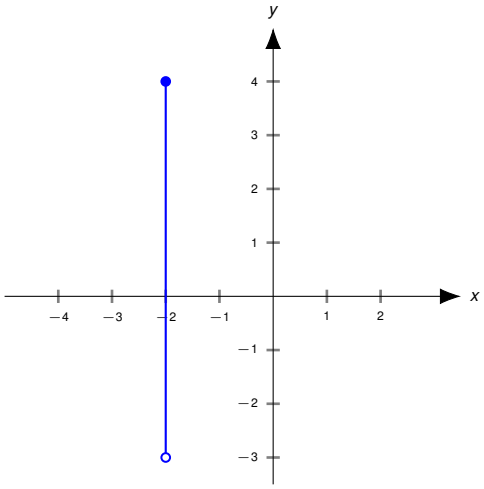
Problem 9 $\{(-1, y) \mid y > 1\}$



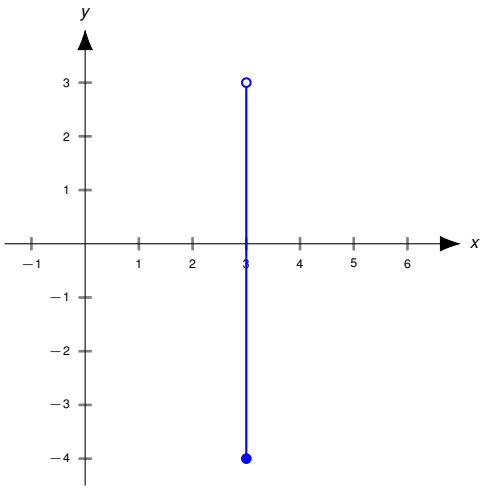
Problem 10 $\{(2, y) \mid y \leq 5\}$



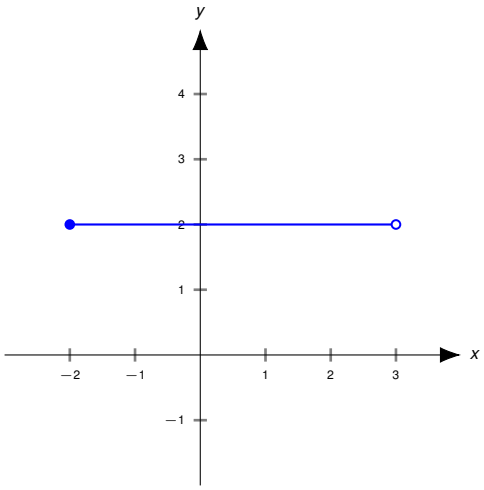
Problem 11 $\{(-2, y) \mid -3 < y \leq 4\}$



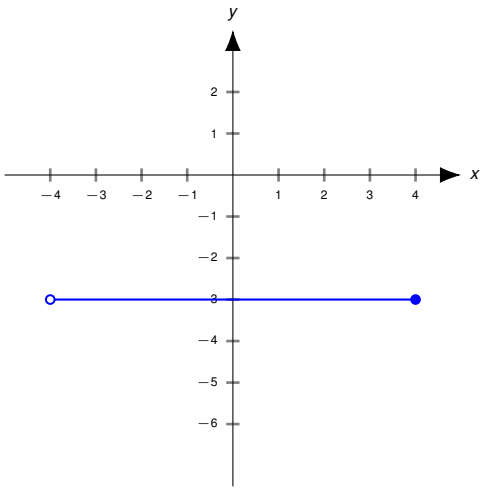
Problem 12 $\{(3, y) \mid -4 \leq y < 3\}$



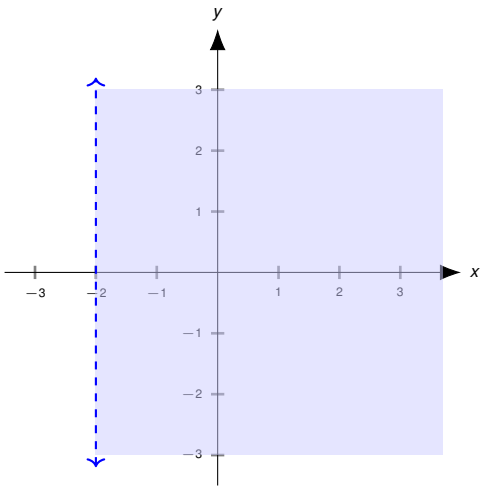
Problem 13 $\{(x, 2) \mid -2 \leq x < 3\}$



