Exercises for Constant and Linear Functions

Question 1 *In Exercises* **??** - **??**, *graph the function. Find the slope and axis intercepts, if any.*

Problem 1.1 $f(x) = 2x - 1$

slope: $m = 2$

y*-intercept:* $(0, -1)$

x*-intercept:* $\left(\frac{1}{2}, 0\right)$

Graph of $f(x) = 2x - 1$

Problem 1.2 $g(t) = 3 - t$

slope: $m = -1$

y*-intercept:* $(0, 3)$

t*-intercept:* $(3, 0)$

Graph of $g(t) = 3 - t$

Problem 1.3 $F(w) = 3$

slope: $m = 0$

y*-intercept:* $(0, 3)$

w*-intercept: none*

Graph of $F(w) = 3$

Problem 1.4 $G(s) = 0$

slope: $m = 0$

y*-intercept:* $(0, 0)$

s*-intercept:* $(0, 0)$

Graph of $G(s) = 0$

Problem 1.5 $h(t) = \frac{2}{3}t + \frac{1}{3}$

slope: $m = \frac{2}{3}$

y*-intercept:* $(0, \frac{1}{3})$

t*-intercept:* $\left(-\frac{1}{2}, 0\right)$

Graph of $h(t) = \frac{2}{3}t + \frac{1}{3}$

Problem 1.6 $j(w) = \frac{1 - w}{2}$

slope: $m = -\frac{1}{2}$

y-intercept: $(0, \frac{1}{3})$

w-intercept: $(1, 0)$

Graph of $j(w) = \dfrac{1-w}{2}$

Question 2 *In Exercises* **??** - **??**, *graph the function. Find the domain, range, and axis intercepts, if any.*

Problem 2.1 $f(x) = \begin{cases} 4-x & \text{if} \quad x \leq 3 \\ 2 & \text{if} \quad x > 3 \end{cases}$

domain: $(-\infty, \infty)$

range: $[1, \infty)$

y-intercept: $(0, 4)$

x-intercept: none

Click here to see the graph on Desmos.

Problem 2.2 $g(x) = \begin{cases} 2-x & \text{if} \quad x < 2 \\ x-2 & \text{if} \quad x \geq 2 \end{cases}$

domain: $(-\infty, \infty)$

range: $[0, \infty)$

y-intercept: $(0, 2)$

x-intercept: $(2, 0)$

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Problem 2.3 $F(t) = \begin{cases} -2t-4 & \text{if} \quad t < 0 \\ 3t & \text{if} \quad t \geq 0 \end{cases}$

domain: $(-\infty, \infty)$

range: $(-4, \infty)$

y-intercept: $(0, 0)$

t-intercepts: $(-2, 0)$, $(0, 0)$

Click here to see the graph on Desmos.

Problem 2.4 $G(t) = \begin{cases} -3 & \text{if} \quad t \leq 0 \\ 2t-3 & \text{if} \quad 0 < t < 3 \\ 3 & \text{if} \quad t \geq 3 \end{cases}$

domain: $(-\infty, \infty)$

range: $[-3, 3]$

y-intercept: $(0, -3)$

t-intercept: $\left(\frac{3}{2}, 0\right) = (1.5, 0)$

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Problem 3 *The **unit step function** is defined as* $U(t) = \begin{cases} 0 & \text{if } t < 0, \\ 1 & \text{if } t \geq 0. \end{cases}$

1. *Graph* $y = U(t)$.

   Click here to see the graph on Desmos.

2. *State the domain and range of* U.

   domain: $(-\infty, \infty)$, *range:* $\{0, 1\}$

3. *List the interval(s) over which* U *is increasing, decreasing, and/or constant.*

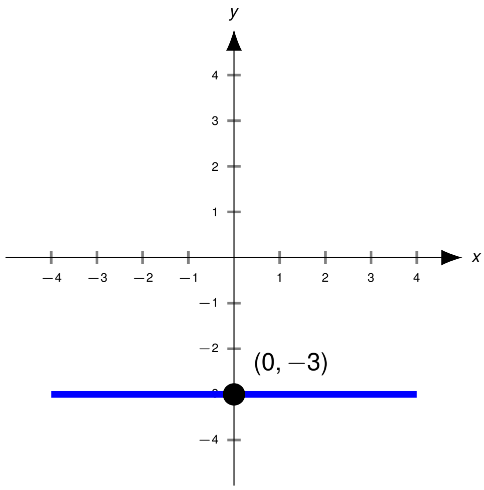
   U *is constant on* $(-\infty, 0)$ *and* $[0, \infty)$.

4. *Write* $U(t-2)$ *as a piecewise defined function and graph.*

   $U(t-2) = \begin{cases} 0 & \text{if } t < 2, \\ 1 & \text{if } t \geq 2. \end{cases}$

   Click here to see the graph on Desmos.

