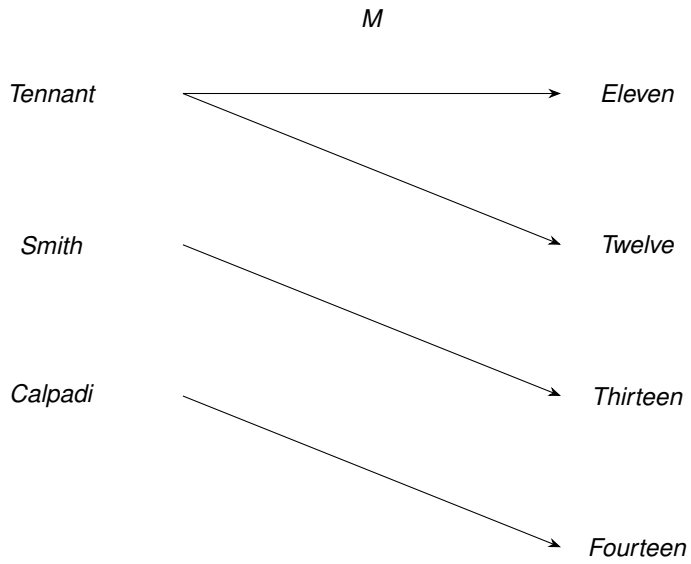
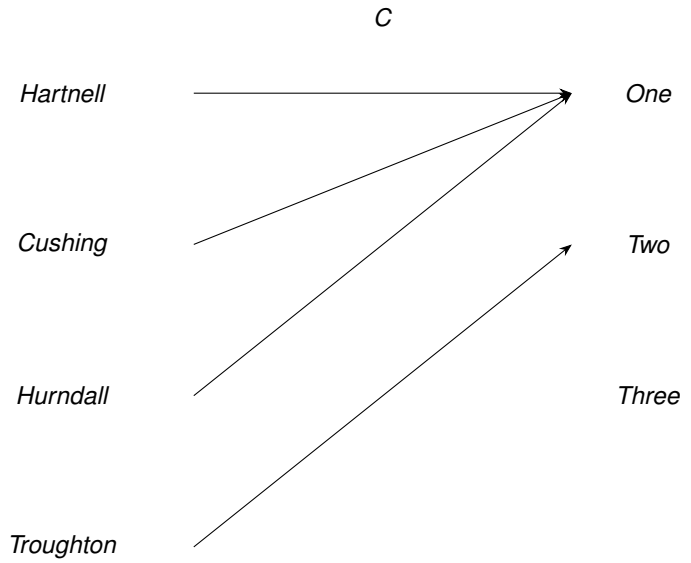
Exercises for Functions and their Representations

Problem 1 *Determine whether or not the mapping diagram represents a function. Explain your reasoning. If the mapping does represent a function, state the domain, range, and represent the function as a set of ordered pairs.*



The mapping M is not a function since 'Tennant' is matched with both 'Eleven' and 'Twelve.'

Problem 2 *Determine whether or not the mapping diagram represents a function. Explain your reasoning. If the mapping does represent a function, state the domain, range, and represent the function as a set of ordered pairs.*



The mapping C is a function since each input is matched with only one output. The domain of C is $\{$*Hartnell*, *Cushing*, *Hurndall*, *Trou*
and the range is $\{$*One*, *Two*$\}$*. We can represent C as the following set of ordered pairs:* $\{$(*Hartnell*, *One*), (*Cushing*, *One*), (*Hurndall*, (

Question 3 *In Exercises* **??** *-* **??***, determine whether or not the data in the given table represents y as a function of x. Explain your reasoning. If the mapping does represent a function, state the domain, range, and represent the function as a set of ordered pairs.*

CC BY-NC-SA 4.0

Problem 3.1

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

In this case, y is a function of x since each x is matched with only one y.

The domain is $\{-3, -2, -1, 0, 1, 2, 3\}$ and the range is $\{0, 1, 2, 3\}$.

As ordered pairs, this function is $\{(-3, 3), (-2, 2), (-1, 1), (0, 0), (1, 1), (2, 2), (3, 3)\}$

Problem 3.2

x	y
0	0
1	1
1	-1
2	2
2	-2
3	3
3	-3

In this case, y is not a function of x since there are x values matched with more than one y value. For instance, 1 is matched both to 1 and -1.

Problem 4 *Suppose W is the set of words in the English language and we set up a mapping from W into the set of natural numbers $\mathbb{N}$ as follows: word $\to$ number of letters in the word. Explain why this mapping is a function. What would you need to know to determine the range of the function?*

The mapping is a function since given any word, there is only one answer to 'how many letters are in the word?' For the range, we would need to know what the length of the longest word is and whether or not we could find words of all the lengths between 1 (the length of the word 'a') and it. See here.

Problem 5 *Suppose L is the set of last names of all the people who have served or are currently serving as the President of the United States. Consider the mapping from L into $\mathbb{N}$ as follows: last name $\to$ number of their presidency. For example, Washington $\to$ 1 and Obama $\to$ 44. Is this mapping a function? What if we use full names instead of just last names?*

Hint: *Research what Grover Cleveland and Donald Trump have in common.*

Since Grover Cleveland was both the 22nd and 24th POTUS, neither mapping described in this exercise is a function. Donald Trump would also have two "outputs".

Problem 6 *Under what conditions would the time of day be a function of the outdoor temperature?*

The outdoor temperature could never be the same for more than two different times - so, for example, it could always be getting warmer or it could always be getting colder.