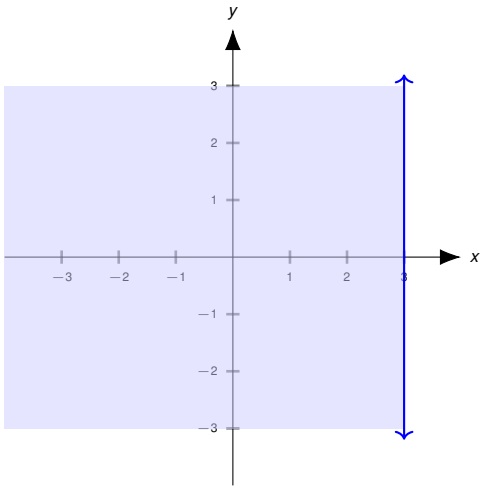
Problem 14 $\{(x, -3) \mid -4 < x \leq 4\}$



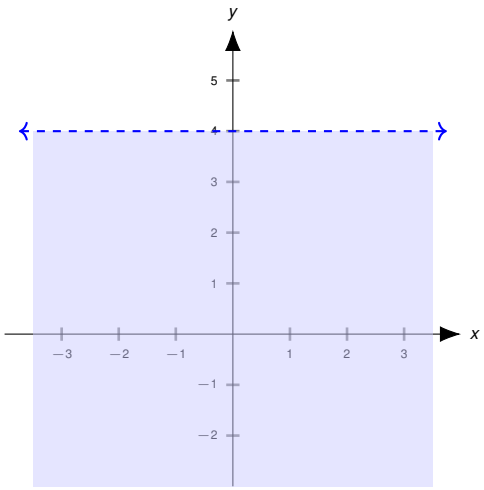
Problem 15 $\{(x, y) \mid x > -2\}$



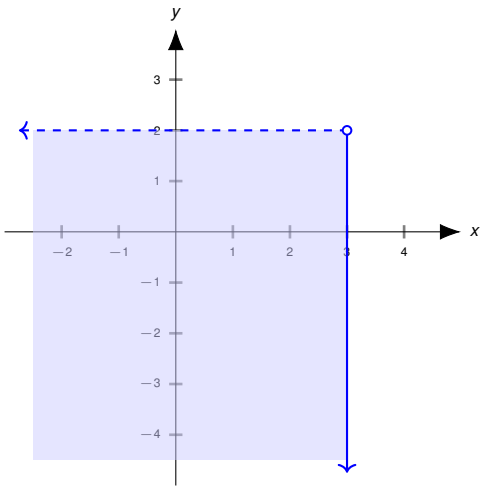
Problem 16 $\{(x, y) \mid x \leq 3\}$



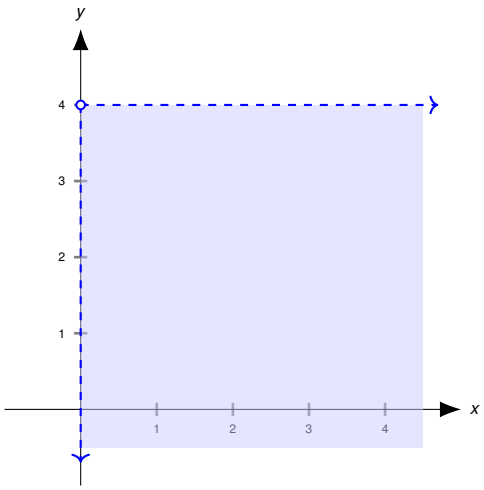
Problem 17 $\{(x, y) \mid y < 4\}$



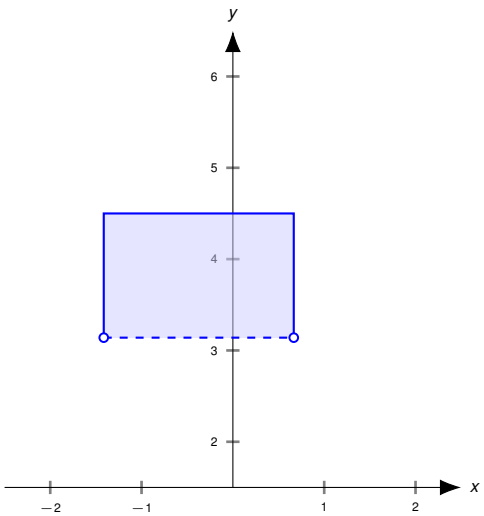
Problem 18 $\{(x, y) \mid x \leq 3,\ y < 2\}$