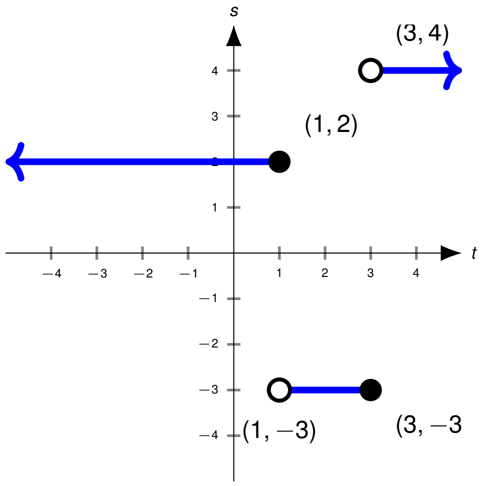
Question 4 *In Exercises* **??** - **??**, *find a formula for the function.*



The graph of $y = f(x)$.

Problem 4.1

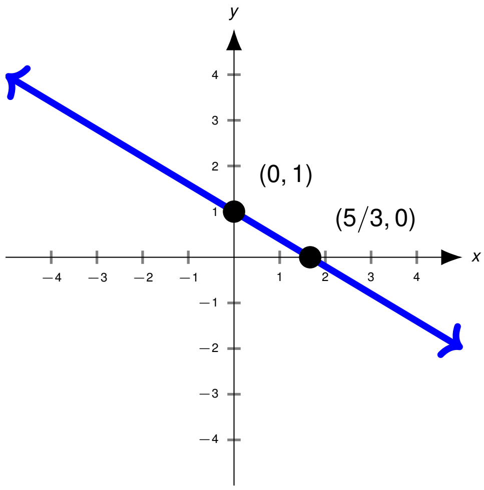
$y = -3$



The graph of $s = F(t)$.

Problem 4.2

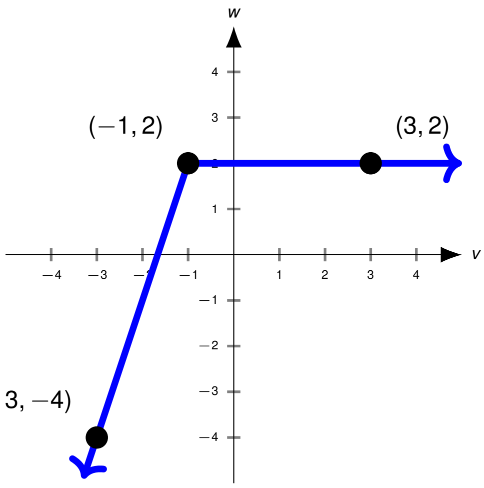
$$F(t) = \begin{cases} 2 & \text{if } t \leq 1, \\ -3 & \text{if } 1 < t \leq 3, \\ 4 & \text{if } t > 3. \end{cases}$$



The graph of $y = L(x)$.

Problem 4.3

$L(x) = -\frac{3}{5}x + 1$



The graph of $w = g(v)$.

Problem 4.4

$$g(v) = \begin{cases} 3v + 5 & \text{if } -3 \leq v < -1, \\ 2 & \text{if } -1 < v \leq 3, \end{cases}$$

Problem 5 *For n copies of the book Me and my Sasquatch, a print on-demand company charges $C(n)$ dollars, where $C(n)$ is determined by the formula*

$$C(n) = \begin{cases} 15n & \text{if} \quad 1 \leq n \leq 25 \\ 13.50n & \text{if} \quad 25 < n \leq 50 \\ 12n & \text{if} \quad n > 50 \end{cases}$$

1. *Find and interpret $C(20)$.*

   $C(20) = 300$. *It costs* \$300 *for 20 copies of the book.*

2. *How much does it cost to order 50 copies of the book? What about 51 copies?*

   $C(50) = 675$, \$675. $C(51) = 612$, \$612.

3. *Your answer to **??** should get you thinking. Suppose a bookstore estimates it will sell 50 copies of the book. How many books can, in fact, be ordered for the same price as those 50 copies? (Round your answer to a whole number of books.)*

   56 books.

Problem 6 *An on-line comic book retailer charges shipping costs according to the following formula*

$$S(n) = \begin{cases} 1.5n + 2.5 & \text{if} \quad 1 \leq n \leq 14 \\ 0 & \text{if} \quad n \geq 15 \end{cases}$$

where n is the number of comic books purchased and $S(n)$ is the shipping cost in dollars.