Question 7 *For the functions f described in Exercises **??** - **??**, find $f(2)$, and find and simplify an expression for $f(x)$ that takes a real number x and performs the following three steps in the order given.*

CC BY-NC-SA 4.0

Problem 7.1 *(1) multiply by 2; (2) add 3; (3) divide by 4.*

$f(2) = \frac{7}{4}$, $f(x) = \frac{2x+3}{4}$

Problem 7.2 *(1) add 3; (2) multiply by 2; (3) divide by 4.*

$f(2) = \frac{5}{2}$, $f(x) = \frac{2(x+3)}{4} = \frac{x+3}{2}$

Problem 7.3 *(1) divide by 4; (2) add 3; (3) multiply by 2.*

$f(2) = 7$, $f(x) = 2\left(\frac{x}{4}+3\right) = \frac{1}{2}x+6$

Problem 7.4 *(1) multiply by 2; (2) add 3; (3) take the square root.*

$f(2) = \sqrt{7}$, $f(x) = \sqrt{2x+3}$

Problem 7.5 *(1) add 3; (2) multiply by 2; (3) take the square root.*

$f(2) = \sqrt{10}$, $f(x) = \sqrt{2(x+3)} = \sqrt{2x+6}$

Problem 7.6 *(1) add 3; (2) take the square root; (3) multiply by 2.*

$f(2) = 2\sqrt{5}$, $f(x) = 2\sqrt{x+3}$

Question 8 *In Exercises* **??** - **??**, *use the given function f to find and simplify the following:*

- $f(3)$
- $f(-1)$
- $f\left(\frac{3}{2}\right)$
- $f(4x)$
- $4f(x)$
- $f(-x)$
- $f(x-4)$
- $f(x)-4$
- $f\left(x^2\right)$

Problem 8.1 $f(x) = 2x+1$

- $f(3) = 7$
- $f(-1) = -1$
- $f\left(\frac{3}{2}\right) = 4$
- $f(4x) = 8x+1$
- $4f(x) = 8x+4$
- $f(-x) = -2x+1$

CC BY-NC-SA 4.0

- $f(x-4) = 2x-7$
- $f(x)-4 = 2x-3$
- $f\left(x^2\right) = 2x^2+1$

Problem 8.2 $f(x) = 3-4x$

For $f(x) = 3-4x$

- $f(3) = -9$
- $f(-1) = 7$
- $f\left(\frac{3}{2}\right) = -3$
- $f(4x) = 3-16x$
- $4f(x) = 12-16x$
- $f(-x) = 4x+3$
- $f(x-4) = 19-4x$
- $f(x)-4 = -4x-1$
- $f\left(x^2\right) = 3-4x^2$

Problem 8.3 $f(x) = 2-x^2$

- $f(3) = -7$
- $f(-1) = 1$
- $f\left(\frac{3}{2}\right) = -\frac{1}{4}$
- $f(4x) = 2-16x^2$
- $4f(x) = 8-4x^2$
- $f(-x) = 2-x^2$
- $f(x-4) = -x^2+8x-14$
- $f(x)-4 = -x^2-2$
- $f\left(x^2\right) = 2-x^4$

Problem 8.4 $f(x) = x^2-3x+2$,

- $f(3) = 2$
- $f(-1) = 6$
- $f\left(\frac{3}{2}\right) = -\frac{1}{4}$
- $f(4x) = 16x^2-12x+2$

CC BY-NC-SA 4.0

- $4f(x) = 4x^2 - 12x + 8$
- $f(-x) = x^2 + 3x + 2$
- $f(x-4) = x^2 - 11x + 30$
- $f(x) - 4 = x^2 - 3x - 2$
- $f\left(x^2\right) = x^4 - 3x^2 + 2$

Problem 8.5 $f(x) = 6$

- $f(3) = 6$
- $f(-1) = 6$
- $f\left(\frac{3}{2}\right) = 6$
- $f(4x) = 6$
- $4f(x) = 24$
- $f(-x) = 6$
- $f(x-4) = 6$
- $f(x) - 4 = 2$
- $f\left(x^2\right) = 6$

Problem 8.6 $f(x) = 0$

- $f(3) = 0$
- $f(-1) = 0$
- $f\left(\frac{3}{2}\right) = 0$
- $f(4x) = 0$
- $4f(x) = 0$
- $f(-x) = 0$
- $f(x-4) = 0$
- $f(x) - 4 = -4$
- $f\left(x^2\right) = 0$

Question 9 *In Exercises* **??** - **??**, *use the given function f to find and simplify the following:*

- $f(2)$
- $f(-2)$
- $f(2a)$
- $2f(a)$

CC BY-NC-SA 4.0

- $f(a+2)$
- $f(a)+f(2)$
- $f\left(\frac{2}{a}\right)$
- $\frac{f(a)}{2}$
- $f(a+h)$

Problem 9.1 $f(x)=2x-5$

- $f(2)=-1$
- $f(-2)=-9$
- $f(2a)=4a-5$
- $2f(a)=4a-10$
- $f(a+2)=2a-1$
- $f(a)+f(2)=2a-6$
- $f\left(\frac{2}{a}\right)=\frac{4}{a}-5$
- $\frac{f(a)}{2}=\frac{2a-5}{2}$
- $f(a+h)=2a+2h-5$

Problem 9.2 $f(t)=5-2t$

- $f(2)=1$
- $f(-2)=9$
- $f(2a)=5-4a$
- $2f(a)=10-4a$
- $f(a+2)=1-2a$
- $f(a)+f(2)=6-2a$
- $f\left(\frac{2}{a}\right)=5-\frac{4}{a}$
- $\frac{f(a)}{2}=\frac{5-2a}{2}$
- $f(a+h)=5-2a-2h$