Problem 19 $\{(x, y) \mid x > 0,\ y < 4\}$

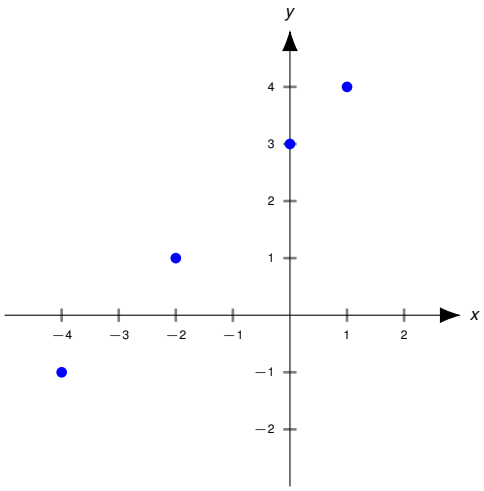


Problem 20 $\{(x, y) \mid -\sqrt{2} \leq x \leq \frac{2}{3},\ \pi < y \leq \frac{9}{2}\}$



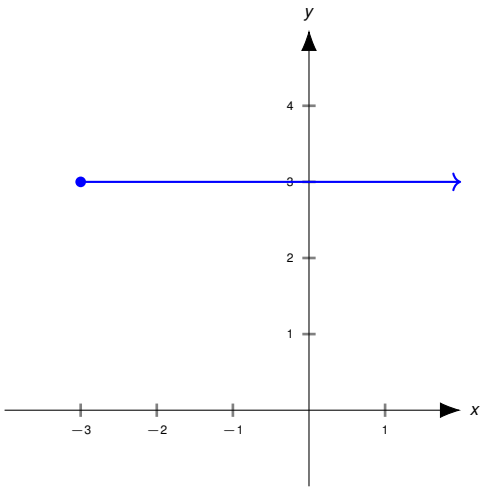
In Exercises **??** - **??**, describe the given relation using either the roster or set-builder method.

