Exercises for Absolute Value Functions

Question 1 *In Exercises* **??** *-* **??***, graph the function using Theorem* **??***. Find the axis intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, the maximum and minimum of each function, if they exist, and list the intervals on which the function is increasing, decreasing or constant.*

Problem 1.1 $f(x) = |x + 4|$

Use the Desmos graph below with the settings $v(x) = |x|,\ h = -4,\ k = 0,\ a = 1$

Desmos link: https://www.desmos.com/calculator/hmwcgnrph3

x*-intercept* $(-4, 0)$
y*-intercept* $(0, 4)$
Domain $(-\infty, \infty)$
Range $[0, \infty)$
Decreasing on $(-\infty, -4]$
Increasing on $[-4, \infty)$
Minimum is 0 *at* $(-4, 0)$
No maximum

Problem 1.2 $f(x) = |x| + 4$

Use the Desmos graph below with the settings $v(x) = |x|,\ h = 0,\ k = 4,\ a = 1$

Desmos link: https://www.desmos.com/calculator/hmwcgnrph3

No x*-intercepts*
y*-intercept* $(0, 4)$
Domain $(-\infty, \infty)$
Range $[4, \infty)$
Decreasing on $(-\infty, 0]$
Increasing on $[0, \infty)$
Minimum is 4 *at* $(0, 4)$
No maximum

Problem 1.3 $f(x) = |4x|$

Note $f(x) = |4x| = 4|x|$.

Use the Desmos graph below with the settings $v(x) = |x|,\ h = 0,\ k = 0,\ a = 4$

Desmos link: https://www.desmos.com/calculator/hmwcgnrph3

x*-intercept* $(0, 0)$
y*-intercept* $(0, 0)$
Domain $(-\infty, \infty)$
Range $[0, \infty)$
Decreasing on $(-\infty, 0]$
Increasing on $[0, \infty)$
Minimum is 0 *at* $(0, 0)$
No maximum

Problem 1.4 $g(t) = -3|t|$

Use the Desmos graph below with the settings $v(x) = |x|,\ h = 0,\ k = 0,\ a = -3$

Desmos link: https://www.desmos.com/calculator/hmwcgnrph3

t-intercept $(0, 0)$
y-intercept $(0, 0)$
Domain $(-\infty, \infty)$
Range $(-\infty, 0]$
Increasing on $(-\infty, 0]$
Decreasing on $[0, \infty)$
Maximum is 0 *at* $(0, 0)$
No minimum

Problem 1.5 $g(t) = 3|t + 4| - 4$

Use the Desmos graph below with the settings $v(x) = |x|,\ h = -4,\ k = -4,\ a = 3$

Desmos link: https://www.desmos.com/calculator/hmwcgnrph3

t-intercepts $\left(-\frac{16}{3}, 0\right)$, $\left(-\frac{8}{3}, 0\right)$
y-intercept $(0, 8)$
Domain $(-\infty, \infty)$
Range $[-4, \infty)$
Decreasing on $(-\infty, -4]$
Increasing on $[-4, \infty)$
Minimum is -4 *at* $(-4, -4)$
No maximum

Problem 1.6 $g(t) = \frac{1}{3}|2t - 1|$

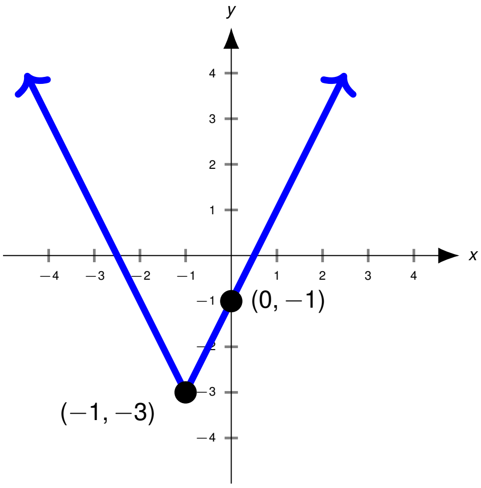
Note $g(t) = \frac{1}{3}|2t - 1| = \frac{2}{3}|t - \frac{1}{2}|$

Use the Desmos graph below with the settings $v(x) = |x|,\ h = \frac{1}{2},\ k = 0,\ a = \frac{2}{3}$

Desmos link: https://www.desmos.com/calculator/hmwcgnrph3

t-intercepts $\left(\frac{1}{2}, 0\right)$
y-intercept $\left(0, \frac{1}{3}\right)$
Domain $(-\infty, \infty)$
Range $[0, \infty)$
Decreasing on $\left(-\infty, \frac{1}{2}\right]$
Increasing on $\left[\frac{1}{2}, \infty\right)$
Minimum is 0 *at* $\left(\frac{1}{2}, 0\right)$
No maximum