Problem 9.3 $f(w)=2w^2-1$

- $f(2)=7$
- $f(-2)=7$

CC BY-NC-SA 4.0

- $f(2a) = 8a^2 - 1$
- $2f(a) = 4a^2 - 2$
- $f(a+2) = 2a^2 + 8a + 7$
- $f(a) + f(2) = 2a^2 + 6$
- $f\left(\frac{2}{a}\right) = \frac{8}{a^2} - 1$
- $\frac{f(a)}{2} = \frac{2a^2 - 1}{2}$
- $f(a+h) = 2a^2 + 4ah + 2h^2 - 1$

Problem 9.4 $f(q) = 3q^2 + 3q - 2$

- $f(2) = 16$
- $f(-2) = 4$
- $f(2a) = 12a^2 + 6a - 2$
- $2f(a) = 6a^2 + 6a - 4$
- $f(a+2) = 3a^2 + 15a + 16$
- $f(a) + f(2) = 3a^2 + 3a + 14$
- $f\left(\frac{2}{a}\right) = \frac{12}{a^2} + \frac{6}{a} - 2$
- $\frac{f(a)}{2} = \frac{3a^2 + 3a - 2}{2}$
- $f(a+h) = 3a^2 + 6ah + 3h^2 + 3a + 3h - 2$

Problem 9.5 $f(r) = 117$

- $f(2) = 117$
- $f(-2) = 117$
- $f(2a) = 117$
- $2f(a) = 234$
- $f(a+2) = 117$
- $f(a) + f(2) = 117$
- $f\left(\frac{2}{a}\right) = 117$
- $\frac{f(a)}{2} = \frac{117}{2}$
- $f(a+h) = 117$

Problem 9.6 $f(z) = \frac{z}{2}$

CC BY-NC-SA 4.0

- $f(2) = 1$
- $f(-2) = -1$
- $f(2a) = a$
- $2f(a) = a$
- $f(a+2) = \dfrac{a+2}{2}$
- $f(a) + f(2) = \dfrac{a}{2} + 1$
- $f\left(\dfrac{2}{a}\right) = \dfrac{1}{a}$
- $\dfrac{f(a)}{2} = \dfrac{a}{4}$
- $f(a+h) = \dfrac{a+h}{2}$

Question 10 *In Exercises* **??** - **??**, *use the given function* f *to find* $f(0)$ *and solve* $f(x) = 0$.

Problem 10.1 $f(x) = 2x - 1$

$f(0) = -1$

Solving $f(x) = 0$ *gives us*

Select All Correct Answers:

1. $x = -1$
2. $x = -\dfrac{1}{2}$
3. $x = 0$
4. $x = \dfrac{1}{2}$ ✓
5. $x = 1$

Problem 10.2 $f(x) = 3 - \dfrac{2}{5}x$

$f(0) = 3$

Solving $f(x) = 0$ *gives us*

Select All Correct Answers:

1. $x = 0$
2. $x = \dfrac{6}{5}$
3. $x = 3$
4. $x = \dfrac{5}{13}$
5. $x = 5$

CC BY-NC-SA 4.0

6. $x = \frac{15}{2}$ ✓

Problem 10.3 $f(x) = 2x^2 - 6$,

$f(0) = -6$

Solving $f(x) = 0$ *gives us*

Select All Correct Answers:

1. $x = -6$
2. $x = -3$
3. $x = -\sqrt{3}$ ✓
4. $x = 0$
5. $x = \sqrt{3}$ ✓
6. $x = 3$
7. $x = 6$

Problem 10.4 $f(x) = x^2 - x - 12$

$f(0) = -12$

Solving $f(x) = 0$ *gives us*

Select All Correct Answers:

1. $x = -12$
2. $x = -4$
3. $x = -3$ ✓
4. $x = 0$
5. $x = 3$
6. $x = 4$ ✓
7. $x = 12$

Question 11 *In Exercises* **??** - **??**, *determine whether or not the following equation represents* y *as a function of* x.

Problem 11.1 $y = x^3 - x$

Multiple Choice:

1. *Yes,* y *is a function of* x. ✓
2. *No,* y *is a not a function of* x.

Problem 11.2 $y = \sqrt{x - 2}$

CC BY-NC-SA 4.0

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

Problem 11.3 $x^3y = -4$

Multiple Choice:

1. *Yes, y is a function of x.* ✓

   Feedback(`attempt`***):*** *Yes, in fact, $y = \dfrac{-4}{x^3}$.*

2. *No, y is a not a function of x.*

Problem 11.4 $x^2 - y^2 = 1$

Multiple Choice:

1. *Yes, y is a function of x.*
2. *No, y is a not a function of x.* ✓

   Feedback(`attempt`***):*** *We cannot solve for y without using $\pm$ when we take the square root of both sides of our equation.*

Problem 11.5 $y = \dfrac{x}{x^2 - 9}$

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

Problem 11.6 $x = -6$

Multiple Choice:

1. *Yes, y is a function of x.*
2. *No, y is a not a function of x.* ✓

Problem 11.7 $x = y^2 + 4$

Multiple Choice:

1. *Yes, y is a function of x.*
2. *No, y is a not a function of x.* ✓

   Feedback(`attempt`***):*** *We cannot solve for y without using $\pm$ when we take the square root of both sides of our equation.*

CC BY-NC-SA 4.0

Problem 11.8 $y = x^2 + 4$

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

Problem 11.9 $x^2 + y^2 = 4$

Multiple Choice:

1. *Yes, y is a function of x.*
2. *No, y is a not a function of x.* ✓

   Feedback(`attempt`***):*** *We cannot solve for y without using $\pm$ when we take the square root of both sides of our equation.*

Problem 11.10 $y = \sqrt{4 - x^2}$

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

Problem 11.11 $x^2 - y^2 = 4$

Multiple Choice:

1. *Yes, y is a function of x.*
2. *No, y is a not a function of x.* ✓

   Feedback(`attempt`***):*** *We cannot solve for y without using $\pm$ when we take the square root of both sides of our equation.*

Problem 11.12 $x^3 + y^3 = 4$

Multiple Choice:

1. *Yes, y is a function of x.* ✓

   Feedback(`attempt`***):*** *Yes, in fact, $y = \sqrt[3]{4 - x^3}$.*

2. *No, y is a not a function of x.*

Problem 11.13 $2x + 3y = 4$

Multiple Choice:

1. *Yes, y is a function of x.* ✓

CC BY-NC-SA 4.0

Feedback(*attempt*): *Yes, in fact,* $y = -\frac{2}{3}x + \frac{4}{3}$.

2. *No,* y *is a not a function of* x.

Problem 11.14 $2xy = 4$

Multiple Choice:

1. *Yes,* y *is a function of* x. ✓

   Feedback(*attempt*): *Yes, in fact,* $y = \frac{4}{2x}$.

2. *No,* y *is a not a function of* x.

Problem 11.15 $x^2 = y^2$

Multiple Choice:

1. *Yes,* y *is a function of* x.
2. *No,* y *is a not a function of* x. ✓

   Feedback(*attempt*): *We cannot solve for y without using* $\pm$ *when we take the square root of both sides of our equation.*

Question 12 *Exercises* **??** - **??** *give a set of points in the xy-plane. Determine if* y *is a function of* x. *If so, state the domain and range.*

Problem 12.1 $\{(-3,9),\ (-2,4),\ (-1,1),\ (0,0),\ (1,1),\ (2,4),\ (3,9)\}$

Multiple Choice:

1. *Yes,* y *is a function of* x. ✓
2. *No,* y *is a not a function of* x.

   Feedback(*attempt*): *domain* $= \{-3, -2, -1, 0, 1, 2, 3\}$
   range $= \{0, 1, 4, 9\}$

Problem 12.2 $\{(-3,0),(1,6),(2,-3),(4,2),(-5,6),(4,-9),(6,2)\}$

Multiple Choice:

1. *Yes,* y *is a function of* x.
2. *No,* y *is a not a function of* x. ✓

   Feedback(*attempt*): *Notice* 4 *has two "outputs".*

Problem 12.3 $\{(-3,0),(-7,6),(5,5),(6,4),(4,9),(3,0)\}$

Multiple Choice:

CC BY-NC-SA 4.0

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

   Feedback(`attempt`***):*** *domain* $= \{-7, -3, 3, 4, 5, 6\}$
   range $= \{0, 4, 5, 6, 9\}$

Problem 12.4 $\{(1, 2), (4, 4), (9, 6), (16, 8), (25, 10), (36, 12), ...\}$

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

   Feedback(`attempt`***):*** *domain* $= \{1, 4, 9, 16, 25, 36, ...\}$
   $= \{x \mid x$ *is a perfect square*$\}$
   range $= \{2, 4, 6, 8, 10, 12, ...\}$
   $= \{y \mid y$ *is a positive even integer*$\}$

Problem 12.5 $\{(x, y) \mid x$ *is an odd integer, and* y *is an even integer*$\}$

Multiple Choice:

1. *Yes, y is a function of x.*
2. *No, y is a not a function of x.* ✓

   Feedback(`attempt`***):*** *Notice that each "input" has many "outputs".*

Problem 12.6 $\{(x, 1) \mid x$ *is an irrational number*$\}$

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

   Feedback(`attempt`***):*** *domain* $= \{x \mid x$ *is irrational*$\}$
   range $= \{1\}$

Problem 12.7 $\{(1, 0), (2, 1), (4, 2), (8, 3), (16, 4), (32, 5), ...\}$

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

   Feedback(`attempt`***):*** *domain* $= \{x \mid 1, 2, 4, 8, ...\}$
   $= \{x \mid x = 2^n$ *for some whole number* $n\}$
   range $= \{0, 1, 2, 3, ...\}$
   $= \{y \mid y$ *is any whole number*$\}$

Problem 12.8 $\{... (-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), ...\}$

CC BY-NC-SA 4.0

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

   Feedback(`attempt`***):*** *domain* $= \{x \mid x \text{ is any integer}\}$
   range $= \{y \mid y \text{ is the square of an integer}\}$

Problem 12.9 $\{(-2, y) \mid -3 < y < 4\}$

Multiple Choice:

1. *Yes, y is a function of x.*
2. *No, y is a not a function of x.* ✓

   Feedback(`attempt`***):*** *Notice* -2 *has many"outputs".*

Problem 12.10 $\{(x, 3) \mid -2 \leq x < 4\}$

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

   Feedback(`attempt`***):*** *domain* $= \{x \mid -2 \leq x < 4\} = [-2, 4)$,
   range $= \{3\}$

Problem 12.11 $\{\left(x, x^2\right) \mid x \text{ is a real number}\}$

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

   Feedback(`attempt`***):*** *domain* $= \{x \mid x \text{ is a real number}\} = (-\infty, \infty)$
   range $= \{y \mid y \geq 0\} = [0, \infty)$