Problem 21 *Relation A:*



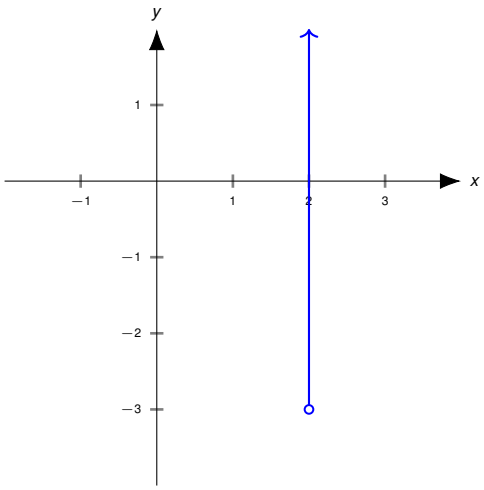
$A = \{(-4,-1),(-2,1),(0,3),(1,4)\}$

Problem 22 *Relation B:*



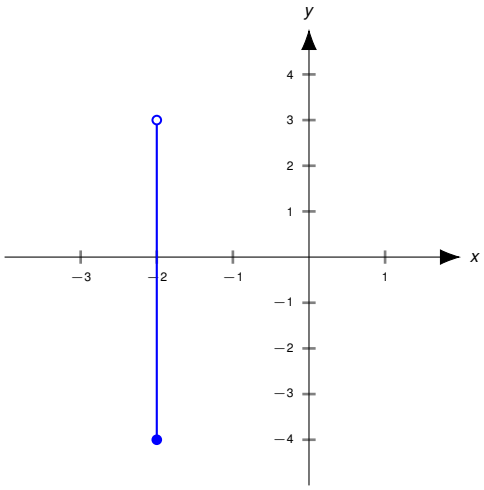
$B = \{(x,3) \mid x \geq -3\}$

Problem 23 *Relation C:*



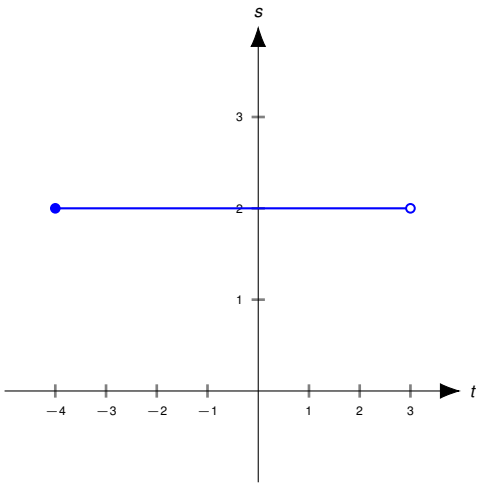
$C = \{(2, y) \mid y > -3\}$

Problem 24 *Relation D:*



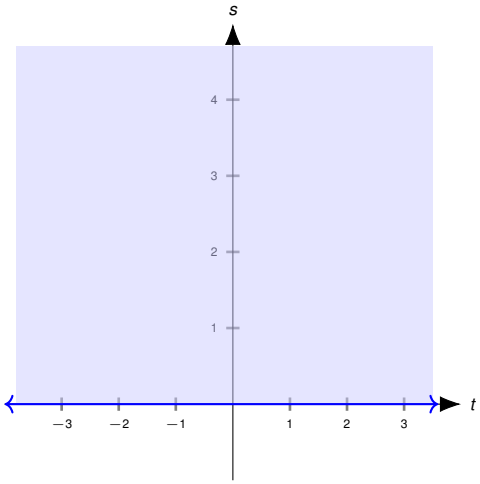
$D = \{(-2, y) \mid -4 \leq y < 3\}$

Problem 25 *Relation E:*



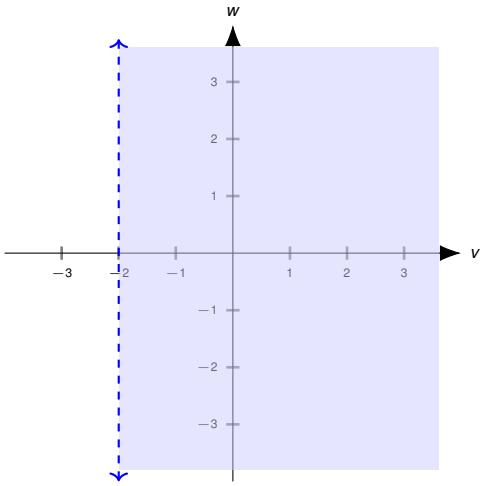
$E = \{(t, 2) \mid -4 \leq t < 3\}$

Problem 26 *Relation F:*



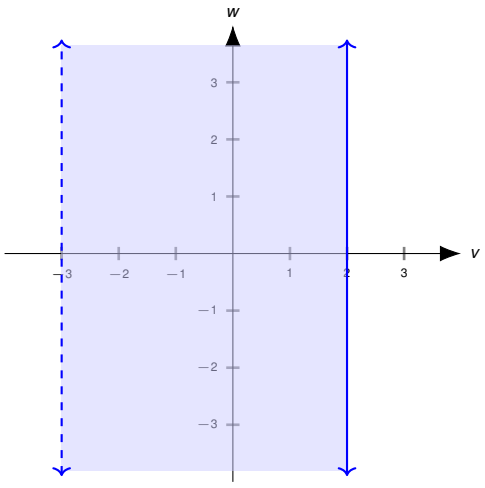
$F = \{(t, s) \mid s \geq 0\}$

Problem 27 *Relation G:*



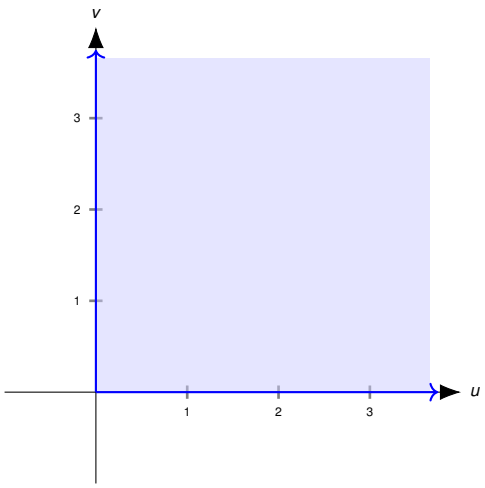
$G = \{(v, w) \mid v > -2\}$

Problem 28 *Relation H:*



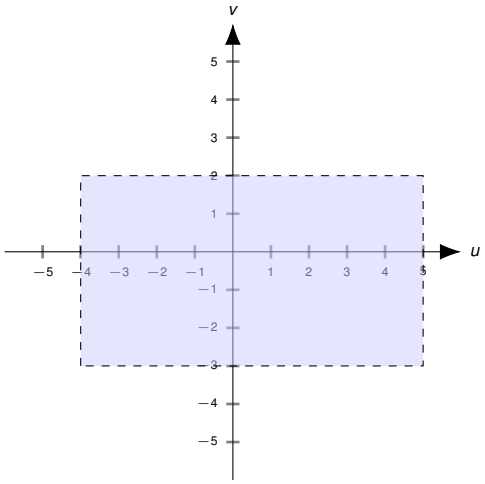
$H = \{(v, w) \mid -3 < v \leq 2\}$

Problem 29 *Relation I:*



$I = \{(u, v) \mid u \geq 0, v \geq 0\}$

Problem 30 *Relation J:*



$J = \{(u, v) \mid -4 < u < 5,\ -3 < v < 2\}$

EXERCISES FOR RELATIONS

Some relations are fairly easy to describe in words or with the roster method but are rather difficult, if not impossible, to graph. Discuss with your classmates how you might graph the relations given in Exercises **??** - **??**. Note that in the notation below we are using the ellipsis, '. . . ,' to denote that the list does not end, but rather, continues to follow the established pattern indefinitely.

