

EXERCISES FOR TRANSFORMATIONS

Question 1 Suppose $(2, -3)$ is on the graph of $y = f(x)$. In Exercises ?? - ??, use Theorem ?? to find a point on the graph of the given transformed function.

Problem 1.1 $y = f(x) + 3$

$(2, 0)$

Problem 1.2 $y = f(x + 3)$

$(-1, -3)$

Problem 1.3 $y = f(x) - 1$

$(2, -4)$

Problem 1.4 $y = f(x - 1)$

$(3, -3)$

Problem 1.5 $y = 3f(x)$

$(2, -9)$

Problem 1.6 $y = f(3x)$

$\left(\frac{2}{3}, -3\right)$

Problem 1.7 $y = -f(x)$

$(2, 3)$

Problem 1.8 $y = f(-x)$

$(-2, -3)$

Problem 1.9 $y = f(x - 3) + 1$

$(5, -2)$

Problem 1.10 $y = 2f(x + 1)$

$(1, -6)$

Problem 1.11 $y = 10 - f(x)$

$(2, 13)$

Problem 1.12 $y = 3f(2x) - 1$

$y = (1, -10)$

Problem 1.13 $y = \frac{1}{2}f(4 - x)$

$\left(2, -\frac{3}{2}\right)$

Problem 1.14 $y = 5f(2x + 1) + 3$

$\left(\frac{1}{2}, -12\right)$

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Problem 1.15 $y = 2f(1 - x) - 1$

$(-1, -7)$

Problem 1.16 $y = f\left(\frac{7 - 2x}{4}\right)$

$\left(-\frac{1}{2}, -3\right)$

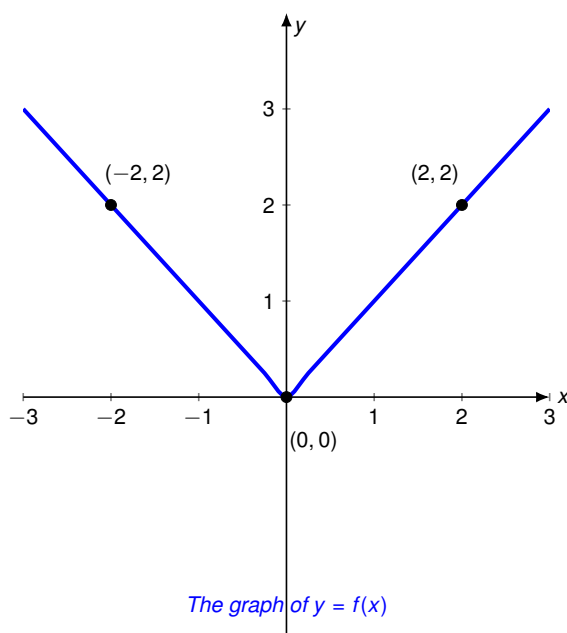
Problem 1.17 $y = \frac{f(3x) - 1}{2}$

$\left(\frac{2}{3}, -2\right)$

Problem 1.18 $y = \frac{4 - f(3x - 1)}{7}$

$(1, 1)$

Question 2 The complete graph of $y = f(x)$ is given below. In Exercises ?? - ??, use it and Theorem ?? to graph the given transformed function.



Problem 2.1 $y = f(x) + 1$

Use the Desmos graph below with the settings $f(x) = |x|$, $h = 0$, $k = 1$, $a = 1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/vgtibem8ig>

Problem 2.2 $y = f(x) - 2$

Use the Desmos graph below with the settings $f(x) = |x|$, $h = 0$, $k = -2$, $a = 1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/vgtibem8ig>

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Problem 2.3 $y = f(x + 1)$

Use the Desmos graph below with the settings $f(x) = |x|$, $h = -1$, $k = 0$, $a = 1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/vgtibem8ig>

Problem 2.4 $y = f(x - 2)$

Use the Desmos graph below with the settings $f(x) = |x|$, $h = 2$, $k = 0$, $a = 1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/vgtibem8ig>

Problem 2.5 $y = 2f(x)$

Use the Desmos graph below with the settings $f(x) = |x|$, $h = 0$, $k = 0$, $a = 2$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/vgtibem8ig>

Problem 2.6 $y = f(2x)$

Use the Desmos graph below with the settings $f(x) = |x|$, $h = 0$, $k = 0$, $a = 1$, $b = 2$