Problem 12.12 $\{\left(x^2, x\right) \mid x \text{ is a real number}\}$

Multiple Choice:

1. *Yes, y is a function of x.*
2. *No, y is a not a function of x.* ✓

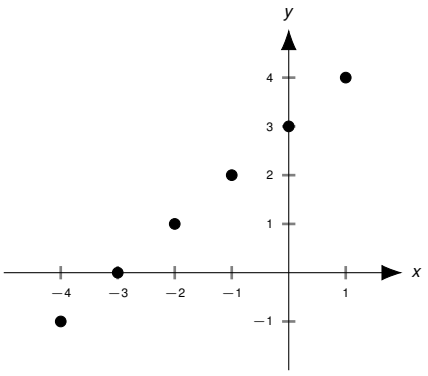
   Feedback(`attempt`***):*** *Notice* $(4, 2)$ *and* $(4, -2)$ *are both in this set, for example.*

Problem 13 *The Vertical Line Test is a quick way to determine from a graph if the vertical axis variable is a function of the horizontal axis variable. If we are given a graph and asked to determine if the horizontal axis variable is a function of the vertical axis variable, we can use horizontal lines instead of vertical lines to check. Using Theorem* **??** *as a guide, formulate a 'Horizontal Line Test.' (We'll refer back to this exercise in Section* **??***.)*

Horizontal Line Test: *A graph on the xy-plane represents x as a function of y if and only if no* ***horizontal*** *line intersects the graph more than once.*

CC BY-NC-SA 4.0

In Exercises **??** - **??**, determine whether or not the graph suggests y is a function of x. For the ones which do, state the domain and range.

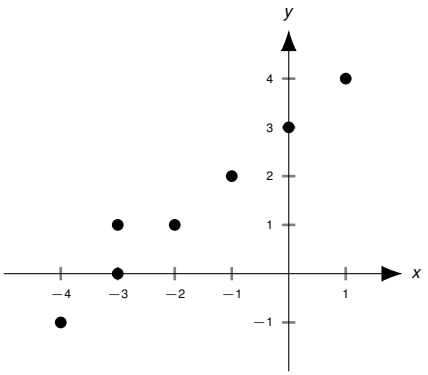


Problem 14

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

   Feedback(`attempt`***):*** *domain* $= \{-4, -3, -2, -1, 0, 1\}$
   range $= \{-1, 0, 1, 2, 3, 4\}$

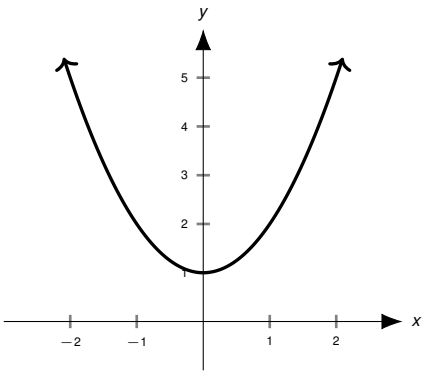


Problem 15

Multiple Choice:

1. *Yes, y is a function of x.*
2. *No, y is a not a function of x.* ✓

   Feedback(`attempt`***):*** *Notice the vertical line test fails for* $x = -3$.



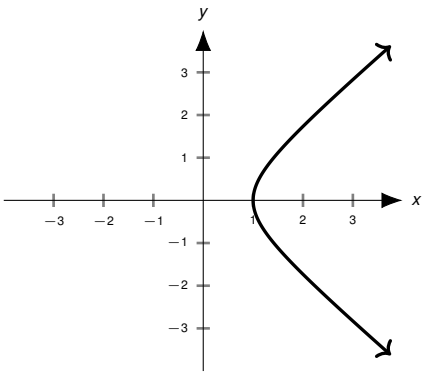
Problem 16

CC BY-NC-SA 4.0

Multiple Choice:

1. *Yes, y is a function of x.* ✓
2. *No, y is a not a function of x.*

Feedback(`attempt`***):*** *domain* = $(-\infty, \infty)$
range = $[1, \infty)$



Problem 17

Multiple Choice:

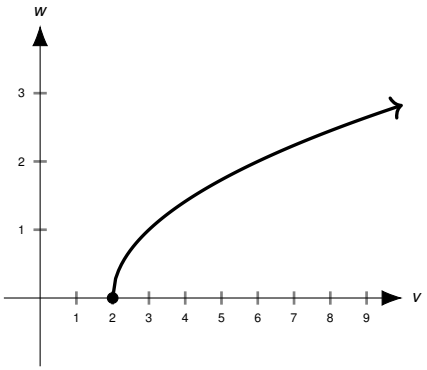
1. *Yes, y is a function of x.*
2. *No, y is a not a function of x.* ✓

Feedback(`attempt`***):*** *Notice the vertical line test fails many places.*

Problem 18 *Determine which of the graphs, if any, in Exercises* **??** *-* **??***, suggest x is a function of y. (Feel free to use Exercise* **??***.) If it does, state the domain and range.*

- *Number* **??** *represents x as a function of y.*
  domain = $\{-1, 0, 1, 2, 3, 4\}$ *and range* = $\{-4, -3, -2, -1, 0, 1\}$
- *Number* **??** *represents x as a function of y.*
  domain = $(-\infty, \infty)$ *and range* = $[1, \infty)$

Question 19 *In Exercises* **??** *-* **??***, determine whether or not the graph suggests w is a function of v. For the ones which do, state the domain and range.*