For the relations in Exercises **??** and **??**, give two examples of points which belong to the relation and two points which do not belong to the relation.

Problem 31 $\{(x, y) \mid x \text{ is an odd integer, and } y \text{ is an even integer.}\}$

Problem 32 $\{(x, 1) \mid x \text{ is an irrational number }\}$

Problem 33 $\{(1, 0), (2, 1), (4, 2), (8, 3), (16, 4), (32, 5), ...\}$

Problem 34 $\{..., (-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), ...\}$

For each equation given in Exercises **??** - **??**:

- Graph the equation in the xy-plane by creating a table of points.
- Find the axis intercepts, if they exist.
- Test the equation for symmetry. If the equation fails a symmetry test, find a point on the graph of the equation whose symmetric point is not on the graph of the equation.
- Determine if the equation describes y as a function of x. If not, describe the graph of the equation using two or more explicit functions of x. Check your answers using a graphing utility.

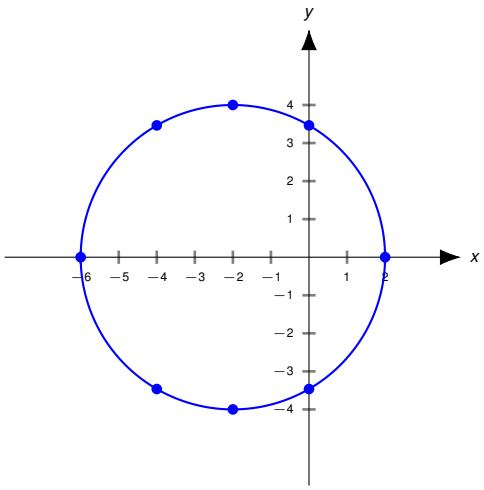
Problem 35 $(x + 2)^2 + y^2 = 16$

Re-write as $y = \pm\sqrt{16 - (x + 2)^2}$.

x-intercepts: $(-6, 0)$, $(2, 0)$

y-intercepts: $\left(0, \pm 2\sqrt{3}\right)$

x	y	(x, y)
-6	0	$(-6, 0)$
-4	$\pm 2\sqrt{3}$	$\left(-4, \pm 2\sqrt{3}\right)$
-2	± 4	$(-2, \pm 4)$
0	$\pm 2\sqrt{3}$	$\left(0, \pm 2\sqrt{3}\right)$
2	0	$(2, 0)$



The graph is symmetric about the x-axis

The graph is not symmetric about the y-axis: $(-6, 0)$ *is on the graph but* $(6, 0)$ *is not.*

The graph is not symmetric about the origin: $(-6, 0)$ *is on the graph but* $(6, 0)$ *is not.*

The equation does not describe y as a function of x.

The graph of the equation is the graphs of $f_1(x) = \sqrt{16 - (x+2)^2}$ *together with* $f_2(x) = -\sqrt{16 - (x+2)^2}$.

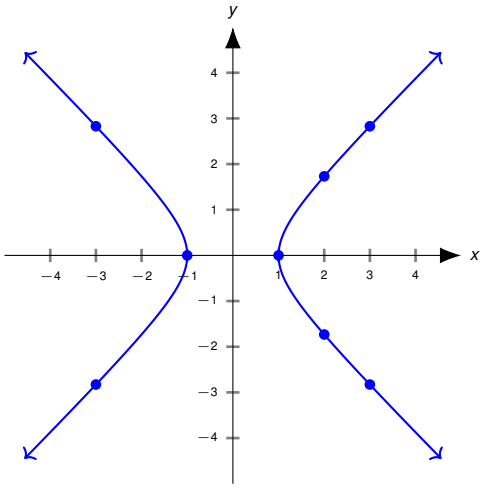
Problem 36 $x^2 - y^2 = 1$

Re-write as: $y = \pm\sqrt{x^2 - 1}$.

x-intercepts: $(-1, 0), (1, 0)$

The graph has no y-intercepts

x	y	(x, y)
-3	$\pm\sqrt{8}$	$(-3, \pm\sqrt{8})$
-2	$\pm\sqrt{3}$	$(-2, \pm\sqrt{3})$
-1	0	$(-1, 0)$
1	0	$(1, 0)$
2	$\pm\sqrt{3}$	$(2, \pm\sqrt{3})$
3	$\pm\sqrt{8}$	$(3, \pm\sqrt{8})$



The graph is symmetric about the x-axis.