1. *What is the cost to ship 10 comic books?*

   \$17.50

2. *What is the significance of the formula $S(n) = 0$ for $n \geq 15$?*

   There is free shipping on orders of 15 *or more comic books.*

Problem 7 *The cost in dollars $C(m)$ to talk m minutes a month on a mobile phone plan is modeled by*

$$C(m) = \begin{cases} 25 & \text{if} \quad 0 \leq m \leq 1000 \\ 25 + 0.1(m - 1000) & \text{if} \quad m > 1000 \end{cases}$$

1. *How much does it cost to talk* 750 *minutes per month with this plan?*

   $25

2. *How much does it cost to talk* 20 *hours a month with this plan?*

   $45

3. *Explain the terms of the plan verbally.*

   It costs $25 *for up to* 1000 *minutes and* 10 *cents per minute for each minute over* 1000 *minutes.*

Problem 8 *Jeff can walk comfortably at* 3 *miles per hour. Find an expression for a linear function $d(t)$ that represents the total distance Jeff can walk in t hours, assuming he doesn't take any breaks.*

$d(t) = 3t$, $t \geq 0$.

Problem 9 *Carl can stuff* 6 *envelopes per minute. Find an expression for a linear function $E(t)$ that represents the total number of envelopes Carl can stuff after t hours, assuming he doesn't take any breaks.*

$E(t) = 360t$, $t \geq 0$.

Problem 10 *A landscaping company charges* $45 *per cubic yard of mulch plus a delivery charge of* $20. *Find an expression for a linear function $C(x)$ which computes the total cost in dollars to deliver x cubic yards of mulch.*

$C(x) = 45x + 20$, $x \geq 0$.

Problem 11 *A plumber charges* $50 *for a service call plus* $80 *per hour. If she spends no longer than 8 hours a day at any one site, find an expression for a linear function $C(t)$ that computes her total daily charges in dollars as a function of the amount of time spent in hours, t at any one given location.*

$C(t) = 80t + 50$, $0 \leq t \leq 8$.

Problem 12 *A salesperson is paid $200 per week plus 5% commission on her weekly sales of x dollars. Find an expression for a linear function $W(x)$ which computes her total weekly pay in dollars as a function of x. What must her weekly sales be in order for her to earn $475.00 for the week?*

$W(x) = 200 + .05x$, $x \geq 0$ *She must make $5500 in weekly sales.*

Problem 13 *An on-demand publisher charges* $22.50 *to print a 600 page book and* $15.50 *to print a 400 page book. Find an expression for a linear function which models the cost of a book in dollars $C(p)$ as a function of the number of pages p. Find and interpret both the slope of the linear function and $C(0)$.*

$C(p) = 0.035p + 1.5$ *The slope* 0.035 *means it costs* 3.5¢ *per page.* $C(0) = 1.5$ *means there is a fixed, or start-up, cost of* $1.50 *to make each book.*

Problem 14 *The Topology Taxi Company charges* $2.50 *for the first fifth of a mile and* $0.45 *for each additional fifth of a mile. Find an expression for a linear function which models the taxi fare $F(m)$ as a function of the number of miles driven, m. Find and interpret both the slope of the linear function and $F(0)$.*

$F(m) = 2.25m + 2.05$ *The slope* 2.25 *means it costs an additional* $2.25 *for each mile beyond the first 0.2 miles.* $F(0) = 2.05$, *so according to the model, it would cost* $2.05 *for a trip of* 0 *miles. Would this ever really happen? Depends on the driver and the passenger, we suppose.*

Problem 15 *Water freezes at 0° Celsius and 32° Fahrenheit and it boils at $100^\circ C$ and $212^\circ F$.*

1. *Find an expression for a linear function $F(T)$ that computes temperature in the Fahrenheit scale as a function of the temperature T given in degrees Celsius. Use this function to convert $20^\circ C$ into Fahrenheit.*
   $F(T) = \frac{9}{5}T + 32$
2. *Find an expression for a linear function $C(T)$ that computes temperature in the Celsius scale as a function of the temperature T given in degrees Fahrenheit. Use this function to convert $110^\circ F$ into Celsius.*
   $C(T) = \frac{5}{9}(T - 32) = \frac{5}{9}T - \frac{160}{9}$
3. *Is there a temperature T such that $F(T) = C(T)$?*
   -40

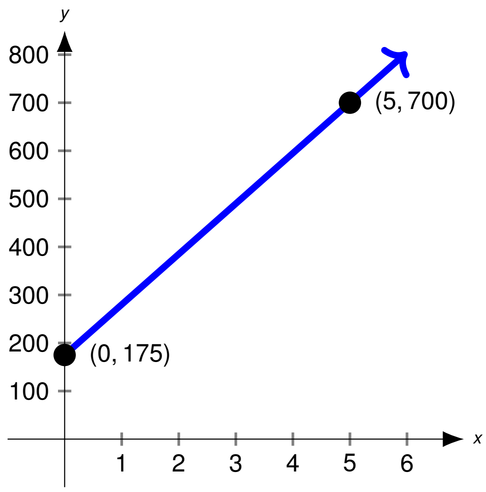
Problem 16 *Legend has it that a bull Sasquatch in rut will howl approximately 9 times per hour when it is $40^\circ F$ outside and only 5 times per hour if it's $70^\circ F$. Assuming that the number of howls per hour, N, can be represented by a linear function of temperature Fahrenheit, find the number of howls per hour he'll make when it's only $20^\circ F$ outside. What troubles do you encounter when trying to determine a reasonable applied domain?*