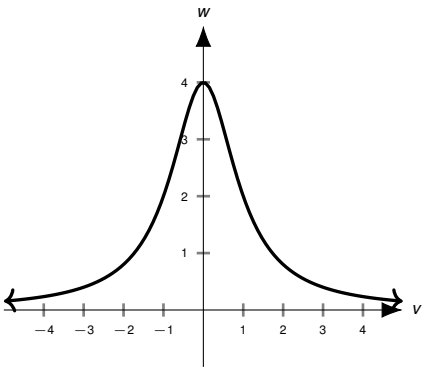
Problem 19.1

CC BY-NC-SA 4.0

Multiple Choice:

1. *Yes, w is a function of v.* ✓
2. *No, w is a not a function of v.*

Feedback(`attempt`***):*** *domain* = $[2, \infty)$
range = $[0, \infty)$

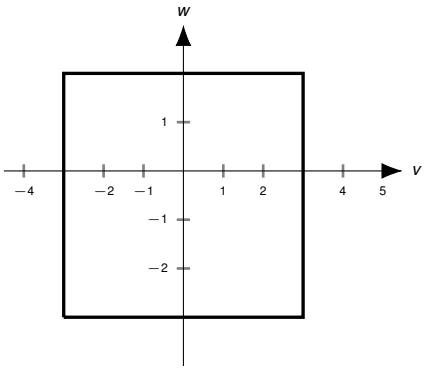


Problem 19.2

Multiple Choice:

1. *Yes, w is a function of v.* ✓
2. *No, w is a not a function of v.*

Feedback(`attempt`***):*** *domain* = $[-\infty, \infty)$
range = $(0, \infty)$



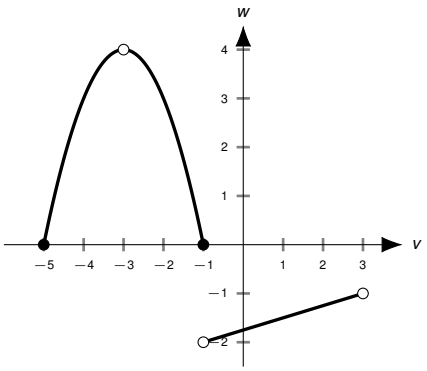
Problem 19.3

Multiple Choice:

1. *Yes, w is a function of v.*
2. *No, w is a not a function of v.* ✓

Feedback(`attempt`***):*** *The vertical line test fails in many places.*

CC BY-NC-SA 4.0



Problem 19.4

Multiple Choice:

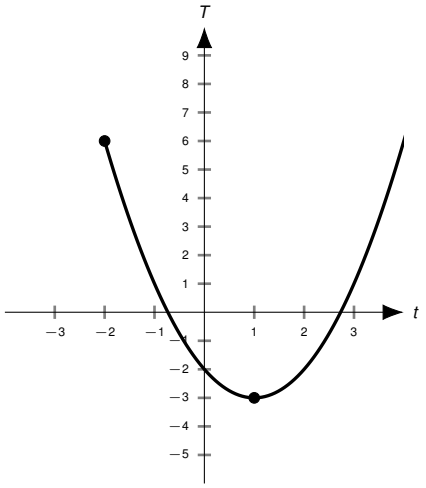
1. *Yes, w is a function of v.* ✓
2. *No, w is a not a function of v.*

   Feedback(`attempt`***):*** *domain* = $[-5,-3)\cup(-3,3)$
   range = $(-2,-1)\cup[0,4)$

Problem 20 *Determine which of the graphs, if any, in Exercises* **??** *-* **??***, suggest v is a function of w. (Feel free to use Exercise* **??***.) If it does, state the domain and range.*

Only number **??** *represents v as a function of w; domain* = $[0,\infty)$ *and range* = $[2,\infty)$

Question 21 *In Exercises* **??** *-* **??***, determine whether or not the graph suggests T is a function of t. For the ones which do, state the domain and range.*



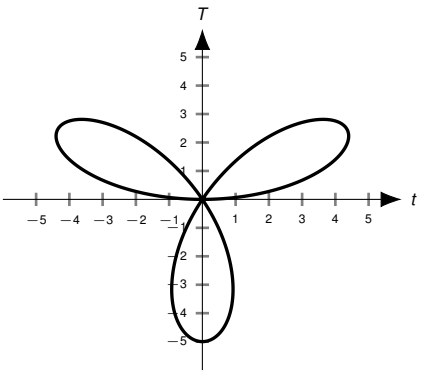
Problem 21.1

Multiple Choice:

1. *Yes, T is a function of t.* ✓
2. *No, T is a not a function of t.*

   Feedback(`attempt`***):*** *domain* = $[-2,\infty)$
   range = $[-3,\infty)$

CC BY-NC-SA 4.0



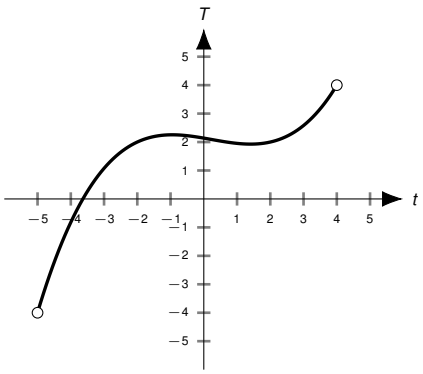
Problem 21.2

Multiple Choice:

1. *Yes, T is a function of t.*
2. *No, T is a not a function of t.* ✓

   Feedback(`attempt`***):*** *The vertical line test fails in many places.*

Problem 21.3 *Determine whether or not the graph suggests T is a function of t If it does, state the domain and range.*