The graph is symmetric about the y-axis.

The graph is symmetric about the origin.

The equation does not describe y as a function of x.

The graph of the equation is the graphs of $f_1(x) = \sqrt{x^2 - 1}$ *together with* $f_2(x) = -\sqrt{x^2 - 1}$.

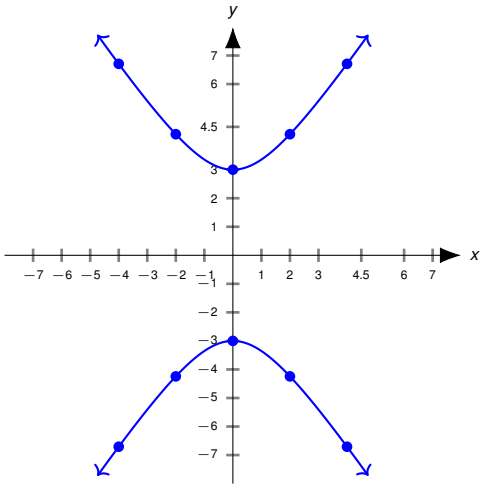
Problem 37 $4y^2 - 9x^2 = 36$

Re-write as: $y = \pm\dfrac{\sqrt{9x^2 + 36}}{2}$.

The graph has no x-intercepts

y-intercepts: $(0, \pm 3)$

x	y	(x, y)
-4	$\pm 3\sqrt{5}$	$\left(-4, \pm 3\sqrt{5}\right)$
-2	$\pm 3\sqrt{2}$	$\left(-2, \pm 3\sqrt{2}\right)$
0	± 3	$(0, \pm 3)$
2	$\pm 3\sqrt{2}$	$\left(2, \pm 3\sqrt{2}\right)$
4	$\pm 3\sqrt{5}$	$\left(4, \pm 3\sqrt{5}\right)$



The graph is symmetric about the x-axis.

The graph is symmetric about the y-axis.

The graph is symmetric about the origin.

The equation does not describe y as a function of x.

The graph of the equation is the graphs of $f_1(x) = \dfrac{\sqrt{9x^2 + 36}}{2}$ *together with* $f_2(x) = -\dfrac{\sqrt{9x^2 + 36}}{2}$.

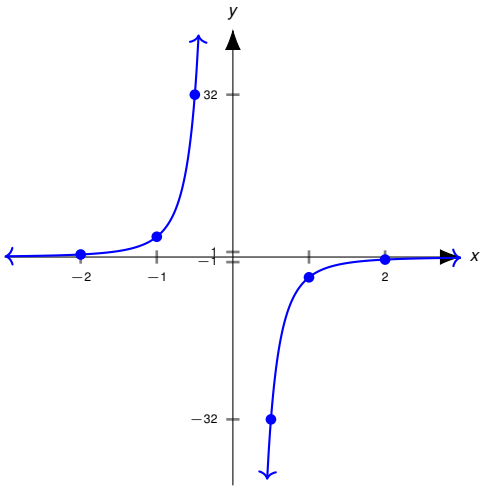
Problem 38 $x^3 y = -4$

Re-write as: $y = -\dfrac{4}{x^3} = -4x^{-3}$.

The graph has no x-intercepts

The graph has no y-intercepts

x	y	(x, y)
-2	$\frac{1}{2}$	$(-2, \frac{1}{2})$
-1	4	$(-1, 4)$
$-\frac{1}{2}$	32	$(-\frac{1}{2}, 32)$
$\frac{1}{2}$	-32	$(\frac{1}{2}, -32)$
1	-4	$(1, -4)$
2	$-\frac{1}{2}$	$(2, -\frac{1}{2})$



The graph is not symmetric about the x-axis: $(1, -4)$ *is on the graph but* $(1, 4)$ *is not.*

The graph is not symmetric about the y-axis: $(1, -4)$ *is on the graph but* $(-1, -4)$ *is not.*

The graph is symmetric about the origin.

The equation does describe y as a function of x, namely $y = f(x) = -4x^{-3}$.