Desmos link: <https://www.desmos.com/calculator/vgtibem8ig>

Problem 2.7 $y = 2 - f(x)$

Use the Desmos graph below with the settings $f(x) = |x|$, $h = 0$, $k = 2$, $a = -1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/vgtibem8ig>

Problem 2.8 $y = f(2 - x)$

Use the Desmos graph below with the settings $f(x) = |x|$, $h = -2$, $k = 0$, $a = 1$, $b = -1$

Desmos link: <https://www.desmos.com/calculator/vgtibem8ig>

Problem 2.9 $y = 2 - f(2 - x)$

Use the Desmos graph below with the settings $f(x) = |x|$, $h = -2$, $k = 2$, $a = -1$, $b = -1$

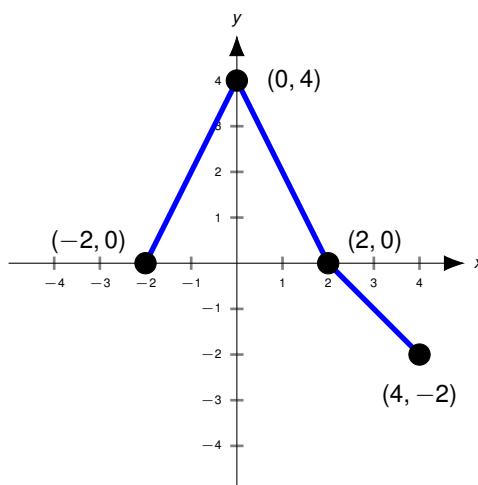
Desmos link: <https://www.desmos.com/calculator/vgtibem8ig>

Problem 3 Some of the answers to Exercises ?? - ?? above should be the same. Which ones match up? What properties of the graph of $y = f(x)$ contribute to the duplication?

Problem 4 The function f used in Exercises ?? - ?? should look familiar. What is $f(x)$? How does this explain some of the duplication in the answers to Exercises ?? - ?? mentioned in Exercise ???

Question 5 The complete graph of $y = g(t)$ is given below. In Exercises ?? - ??, use it and Theorem ?? to graph the given transformed function.

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Problem 5.1 $y = g(t) - 1$

Use the Desmos graph below with the settings $h = 0$, $k = -1$, $a = 1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/bbgtvjhnpc>

Problem 5.2 $y = g(t + 1)$

Use the Desmos graph below with the settings $h = 0$, $k = -1$, $a = 1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/bbgtvjhnpc>

Problem 5.3 $y = \frac{1}{2}g(t)$

Use the Desmos graph below with the settings $h = 0$, $k = 0$, $a = \frac{1}{2}$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/bbgtvjhnpc>

Problem 5.4 $y = g(2t)$

Use the Desmos graph below with the settings $h = 0$, $k = 0$, $a = 1$, $b = 2$

Desmos link: <https://www.desmos.com/calculator/bbgtvjhnpc>

Problem 5.5 $y = -g(t)$

Use the Desmos graph below with the settings $h = 0$, $k = 0$, $a = -1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/bbgtvjhnpc>

Problem 5.6 $y = g(-t)$

Use the Desmos graph below with the settings $h = 0$, $k = 0$, $a = 1$, $b = -1$

Desmos link: <https://www.desmos.com/calculator/bbgtvjhnpc>

Problem 5.7 $y = g(t + 1) - 1$

Use the Desmos graph below with the settings $h = -1$, $k = -1$, $a = 1$, $b = 1$

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Desmos link: <https://www.desmos.com/calculator/bbgtvjhnpc>

Problem 5.8 $y = 1 - g(t)$

Use the Desmos graph below with the settings $h = 0$, $k = 1$, $a = -1$, $b = 1$

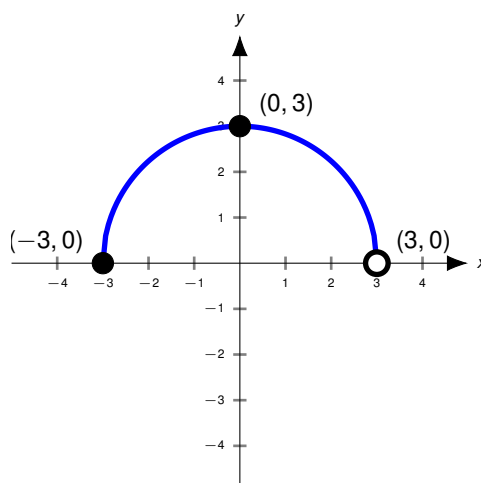
Desmos link: <https://www.desmos.com/calculator/bbgtvjhnpc>