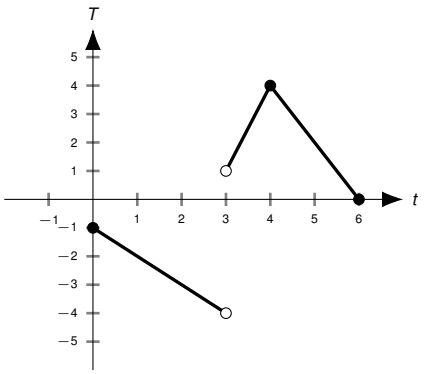
Multiple Choice:

1. *Yes, T is a function of t.* ✓
2. *No, T is a not a function of t.*

   Feedback(`attempt`***):*** *domain* = $(-5, 4)$
   range = $(-4, 4)$

Problem 21.4 *Determine whether or not the graph suggests T is a function of t If it does, state the domain and range.*

CC BY-NC-SA 4.0



Multiple Choice:

1. *Yes, T is a function of t.* ✓
2. *No, T is a not a function of t.*

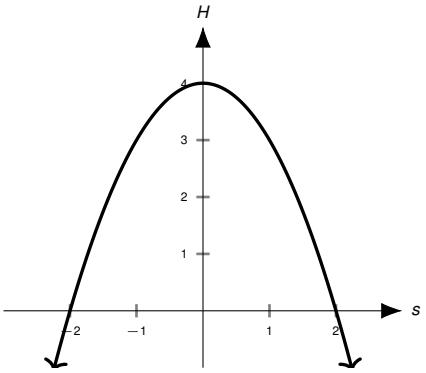
Feedback(`attempt`***):*** *domain* = $[0, 3) \cup (3, 6]$
range = $(-4, -1] \cup [0, 4]$

Problem 22 *Determine which of the graphs, if any, in Exercises* **??** - **??**, *suggest t is a function of T. (Feel free to use Exercise* **??**.*) If it does, state the domain and range.*

None of numbers **??** - **??** *represent t as a function of T.*

Question 23 *In Exercises* **??** - **??**, *determine whether or not the graph suggests H is a function of s. For the ones which do, state the domain and range.*

Problem 23.1

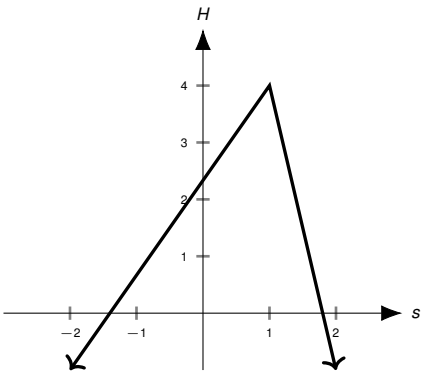


Multiple Choice:

1. *Yes, H is a function of s.* ✓
2. *No, H is a not a function of s.*

Feedback(`attempt`***):*** *domain* = $(-\infty, \infty)$
range = $(-\infty, 4]$

CC BY-NC-SA 4.0

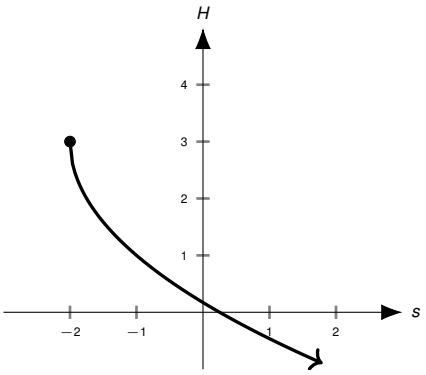


Problem 23.2

Multiple Choice:

1. *Yes, H is a function of s.* ✓
2. *No, H is a not a function of s.*

   Feedback(`attempt`***):*** *domain* = $(-\infty, \infty)$
   range = $(-\infty, 4]$

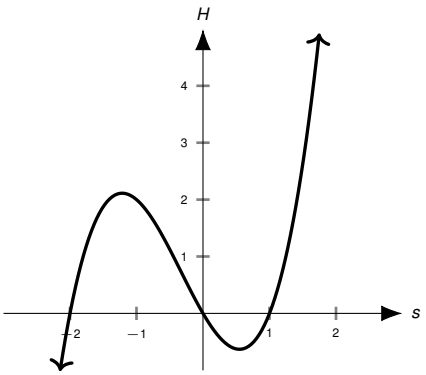


Problem 23.3

Multiple Choice:

1. *Yes, H is a function of s.* ✓
2. *No, H is a not a function of s.*

   Feedback(`attempt`***):*** *domain* = $[-2, \infty)$
   range = $(-\infty, 3]$



Problem 23.4

CC BY-NC-SA 4.0

Multiple Choice:

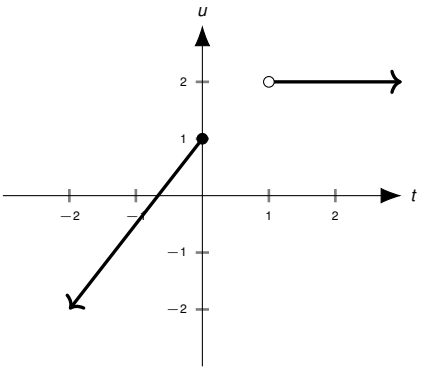
1. *Yes, H is a function of s.* ✓
2. *No, H is a not a function of s.*

Feedback(`attempt`***):*** *domain* = $(-\infty, \infty)$
range = $(-\infty, \infty)$