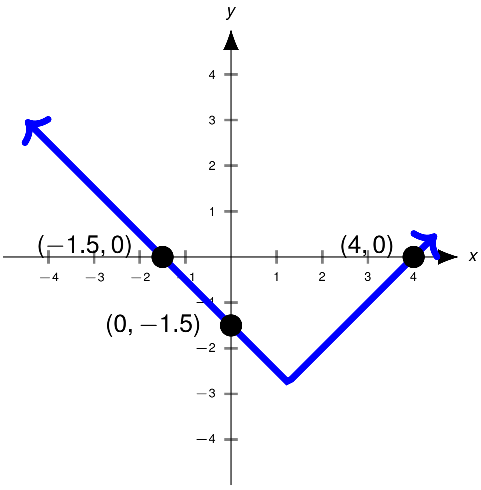
Question 2 *In Exercises **??** - **??**, find a formula for each function below in the form* $F(x) = a|x - h| + k$.



The graph of $y = F(x)$.

Problem 2.1

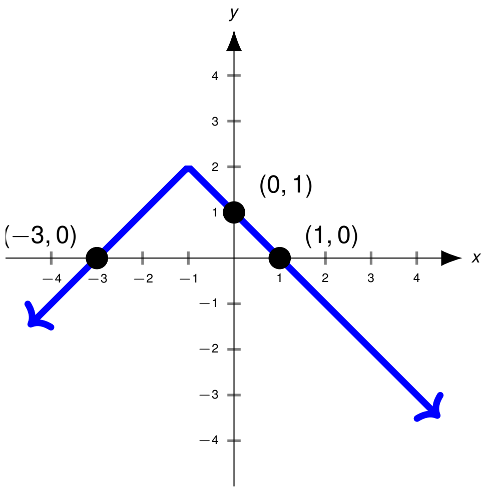
$F(x) = 2|x + 1| - 3$



The graph of $y = F(x)$.

Problem 2.2

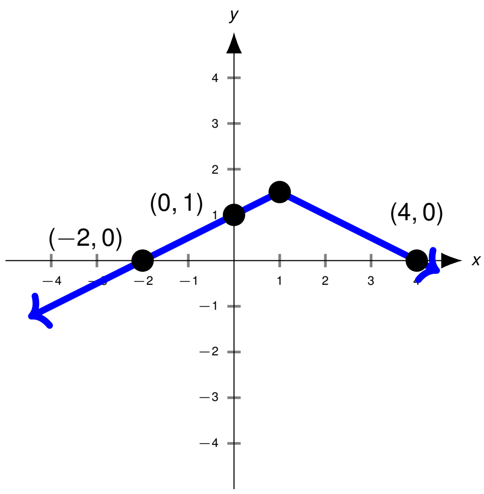
$F(x) = |x - 1.25| - 2.75$



The graph of $y = F(x)$.

Problem 2.3

$F(x) = -|x+1| + 2$



The graph of $y = F(x)$.

Problem 2.4

$F(x) = -\frac{1}{2}|x+1| + \frac{3}{2}$

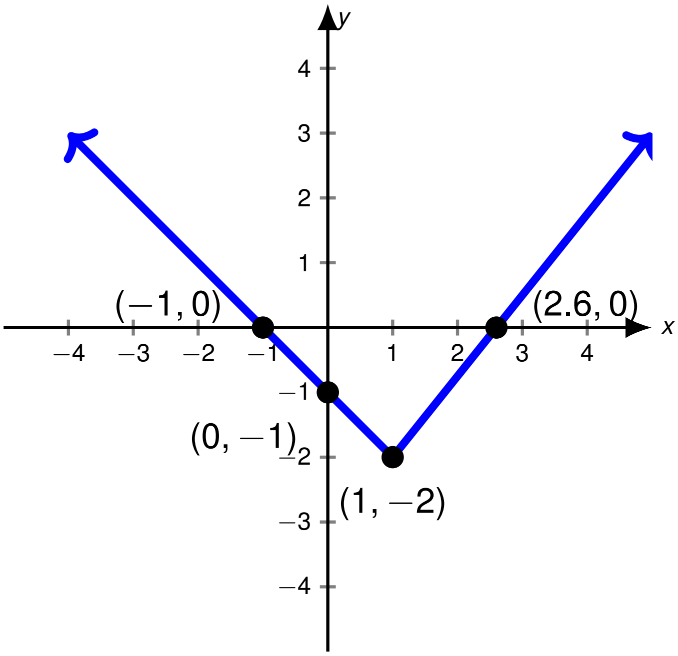
Problem 3 *Usethis Desmos link to graph the following pairs of functions on the same set of axes. (The first one is done for you, and you can just modify f(x) to get the others.)*