Problem 5.9 $y = \frac{1}{2}g(t+1) - 1$

Use the Desmos graph below with the settings $h = -1$, $k = -1$, $a = \frac{1}{2}$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/bbgtvjhnpc>

Question 6 The complete graph of $y = f(x)$ is given below. In Exercises ?? - ??, use it and Theorem ?? to graph the given transformed function.



Problem 6.1 $g(x) = f(x) + 3$

Use the Desmos graph below with the settings $h = 0$, $k = 3$, $a = 1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/jcwoolfhykb>

Problem 6.2 $h(x) = f(x) - \frac{1}{2}$

Use the Desmos graph below with the settings $h = 0$, $k = -\frac{1}{2}$, $a = 1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/jcwoolfhykb>

Problem 6.3 $j(x) = f\left(x - \frac{2}{3}\right)$

Use the Desmos graph below with the settings $h = 0$, $k = \frac{2}{3}$, $a = 1$, $b = 1$

Desmos link: <https://www.desmos.com/calculator/jcwoolfhykb>

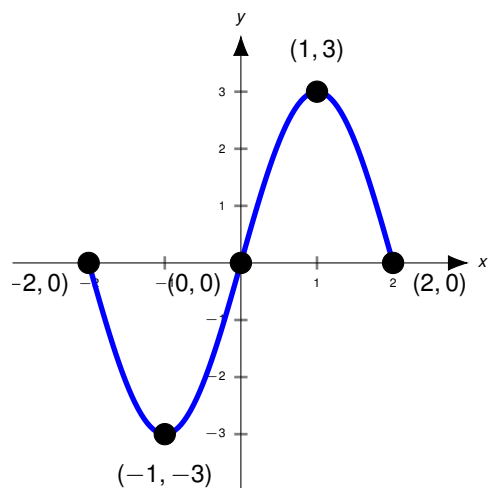
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Problem 6.4 $a(x) = f(x + 4)$ Use the Desmos graph below with the settings $h = -4$, $k = 0$, $a = 1$, $b = 1$ Desmos link: <https://www.desmos.com/calculator/jcwoffhykb>**Problem 6.5** $b(x) = f(x + 1) - 1$ Use the Desmos graph below with the settings $h = -1$, $k = -1$, $a = 1$, $b = 1$ Desmos link: <https://www.desmos.com/calculator/jcwoffhykb>**Problem 6.6** $c(x) = \frac{3}{5}f(x)$ Use the Desmos graph below with the settings $h = 0$, $k = 0$, $a = \frac{3}{5}$, $b = 1$ Desmos link: <https://www.desmos.com/calculator/jcwoffhykb>**Problem 6.7** $d(x) = -2f(x)$ Use the Desmos graph below with the settings $h = 0$, $k = 0$, $a = -2$, $b = 1$ Desmos link: <https://www.desmos.com/calculator/jcwoffhykb>**Problem 6.8** $k(x) = f\left(\frac{2}{3}x\right)$ Use the Desmos graph below with the settings $h = 0$, $k = 0$, $a = 1$, $b = \frac{2}{3}$ Desmos link: <https://www.desmos.com/calculator/jcwoffhykb>**Problem 6.9** $m(x) = -\frac{1}{4}f(3x)$ Use the Desmos graph below with the settings $h = 0$, $k = 0$, $a = -\frac{1}{4}$, $b = 3$ Desmos link: <https://www.desmos.com/calculator/jcwoffhykb>**Problem 6.10** $n(x) = 4f(x - 3) - 6$ Use the Desmos graph below with the settings $h = 3$, $k = -6$, $a = 4$, $b = 1$ Desmos link: <https://www.desmos.com/calculator/jcwoffhykb>**Problem 6.11** $p(x) = 4 + f(1 - 2x)$ Use the Desmos graph below with the settings $h = -1$, $k = 4$, $a = 1$, $b = -2$ Desmos link: <https://www.desmos.com/calculator/jcwoffhykb>**Problem 6.12** $q(x) = -\frac{1}{2}f\left(\frac{x+4}{2}\right) - 3$ Use the Desmos graph below with the settings $h = -2$, $k = -3$, $a = -\frac{1}{2}$, $b = \frac{1}{2}$