For each equation given in Exercises **??** - **??**:

- Graph the equation in the vw-plane by creating a table of points.
- Find the axis intercepts, if they exist.
- Test the equation for symmetry. If the equation fails a symmetry test, find a point on the graph of the equation whose symmetric point is not on the graph of the equation.
- Determine if the equation describes w as a function of v. If not, describe the graph of the equation using two or more explicit functions of v. Check your answers using a graphing utility.

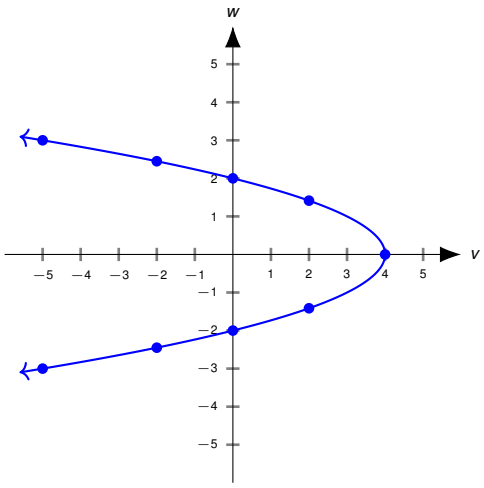
Problem 39 $v + w^2 = 4$

Re-write as $w = \pm\sqrt{4 - v}$.

v-intercept: $(4, 0)$

w-intercepts: $(0, \pm 2)$

v	w	(v, w)
-5	± 3	$(-5, \pm 3)$
-2	$\pm\sqrt{6}$	$\left(-2, \pm\sqrt{6}\right)$
0	± 2	$(0, \pm 2)$
2	$\pm\sqrt{2}$	$\left(1, \pm\sqrt{3}\right)$
4	0	$(4, 0)$



The graph is symmetric about the v-axis

The graph is not symmetric about the w-axis: $(4, 0)$ *is on the graph but* $(-4, 0)$ *is not.*

The graph is not symmetric about the origin: $(4, 0)$ *is on the graph but* $(-4, 0)$ *is not.*

The equation does not describe w as a function of v.

The graph of the equation is the graphs of $f_1(v) = \sqrt{4 - v}$ *together with* $f_2(v) = -\sqrt{4 - v}$.

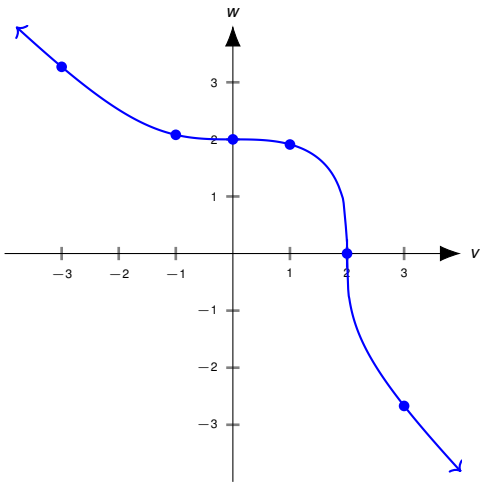
Problem 40 $v^3 + w^3 = 8$

Re-write as: $w = \sqrt[3]{8 - v^3}$.

v-intercept: $(2, 0)$

w-intercept: $(0, 2)$

v	w	(v, w)
-3	$\sqrt[3]{35}$	$(-3, \sqrt[3]{35})$
-1	$\sqrt[3]{9}$	$(-1, \sqrt[3]{9})$
0	2	$(0, 2)$
1	$\sqrt[3]{7}$	$(1, \sqrt[3]{7})$
2	0	$(2, 0)$
3	$-\sqrt[3]{19}$	$(3, -\sqrt[3]{19})$



The graph is not symmetric about the v-axis: $(0,2)$ *is on the graph but* $(0,-2)$ *is not.*

The graph is not symmetric about the w-axis: $(2,0)$ *is on the graph but* $(-2,0)$ *is not.*

The graph is not symmetric about the origin: $(0,2)$ *is on the graph but* $(0,-2)$ *is not.*

The equation does describe w as a function of v, namely $w = f(v) = \sqrt[3]{8 - v^3}$.

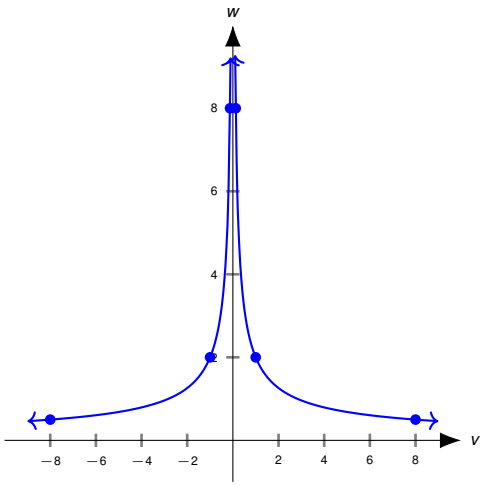
Problem 41 $v^2 w^3 = 8$

Re-write as $w = \dfrac{2}{\sqrt[3]{v^2}} = 2v^{-\frac{2}{3}}$.