$N(T) = -\frac{2}{15}T + \frac{43}{3}$ *and* $N(20) = \frac{35}{3} \approx 12$ *howls per hour.*

Having a negative number of howls makes no sense and since $N(107.5) = 0$ we can put an upper bound of $107.5^\circ F$ on the domain. The lower bound is trickier because there's nothing other than common sense to go on. As it gets colder, he howls more often. At some point it will either be so cold that he freezes to death or he's howling non-stop. So we're going to say that he can withstand temperatures no lower than $-42^\circ F$ so that the applied domain is $[-42, 107.5]$.

Problem 17 *Economic forces have changed the cost function for PortaBoys to $C(x) = 105x + 175$. Rework Example **??** with this new cost function.*

1. $C(0) = 175$, *so our start-up costs are* \$175. $C(5) = 700$, *so to produce* 5 *systems, it costs* \$700.



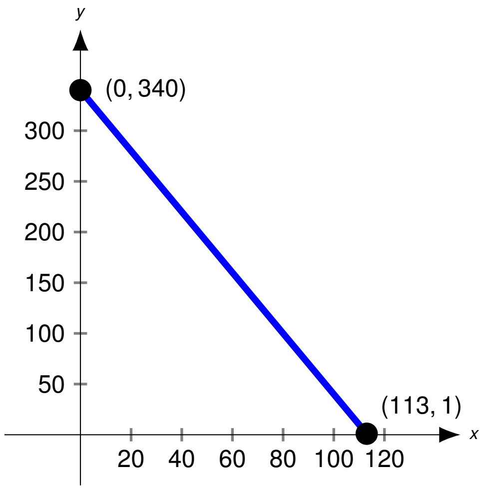
$y = C(x)$.

2. *Since we can't make a negative number of game systems,* $x \geq 0$.
3. *The slope is* $m = 105$ *so for each additional system produced, it costs an additional* \$105.

4. *Solving* $C(x) = 15000$ *gives* $x \approx 141.19$ *so* 141 *can be produced for* \$15,000.

Problem 18 *In response to the economic forces in Exercise* **??** *above, the local retailer sets the selling price of a PortaBoy at \$250. Remarkably, 30 units were sold each week. When the systems went on sale for \$220, 40 units per week were sold. Rework Example* **??** *with this new data.*

1. $p(x) = -3x + 340$, $0 \le x \le 113$.



$y = p(x)$.

2. *The slope is* $m = -3$ *so for each* \$3 *drop in price, we sell one additional game system.*
3. *Since* $x = 150$ *is not in the domain of* p, $p(150)$ *is not defined. (In other words, under these conditions, it is impossible to sell 150 game systems.)*
4. *Solving* $p(x) = 150$ *gives* $x \approx 63.33$ *so if the price* \$150 *per system, we would sell* 63 *systems.*

Problem 19 *A local pizza store offers medium two-topping pizzas delivered for* \$6.00 *per pizza plus a* \$1.50 *delivery charge per order. On weekends, the store runs a 'game day' special: if six or more medium two-topping pizzas are ordered, they are* \$5.50 *each with no delivery charge. Write a piecewise-defined linear function which calculates the cost in dollars* $C(p)$ *of* p *medium two-topping pizzas delivered during a weekend.*

$$C(p) = \begin{cases} 6p + 1.5 & \text{if } 1 \le p \le 5 \\ 5.5p & \text{if } p \ge 6 \end{cases}$$

Problem 20 *A restaurant offers a buffet which costs* \$15 *per person. For parties of* 10 *or more people, a group discount applies, and the cost is* \$12.50 *per person. Write a piecewise-defined linear function which calculates the total bill* $T(n)$ *of a party of* n *people who all choose the buffet.*

$$T(n) = \begin{cases} 15n & \text{if } 1 \le n \le 9 \\ 12.5n & \text{if } n \ge 10 \end{cases}$$

Problem 21 *A mobile plan charges a base monthly rate of* \$10 *for the first* 500 *minutes of air time plus a charge of* 15¢ *for each additional minute. Write a piecewise-defined linear function which calculates the monthly cost in dollars* $C(m)$ *for using* m *minutes of air time.*