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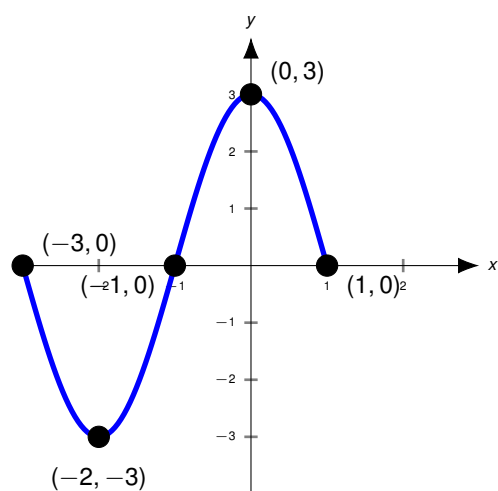
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Question 7 The complete graph of $y = S(t)$ is given below.



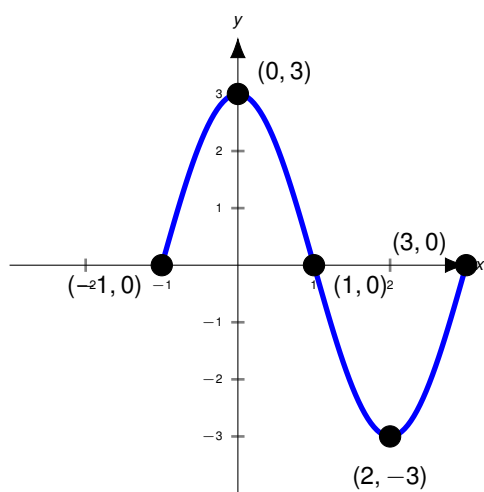
The purpose of Exercises ?? - ?? is to build up to the graph of $y = \frac{1}{2}S(-t + 1) + 1$ one step at a time.

Problem 7.1 $y = S_1(t) = S(t + 1)$

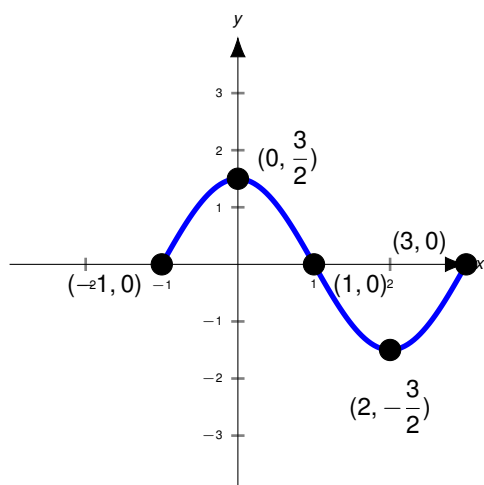


Problem 7.2 $y = S_2(t) = S_1(-t) = S(-t + 1)$

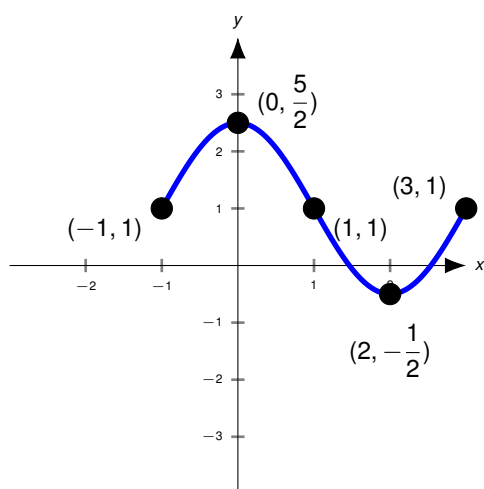
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Problem 7.3 $y = S_3(t) = \frac{1}{2} S_2(t) = \frac{1}{2} S(-t + 1)$



Problem 7.4 $y = S_4(t) = S_3(t) + 1 = \frac{1}{2} S(-t + 1) + 1$



Question 8 Let $f(x) = \sqrt{x}$. Find a formula for a function g whose graph is obtained from f from the given

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sequence of transformations.