- $f(x) = 2 - x$ *and* $g(x) = |2 - x|$
- $f(x) = x^2 - 4$ *and* $g(x) = |x^2 - 4|$
- $f(x) = x^3$ *and* $g(x) = |x^3|$
- $f(x) = \sqrt{x} - 4$ *and* $g(x) = |\sqrt{x} - 4|$

*Choose more functions $f(x)$ and graph $y = f(x)$ alongside $y = |f(x)|$ until you can explain how, in general, one would obtain the graph of $y = |f(x)|$ given the graph of $y = f(x)$. How does your explanation tie in with with Definition **??**?*

*In each case, the graph of g can be obtained from the graph of f by reflecting the portion of the graph of f which lies below the x-axis about the x-axis. This meshes with Definition **??** since what we are doing algebraically is making the negative y-values positive.*

Problem 4 *Explain why the function below cannot be written in the form $F(x) = a|x - h| + k$. Write $F(x)$ as a piecewise-defined linear function.*



If $F(x) = a|x - h| + k$, then for the vertex to be at $(1, -2)$, $h = 1$ and $k = -2$ so $F(x) = a|x - 1| - 2$. Since $(0, -1)$ is on the graph, $F(0) = -1$ so $-1 = a|0 - 1| - 2$ which means $a = 1$. This means $F(x) = |x - 1| - 2$. However, $(2.6, 0)$ is also on the graph, so it should work out that $F(2.6) = 0$. However, we find $F(2.6) = |2.6 - 1| - 2 = -0.4 \neq 0$.

$$F(x) = \begin{cases} -x - 1 & \text{if } x \leq 1, \\ \frac{5}{4}x - \frac{13}{4} & \text{if } x \geq 1, \end{cases}$$

Question 5 *In Exercises **??** - **??**, graph the function by rewriting each function as a piecewise defined function using Definition **??**. Find the axis intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, the maximum and minimum of each function, if they exist, and list the intervals on which the function is increasing, decreasing or constant.*

Problem 5.1 $f(x) = x + |x| - 3$

Re-write $f(x) = x + |x| - 3$ *as*

$$f(x) = \begin{cases} -3 & \text{if} \quad x < 0 \\ 2x - 3 & \text{if} \quad x \geq 0 \end{cases}$$

x-intercept $\left(\frac{3}{2}, 0\right)$
y-intercept $(0, -3)$
Domain $(-\infty, \infty)$
Range $[-3, \infty)$
Increasing on $[0, \infty)$
Constant on $(-\infty, 0]$
Minimum is -3 *at* $(x, -3)$ *where* $x \leq 0$
No maximum

Graph of $x + abs(x) + 3$

Problem 5.2 $f(x) = |x+2| - x$

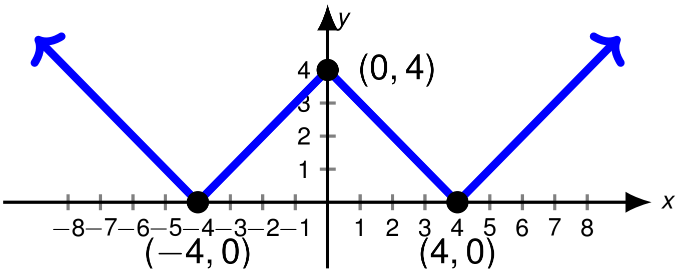
Problem 5.3 $f(x) = |x+2| - |x|$

Problem 5.4 $g(t) = |t+4| + |t-2|$

Problem 5.5 $g(t) = \dfrac{|t+4|}{t+4}$

Problem 5.6 $g(t) = \dfrac{|2-t|}{2-t}$

Problem 6 *With the help of your classmates, find an absolute value function whose graph is given below.*



$f(x) = ||x| - 4|$

Question 7 *In Exercises* **??** - **??**, *solve the equation.*

Problem 7.1 $|x| = 6$

Select All Correct Answers:

1. $x = -6$ ✓
2. $x = 6$ ✓

Problem 7.2 $|3x - 1| = 10$

Select All Correct Answers:

1. $x = -3$ ✓
2. $x = -\dfrac{11}{3}$
3. $x = 0$
4. $x = \dfrac{11}{3}$ ✓
5. $x = 3$

Problem 7.3 $|4 - x| = 7$

Select All Correct Answers:

1. $x = -11$
2. $x = -3$ ✓
3. $x = 0$
4. $x = 3$
5. $x = 11$ ✓

Problem 7.4 $4 - |t| = 3$

Select All Correct Answers:

1. $t = -7$
2. $t = -1$ ✓
3. $t = 0$
4. $t = 1$ ✓
5. $t = 7$

Problem 7.5 $2|5t + 1| - 3 = 0$

Select All Correct Answers:

1. $t = -\frac{1}{2}$ ✓
2. $t = -\frac{1}{10}$
3. $t = \frac{1}{10}$ ✓
4. $t = \frac{1}{2}$
5. *No real solutions.*

Problem 7.6 $|7t - 1| + 2 = 0$

Select All Correct Answers:

1. $t = -\frac{3}{7}$
2. $t = -\frac{1}{7}$
3. $t = \frac{1}{7}$
4. $t = \frac{3}{7}$

5. *No solution* ✓

Problem 7.7 $\frac{5-|w|}{2} = 1$

Select All Correct Answers:

1. $w = -7$
2. $w = -3$ ✓
3. $w = 3$ ✓
4. $w = 7$
5. *No solution*

Problem 7.8 $\frac{2}{3}|5-2w| - \frac{1}{2} = 5$

$w = -\frac{13}{8}$ *or* $w = \frac{53}{8}$

Problem 7.9 $|w| = w + 3$

$w = -\frac{3}{2}$

Problem 7.10 $|2x-1| = x + 1$

$x = 0$ *or* $x = 2$

Problem 7.11 $4 - |x| = 2x + 1$

$x = 1$

Problem 7.12 $|x-4| = x - 5$

no solution

Question 8 *Solve the equations in Exercises* **??** - **??** *using the property that if* $|a| = |b|$ *then* $a = \pm b$.

Problem 8.1 $|3x-2| = |2x+7|$

$x = -1$ *or* $x = 9$

Problem 8.2 $|3x+1| = |4x|$

$x = -\frac{1}{7}$ *or* $x = 1$

Problem 8.3 $|1-2x| = |x+1|$

$x = 0$ *or* $x = 2$

Problem 8.4 $|4-t| - |t+2| = 0$

$t = 1$

Problem 8.5 $|2-5t| = 5|t+1|$

$t = -\frac{3}{10}$

Problem 8.6 $3|t-1| = 2|t+1|$

$t = \frac{1}{5}$ *or* $t = 5$

Question 9 *In Exercises* **??** *-* **??***, solve the inequality. Write your answer using interval notation.*

Problem 9.1 $|3x-5| \leq 4$

$\left[\frac{1}{3}, 3\right]$

Problem 9.2 $|7x+2| > 10$

$\left(-\infty, -\frac{12}{7}\right) \cup \left(\frac{8}{7}, \infty\right)$

Problem 9.3 $|2t+1| - 5 < 0$

$(-3, 2)$

Problem 9.4 $|2-t| - 4 \geq -3$

$(-\infty, 1] \cup [3, \infty)$

Problem 9.5 $|3w+5| + 2 < 1$

No solution

Problem 9.6 $2|7-w| + 4 > 1$

$(-\infty, \infty)$

Problem 9.7 $2 \leq |4-x| < 7$

$(-3, 2] \cup [6, 11)$

Problem 9.8 $1 < |2x-9| \leq 3$

$[3, 4) \cup (5, 6]$

Problem 9.9 $|t+3| \geq |6t+9|$

$\left[-\frac{12}{7}, -\frac{6}{5}\right]$

Problem 9.10 $|t-3| - |2t+1| < 0$

$(-\infty, -4) \cup \left(\frac{2}{3}, \infty\right)$

Problem 9.11 $|1-2x| \geq x+5$

$\left(-\infty, -\frac{4}{3}\right] \cup [6, \infty)$

Problem 9.12 $x+5 < |x+5|$

$(-\infty, -5)$

Problem 9.13 $x \geq |x+1|$

No Solution.

Problem 9.14 $|2x+1| \leq 6-x$

$\left[-7, \dfrac{5}{3}\right]$

Problem 9.15 $t+|2t-3| < 2$

$\left(1, \dfrac{5}{3}\right)$

Problem 9.16 $|3-t| \geq t-5$

$(-\infty, \infty)$

Problem 10 *Show that if δ is a real number with $\delta > 0$, the solution to $|x-a| < \delta$ is the interval: $(a-\delta, a+\delta)$. That is, an interval centered at a with 'radius' δ.*

Problem 11 *The Triangle Inequality for real numbers states that for all real numbers x and a, $|x+a| \leq |x|+|a|$ and, moreover, $|x+a| = |x|+|a|$ if and only if x and a are both positive, both negative, or one or the other is 0. Graph each pair of functions below on the same pair of axes and use the graphs to verify the triangle inequality in each instance.*

- $f(x) = |x+2|$ *and* $g(x) = |x|+2$.
- $f(x) = |x+4|$ *and* $g(x) = |x|+4$.