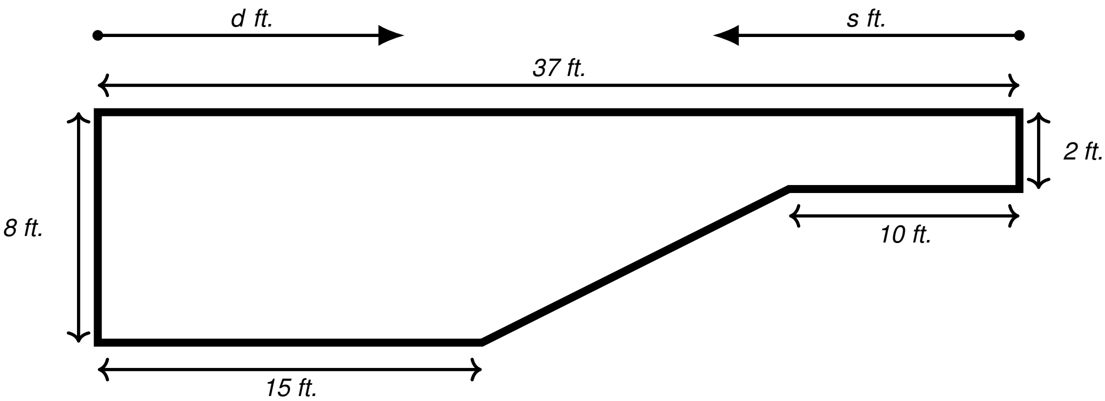
Hint: *You may wish to refer to number* **??** *for inspiration.*

$$C(m) = \begin{cases} 10 & \text{if } 0 \le m \le 500 \\ 10 + 0.15(m - 500) & \text{if } m > 500 \end{cases}$$

Problem 22 *The local pet shop charges* 12¢ *per cricket up to 100 crickets, and* 10¢ *per cricket thereafter. Write a piecewise-defined linear function which calculates the price in dollars $P(c)$ of purchasing c crickets.*

$$P(c) = \begin{cases} 0.12c & \text{if} \quad 1 \le c \le 100 \\ 12 + 0.1(c - 100) & \text{if} \quad c > 100 \end{cases}$$

Problem 23 *Here is a cross-section of a swimming pool.*



Write a piecewise-defined linear function which describes the depth of the pool, D (in feet) as a function of:

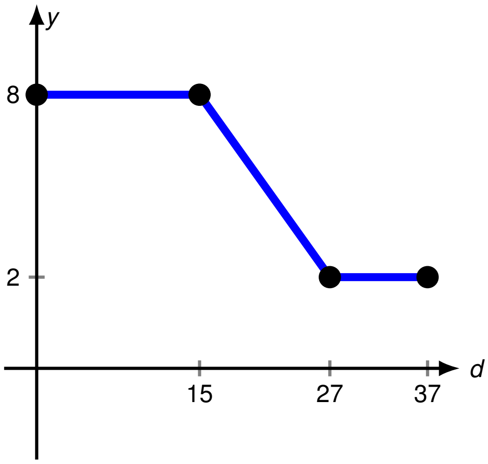
1. *the distance (in feet) from the edge of the shallow end of the pool, d.*

$$D(d) = \begin{cases} 8 & \text{if} \quad 0 \le d \le 15 \\ -\frac{1}{2}d + \frac{31}{2} & \text{if} \quad 15 \le d \le 27 \\ 2 & \text{if} \quad 27 \le d \le 37 \end{cases}$$

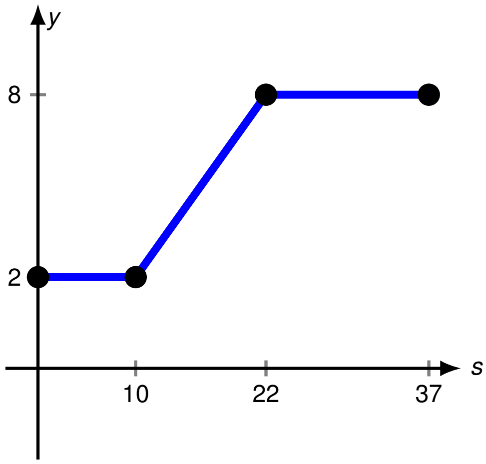
2. *the distance (in feet) from the edge of the deep end of the pool, s.*

$$D(s) = \begin{cases} 2 & \text{if} \quad 0 \le s \le 10 \\ \frac{1}{2}s - 3 & \text{if} \quad 10 \le s \le 22 \\ 8 & \text{if} \quad 22 \le s \le 37 \end{cases}$$

3. *Graph each of the functions in (a) and (b). Discuss with your classmates how to transform one into the other and how they relate to the diagram of the pool.*



$y = D(d)$



$y = D(s)$

Problem 24 *The function defined by $I(x) = x$ is called the Identity Function. Thinking from a procedural perspective, explain a possible origin of this name.*

Since $I(x) = x$ for all real numbers x, the function I doesn't change the 'identity' of the input at all.