Problem 8.1 (1) shift right 2 units; (2) shift down 3 units

$$g(x) = \sqrt{x-2} - 3$$

Problem 8.2 (1) shift down 3 units; (2) shift right 2 units

$$g(x) = \sqrt{x-2} - 3$$

Problem 8.3 (1) reflect across the x -axis; (2) shift up 1 unit

$$g(x) = -\sqrt{x} + 1$$

Problem 8.4 (1) shift up 1 unit; (2) reflect across the x -axis

$$g(x) = -(\sqrt{x} + 1) = -\sqrt{x} - 1$$

Problem 8.5 (1) shift left 1 unit; (2) reflect across the y -axis; (3) shift up 2 units

$$g(x) = \sqrt{-x+1} + 2$$

Problem 8.6 (1) reflect across the y -axis; (2) shift left 1 unit; (3) shift up 2 units

$$g(x) = \sqrt{-(x+1)} + 2 = \sqrt{-x-1} + 2$$

Problem 8.7 (1) shift left 3 units; (2) vertical stretch by a factor of 2; (3) shift down 4 units

$$g(x) = 2\sqrt{x+3} - 4$$

Problem 8.8 (1) shift left 3 units; (2) shift down 4 units; (3) vertical stretch by a factor of 2

$$g(x) = 2(\sqrt{x+3} - 4) = 2\sqrt{x+3} - 8$$

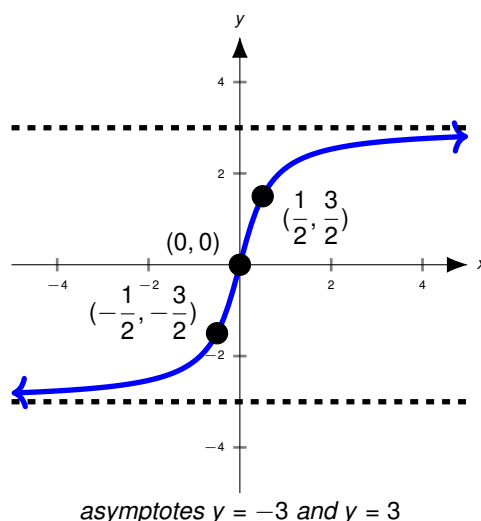
Problem 8.9 (1) shift right 3 units; (2) horizontal shrink by a factor of 2; (3) shift up 1 unit

$$g(x) = \sqrt{2x-3} + 1$$

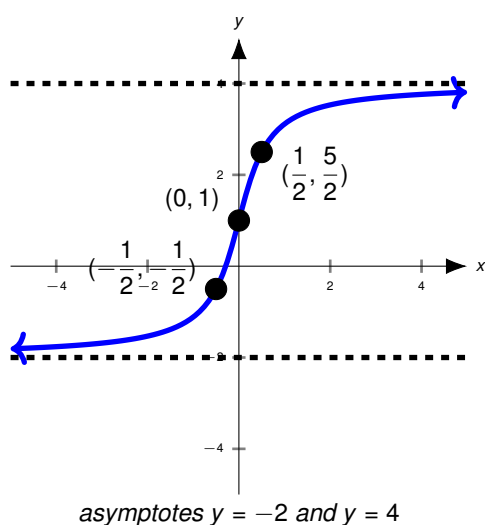
Problem 8.10 (1) horizontal shrink by a factor of 2; (2) shift right 3 units; (3) shift up 1 unit

$$g(x) = \sqrt{2(x-3)} + 1 = \sqrt{2x-6} + 1$$

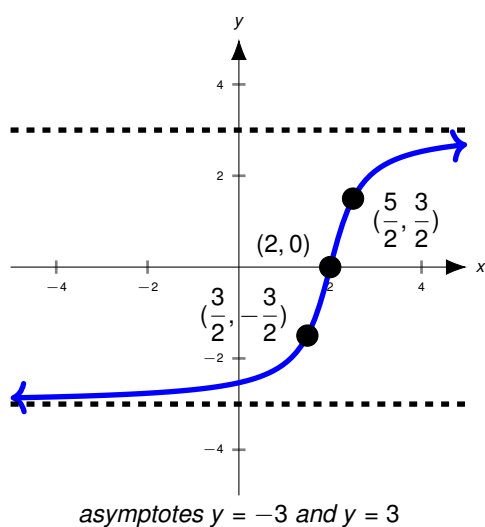
Question 9 For Exercises ?? - ??, use the graph of $y = f(x)$ to write each function in terms of $f(x)$.