The graph has no v-intercepts.

The graph has no w-intercepts.

v	w	(v, w)
-8	$\frac{1}{2}$	$\left(-8, \frac{1}{2}\right)$
-1	2	$(-1, 2)$
$-\frac{1}{8}$	8	$\left(-\frac{1}{8}, 8\right)$
$\frac{1}{8}$	8	$\left(\frac{1}{8}, 8\right)$
1	2	$(1, 2)$
8	$\frac{1}{2}$	$\left(8, \frac{1}{2}\right)$



The graph is not symmetric about the v-axis: $(-1, 2)$ *is on the graph but* $(-1, -2)$ *is not.*

The graph is symmetric about the w-axis.

The graph is not symmetric about the origin: $(-1, 2)$ *is on the graph but* $(-1, -2)$ *is not.*

The equation does describe w as a function of v, namely $w = f(v) = 2v^{-\frac{2}{3}}$.

Problem 42 $v^4 - 2v^2w + w^2 = 16$

HINT: $v^4 - 2v^2w + w^2 = \left(v^2 - w\right)^2 \ldots$

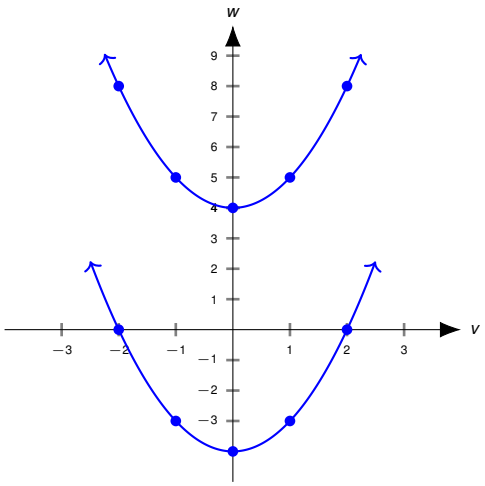
Re-write as: $\left(v^2 - w\right)^2 = 16$

Extracting square roots gives: $w = v^2 + 4$ *and* $w = v^2 - 4$

v-intercepts: $(-2, 0), (2, 0)$

w-intercepts: $(0, -4), (0, 4)$

v	w	(v, w)
-2	8	$(-2, 8)$
-2	0	$(-2, 0)$
-1	5	$(-1, 5)$
-1	-3	$(-1, -3)$
0	± 4	$(0, \pm 4)$
1	5	$(1, 5)$
1	-3	$(1, -3)$
2	8	$(2, 8)$
2	0	$(2, 0)$



The graph is not symmetric about the v-axis: $(1, 5)$ *is on the graph but* $(1, -5)$ *is not.*

The graph is symmetric about the w-axis.

The graph is not symmetric about the origin: $(1, 5)$ *is on the graph but* $(-1, -5)$ *is not.*

The equation does not describe w as a function of v.

The graph of the equation is the graphs of $f_1(v) = v^2 + 4$ *together with* $f_2(v) = v^2 - 4$.

The procedures which we have outlined in the Examples of this section and used in Exercises **??** - **??** all rely on the fact that the equations were "well-behaved". Not everything in Mathematics is quite so tame, as the following equations will show you. Discuss with your classmates how you might approach graphing the equations given in Exercises **??** - **??**. What difficulties arise when trying to apply the various tests and procedures given in this section? For more information, including pictures of the curves, each curve name is a link to its page at www.wikipedia.org. For a much longer list of fascinating curves, click here.

Problem 43 $x^3 + y^3 - 3xy = 0$ *Folium of Descartes*

Problem 44 $x^4 = x^2 + y^2$ *Kampyle of Eudoxus*

Problem 45 $y^2 = x^3 + 3x^2$ *Tschirnhausen cubic*

Problem 46 $(x^2 + y^2)^2 = x^3 + y^3$ *Crooked egg*

Problem 47 *With the help of your classmates, find examples of equations whose graphs possess*

- *symmetry about the x-axis only*
- *symmetry about the y-axis only*
- *symmetry about the origin only*
- *symmetry about the x-axis, y-axis, and origin*
- *Can you find an example of an equation whose graph possesses exactly two of the symmetries listed above? Why or why not?*