Problem 25 *Why must the graph of a function $y = f(x)$ have at most one y-intercept?*

Hint: *Consider what would happen graphically if there were more than one . . .*

If a graph contains more than one y-intercept, it would violate the Vertical Line Test since $x = 0$ would be matched with (at least) two different y-values.

Problem 26 *Why is a discussion of vertical lines omitted when discussing functions?*

Vertical Lines fail the Vertical Line Test.

Problem 27 *Find a formula for the x-intercept of the graph of $f(x) = mx + b$. Assume $m \neq 0$.*

$\left(-\dfrac{b}{m}, 0\right)$. *(Note the importance here of $m \neq 0$.)*

Problem 28 *Suppose $(c, 0)$ is the x-intercept of a linear function f. Use the point-slope form of a linear function, Equation* **??** *to show $f(x) = m(x - c)$. This is the 'slope x-intercept' form of the linear function.*

Plugging in $(c, 0)$ for $(x_0, f(x_0))$, we get $f(x) = f(x_0) + m(x - x_0) = 0 + m(x - c)$ or $f(x) = m(x - c)$.

Problem 29 *Prove that for all linear functions L with with slope 3, $L(120) = L(100) + 60$.*

Since L is linear with slope 3, $L(x) = L(x_0) + m\Delta x = L(100) + (3)(120 - 100) = L(100) + 60$.

Problem 30 *Find the slopes between the following points from the data set given in Example* **??** *and compare them with the slope of the corresponding regression line:*

1. $(0, 64), (4, 75)$

   $m = \dfrac{75 - 64}{4 - 0} = 2.75$

2. $(4, 75), (8, 83)$

   $m = 2$

3. $(8, 83)$, $(10, 83)$

   $m = 0$

4. $(10, 83)$, $(12, 82)$

   $m = -\frac{1}{2}$

The first two points contributed to a regression line slope of $m = 2.55$; the last two points contributed to a regression line slope of $m = -0.25$.

Problem 31 *According to this website[1], the census data for Lake County, Ohio is:*

Year	1970	1980	1990	2000
Population	197200	212801	215499	227511

1. *Find the least squares regression line for these data and comment on the goodness of fit.[2] Interpret the slope of the line of best fit.*

   $y = 936.31x - 1645322.6$ *with* $r = 0.9696$ *which indicates a good fit. The slope* 936.31 *indicates Lake County's population is increasing at a rate of (approximately) 936 people per year.*

2. *Use the regression line to predict the population of Lake County in 2010. (The recorded figure from the 2010 census is* 230,041*)*

   According to the model, the population in 2010 will be 236,660*.*

3. *Use the regression line to predict when the population of Lake County will reach* 250,000*.*

   According to the model, the population of Lake County will reach 250,000 *sometime between 2024 and 2025.*

Problem 32 *According to this website[3], the census data for Lorain County, Ohio is:*

Year	1970	1980	1990	2000
Population	256843	274909	271126	284664

1. *Find the least squares regression line for these data and comment on the goodness of fit. Interpret the slope of the line of best fit.*

   $y = 796.8x - 1309762.5$ *with* $r = 0.8916$ *which indicates a reasonable fit. The slope* 796.8 *indicates Lorain County's population is increasing at a rate of (approximately) 797 people per year.*

2. *Use the regression line to predict the population of Lorain County in 2010. (The recorded figure from the 2010 census is* 301,356*)*

   According to the model, the population in 2010 will be 291,805*.*

3. *Use the regression line to predict when the population of Lake County will reach* 325,000*.*

   According to the model, the population of Lake County will reach 325,000 *sometime between 2051 and 2052.*

Problem 33 *The chart below contains a portion of the fuel consumption information for a 2002 Toyota Echo that Jeffrey used to own. The first row is the cumulative number of gallons of gasoline that I had used and the second row is the odometer reading when I refilled the gas tank. So, for example, the fourth entry is the point*

[1] http://www.ohiobiz.com/census/Lake.pdf

[2] We'll develop more sophisticated models for the growth of populations in Chapter **??**. For the moment, we use a theorem from Calculus to approximate those functions with lines.

[3] http://www.ohiobiz.com/census/Lorain.pdf

(28.25, 1051) which says that I had used a total of 28.25 gallons of gasoline when the odometer read 1051 miles.

Gasoline Used (Gallons)	0	9.26	19.03	28.25	36.45	44.64	53.57	62.62	71.93	81.69	90.43
Odometer (Miles)	41	356	731	1051	1347	1631	1966	2310	2670	3030	3371

Find the least squares line for this data. Is it a good fit? What does the slope of the line represent? Do you and your classmates believe this model would have held for ten years had I not crashed the car on the Turnpike a few years ago?