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Problem 9.1 $y = g(x)$ 

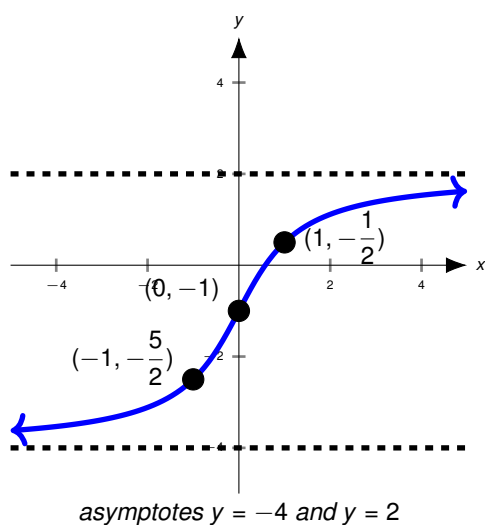
$$g(x) = f(x) + 1$$

Problem 9.2 $y = h(x)$ 

$$h(x) = f(x - 2)$$

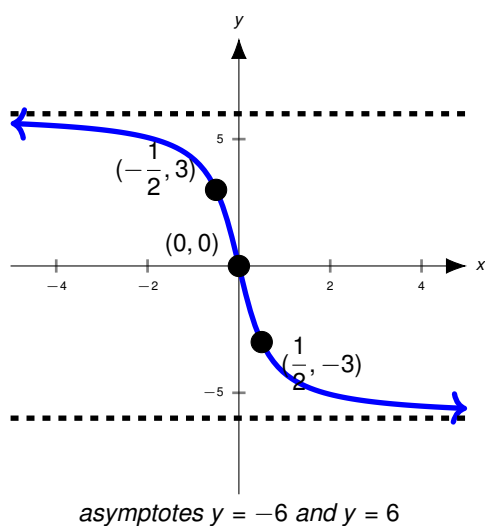
Problem 9.3 $y = p(x)$

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$$p(x) = f\left(\frac{x}{2}\right) - 1$$

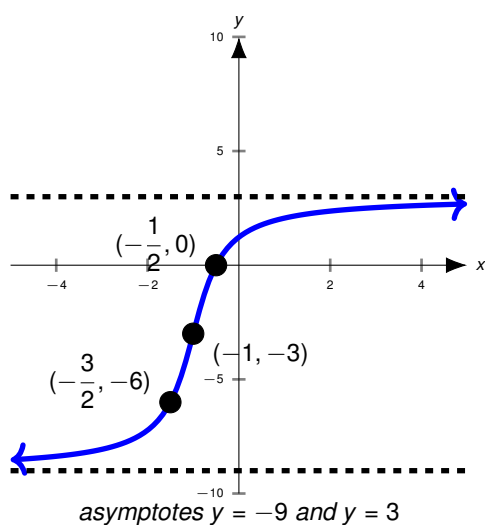
Problem 9.4 $y = q(x)$



$$q(x) = -2f(x) = 2f(-x)$$

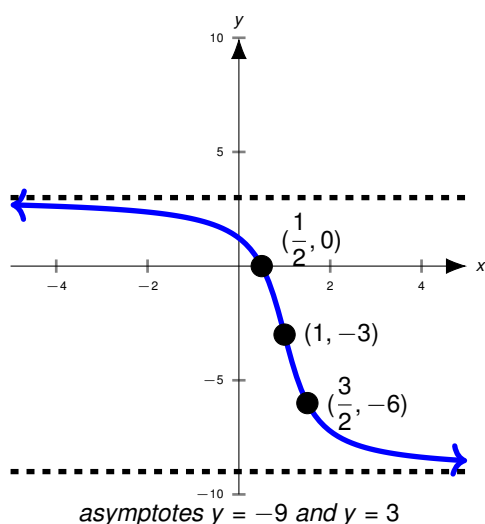
Problem 9.5 $y = r(x)$

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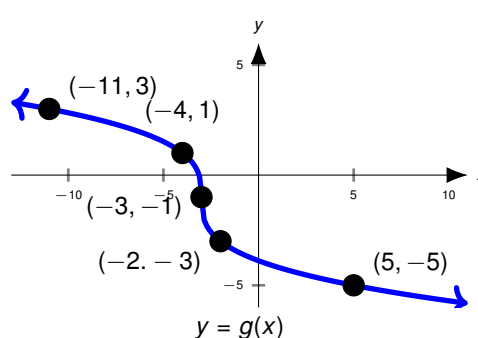
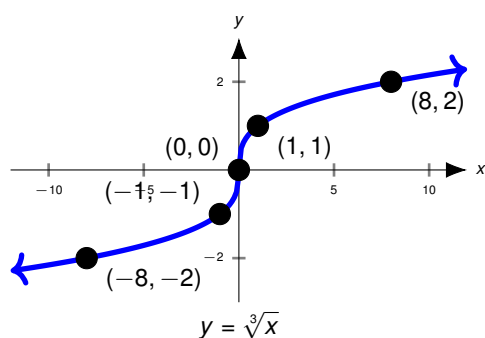
$$r(x) = 2f(x+1) - 3$$

Problem 9.6 $y = s(x)$



$$s(x) = 2f(-x+1) - 3 = -2f(x-1) + 3$$

Problem 10 The graph of $y = f(x) = \sqrt[3]{x}$ is given below on the left and the graph of $y = g(x)$ is given on the right. Find a formula for g based on transformations of the graph of f . Check your answer by confirming that the points shown on the graph of g satisfy the equation $y = g(x)$.



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$$g(x) = -2\sqrt[3]{x+3} - 1 \text{ or } g(x) = 2\sqrt[3]{-x-3} - 1$$

Problem 11 Show that the composition of two linear functions is a linear function. Hence any (finite) sequence of transformations discussed in this section can be combined into the form given in Theorem ??.

Hint: Let $f(x) = ax + b$ and $g(x) = cx + d$. Find $(f \circ g)(x)$.

Problem 12 For many common functions, the properties of Algebra make a horizontal scaling the same as a vertical scaling by (possibly) a different factor. For example, $\sqrt{9x} = 3\sqrt{x}$, so a horizontal compression of $y = \sqrt{x}$ by a factor of 9 results in the same graph as a vertical stretch of $y = \sqrt{x}$ by a factor of 3.

With the help of your classmates, find the equivalent vertical scaling produced by the horizontal scalings

$$y = (2x)^3, y = |5x|, y = \sqrt[3]{27x} \text{ and } y = \left(\frac{1}{2}x\right)^2.$$

$$\text{What about } y = (-2x)^3, y = |-5x|, y = \sqrt[3]{-27x} \text{ and } y = \left(-\frac{1}{2}x\right)^2?$$

Problem 13 Discuss the following questions with your classmates.

- If f is even, what happens when you reflect the graph of $y = f(x)$ across the y -axis?
- If f is odd, what happens when you reflect the graph of $y = f(x)$ across the y -axis?
- If f is even, what happens when you reflect the graph of $y = f(x)$ across the x -axis?
- If f is odd, what happens when you reflect the graph of $y = f(x)$ across the x -axis?
- How would you describe symmetry about the origin in terms of reflections?

Problem 14 We mentioned earlier in the section that, in general, the order in which transformations are applied matters, yet in our first example with two transformations the order did not matter. (You could perform the shift to the left followed by the shift down or you could shift down and then left to achieve the same result.) With the help of your classmates, determine the situations in which order does matter and those in which it does not.

Problem 15 This Exercise is a follow-up to Exercise ?? in Section ??.

1. For each of the following functions, use a graphing utility to compare the graph of $y = f(x)$ with the graphs of $y = |f(x)|$ and $y = f(|x|)$.
 - $f(x) = 3 - x$
 - $f(x) = x^2 - x - 6$
 - $f(x) = \sqrt{x+3} - 1$
2. In general, how does the graph of $y = |f(x)|$ compare with that of $y = f(x)$? What about the graph of $y = f(|x|)$ and $y = f(x)$?
3. Referring to the functions f and g graphed on page ??, write g in terms of f .