The regression line is $y = 36.8x + 16.39$ *with* $r = .99987$*, so this is an excellent fit. The slope* 36.8 *represents mileage in miles per gallon.*

Problem 34 *Using the energy production data given below*

Year	1950	1960	1970	1980	1990	2000
Production (in Quads)	35.6	42.8	63.5	67.2	70.7	71.2

1. *Plot the data using a graphing utility and explain why it does not appear to be linear.*
2. *Discuss with your classmates why ignoring the first two data points may be justified from a historical perspective.*
3. *Find the least squares regression line for the last four data points and comment on the goodness of fit. Interpret the slope of the line of best fit.*

   $y = 0.266x - 459.86$ *with* $r = 0.9607$ *which indicates a good fit. The slope* 0.266 *indicates the country's energy production is increasing at a rate of* 0.266 *Quad per year.*
4. *Use the regression line to predict the annual US energy production in the year* 2010*.*

   According to the model, the production in 2010 will be 74.8 *Quad.*
5. *Use the regression line to predict when the annual US energy production will reach* 100 *Quads.*

   According to the model, the production will reach 100 *Quad in the year 2105.*

Question 35 *In Exercises* **??** - **??***, compute the average rate of change of the function over the specified interval.*

Problem 35.1 $f(x) = x^3,\ [-1, 2]$

$$\frac{2^3 - (-1)^3}{2 - (-1)} = 3$$

Problem 35.2 $g(x) = \frac{1}{x},\ [1, 5]$

$$\frac{\frac{1}{5} - \frac{1}{1}}{5 - 1} = -\frac{1}{5}$$

Problem 35.3 $f(t) = \sqrt{t},\ [0, 16]$

$$\frac{\sqrt{16} - \sqrt{0}}{16 - 0} = \frac{1}{4}$$

Problem 35.4 $g(t) = x^2,\ [-3, 3]$

$$\frac{3^2 - (-3)^2}{3 - (-3)} = 0$$

Problem 35.5 $F(s) = \dfrac{s+4}{s-3},\ [5, 7]$

$$\frac{\frac{7+4}{7-3} - \frac{5+4}{5-3}}{7 - 5} = -\frac{7}{8}$$

Problem 35.6 $G(s) = 3s^2 + 2s - 7,\ [-4, 2]$

$$\frac{(3(2)^2 + 2(2) - 7) - (3(-4)^2 + 2(-4) - 7)}{2 - (-4)} = -4$$

Problem 36 *The height of an object dropped from the roof of a building is modeled by:* $h(t) = -16t^2 + 64$, *for* $0 \le t \le 2$. *Here,* $h(t)$ *is the height of the object off the ground in feet* t *seconds after the object is dropped. Find and interpret the average rate of change of* h *over the interval* $[0, 2]$.

The average rate of change is $\dfrac{h(2) - h(0)}{2 - 0} = -32$. *During the first two seconds after it is dropped, the object has fallen at an average rate of* 32 *feet per second.*

Problem 37 *Using data from Bureau of Transportation Statistics, the average fuel economy* $F(t)$ *in miles per gallon for passenger cars in the US can be modeled by* $F(t) = -0.0076t^2 + 0.45t + 16$, $0 \le t \le 28$, *where* t *is the number of years since* 1980. *Find and interpret the average rate of change of* F *over the interval* $[0, 28]$.

The average rate of change is $\dfrac{F(28) - F(0)}{28 - 0} = 0.2372$. *From 1980 to 2008, the average fuel economy of passenger cars in the US increased, on average, at a rate of* 0.2372 *miles per gallon per year.*

Problem 38 *The temperature* $T(t)$ *in degrees Fahrenheit* t *hours after 6 AM is given by:*

$$T(t) = -\frac{1}{2}t^2 + 8t + 32, \quad 0 \le t \le 12$$

1. *Find and interpret* $T(4)$, $T(8)$ *and* $T(12)$.

   $T(4) = 56$, *so at 10 AM (4 hours after 6 AM), it is* 56° *F.* $T(8) = 64$, *so at 2 PM (8 hours after 6 AM), it is* 64° *F.* $T(12) = 56$, *so at 6 PM (12 hours after 6 AM), it is* 56° *F.*

2. *Find and interpret the average rate of change of* T *over the interval* $[4, 8]$.

   The average rate of change is $\dfrac{T(8) - T(4)}{8 - 4} = 2$. *Between 10 AM and 2 PM, the temperature increases, on average, at a rate of* 2° *F per hour.*

3. *Find and interpret the average rate of change of* T *from* $t = 8$ *to* $t = 12$.

   The average rate of change is $\dfrac{T(12) - T(8)}{12 - 8} = -2$. *Between 2 PM and 6 PM, the temperature decreases, on average, at a rate of* 2° *F per hour.*

4. *Find and interpret the average rate of temperature change between 10 AM and 6 PM.*

   The average rate of change is $\dfrac{T(12) - T(4)}{12 - 4} = 0$. *Between 10 AM and 6 PM, the temperature, on average, remains constant.*

Problem 39 *Suppose $C(x) = x^2 - 10x + 27$ represents the costs, in hundreds, to produce x thousand pens. Find and interpret the average rate of change as production is increased from making 3000 to 5000 pens.*

The average rate of change is $\dfrac{C(5) - C(3)}{5 - 3} = -2$. As production is increased from 3000 to 5000 pens, the cost decreases at an average rate of \$200 per 1000 pens produced (20¢ per pen.)

Problem 40 *Recall from Example **??** The formula $s(t) = -5t^2 + 100t$ for $0 \leq t \leq 20$ gives the height, $s(t)$, measured in feet, of a model rocket above the Moon's surface as a function of the time after lift-off, t, in seconds.*

1. *Find and interpret the average rate of change of s over the following intervals:*

   (i) $[14.9, 15]$
   -49.5 *so the average velocity of the rocket between* 14.9 *and* 15 *seconds after lift off is* -49.5 *feet per second (*49.5 *feet per second directed downwards.)*

   (ii) $[15, 15.1]$
   -50.5 *so the average velocity of the rocket between* 14 *and* 15.1 *seconds after lift off is* -50.5 *feet per second. (*50.5 *feet per second directed downwards.)*

   (iii) $[14.99, 15]$
   -49.95 *so the average velocity of the rocket between* 14.99 *and* 15 *seconds after lift off is* -49.95 *feet per second. (*49.95 *feet per second directed downwards.)*

   (iv) $[15, 15.01]$
   -50.05 *so the average velocity of the rocket between* 15.01 *and* 15 *seconds after lift off is* -50.05 *feet per second. (*50.05 *feet per second directed downwards.)*

2. *What value does the average rate of change appear to be approaching as the interval shrinks closer to the value $t = 15$?*

   The average rate of change seems to be approaching -50.

3. *Find the equation of the line containing $(15, 375)$ with slope $m = -50$ and graph it along with s on the same set of axes using a graphing utility. What happens as you zoom in near $(15, 375)$?*

   Line: $y = -50(t - 15) + 375$ or $y = -50t + 1125$. Graphing this line along with the s on a graphing utility we find the two graphs become indistinguishable as we zoom in near $(15, 375)$.

Problem 41 *Show the average rate of change of a function of the form $f(x) = mx + b$ over any interval is m.*

Problem 42 *Why doesn't the graph of the vertical line $x = b$ in the xy-plane represent y as a function of x?*

Problem 43 *With help from a graphing utility, graph the following pairs of functions on the same set of axes:*[4]

- $f(x) = 2 - x$ *and* $g(x) = \lfloor 2 - x \rfloor$
- $f(x) = x^2 - 4$ *and* $g(x) = \lfloor x^2 - 4 \rfloor$
- $f(x) = x^3$ *and* $g(x) = \lfloor x^3 \rfloor$
- $f(x) = \sqrt{x} - 4$ *and* $g(x) = \lfloor \sqrt{x} - 4 \rfloor$

Choose more functions $f(x)$ and graph $y = f(x)$ alongside $y = \lfloor f(x) \rfloor$ until you can explain how, in general, one would obtain the graph of $y = \lfloor f(x) \rfloor$ given the graph of $y = f(x)$.

[4] See Example **??** for the definition of $\lfloor x \rfloor$.

Problem 44 *The Lagrange Interpolate function L for two points (x_0, y_0) and (x_1, y_1) where $x_0 \neq x_1$ is given by:*

$$L(x) = y_0 \frac{x - x_1}{x_0 - x_1} + y_1 \frac{x - x_0}{x_1 - x_0}$$

1. *For each of the following pairs of points, find $L(x)$ using the formula above and verify each of the points lies on the graph of $y = L(x)$.*
   - (i) $(-1, 3)$, $(2, 3)$
     $L(x) = 3$
   - (ii) $(-3, -2)$, $(5, -2)$
     $L(x) = -2$
   - (iii) $(-3, -2)$, $(0, 1)$
     $L(x) = x + 1$
   - (iv) $(-1, 5)$, $(2, -1)$
     $L(x) = -2x + 3$
2. *Verify that, in general, $L(x_0) = y_0$ and $L(x_1) = y_1$.*
3. *Show the point-slope form of a linear function, Equation* **??** *is equivalent to the formula given for $L(x)$ after making the identifications: $f(x_0) = y_0$ and $m = \dfrac{y_1 - y_0}{x_1 - x_0}$.*