Problem 24 *Determine which, if any, of the graphs in numbers* **??** *-* **??** *represent s as a function of H. For the ones which do, state the domain and range. (Feel free to use Exercise* **??***.)*

Only number **??** *represents s as a function of H; domain* = $(-\infty, 3]$ *and range* = $[-2, \infty)$

Question 25 *In Exercises* **??** *-* **??***, determine whether or not the graph suggests u is a function of t. For the ones which do, state the domain and range.*

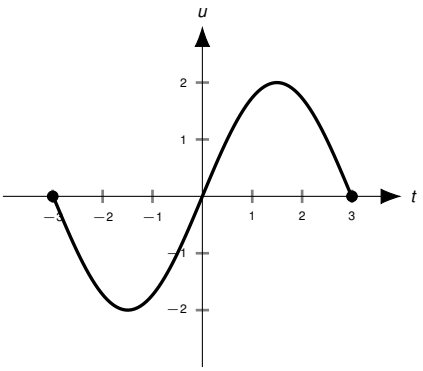


Problem 25.1

Multiple Choice:

1. *Yes, u is a function of t.* ✓
2. *No, u is a not a function of t.*

Feedback(`attempt`***):*** *domain* = $(-\infty, 0] \cup (1, \infty)$
range = $(-\infty, 1] \cup \{2\}$



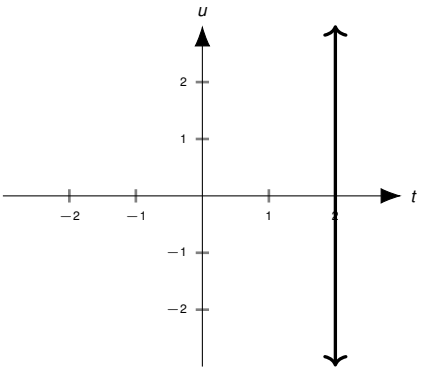
Problem 25.2

Multiple Choice:

1. *Yes, u is a function of t.* ✓

2. *No, u is a not a function of t.*

   Feedback(`attempt`***):*** *domain* = $[-3, 3]$
   range = $[-2, 2]$

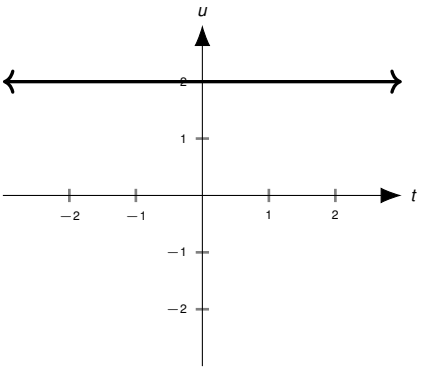


Problem 25.3

Multiple Choice:

1. *Yes, u is a function of t.*
2. *No, u is a not a function of t.* ✓

   Feedback(`attempt`***):*** *The vertical line test fails at t* = 2.



Problem 25.4

Multiple Choice:

1. *Yes, u is a function of t.* ✓
2. *No, u is a not a function of t.*

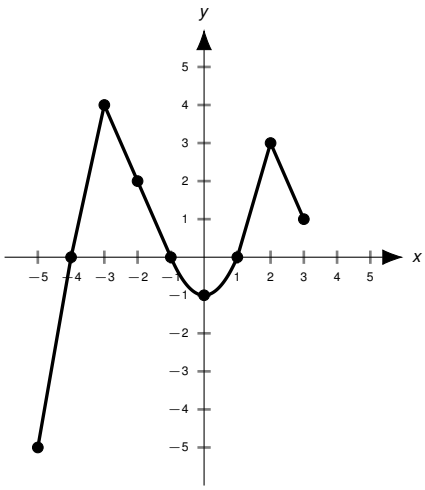
   Feedback(`attempt`***):*** *domain* = $(-\infty, \infty)$
   range = $\{2\}$

Problem 26 *Determine which, if any, of the graphs in numbers* **??** *-* **??** *represent t as a function of u. For the ones which do, state the domain and range. (Feel free to use Exercise* **??***.)*

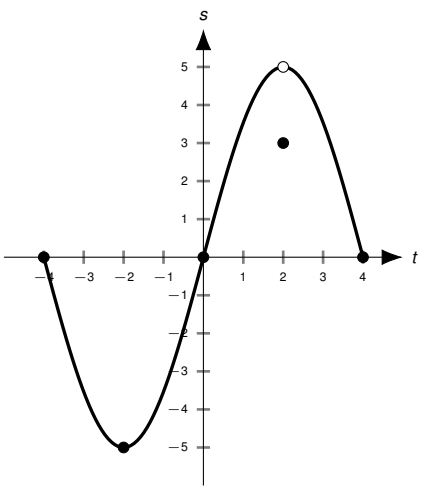
Only number **??** *represents t as a function of u; domain* = $(-\infty, \infty)$ *and range*=$\{2\}$.

Question 27 *In Exercises* **??** *-* **??***, use the graphs of f and g below to find the indicated values.*

CC BY-NC-SA 4.0



The graph of $y = f(x)$.



The graph of $s = g(t)$.

Problem 27.1 $f(-2) = 2$

Problem 27.2 $g(-2) = -5$

Problem 27.3 $f(2) = 3$

Problem 27.4 $g(2) = 3$

Problem 27.5 $f(0) = -1$

Problem 27.6 $g(0) = 0$

Problem 27.7 *Solving $f(x) = 0$ gives us*

Select All Correct Answers:

1. $x = -4$ ✓
2. $x = -3$
3. $x = -1$ ✓
4. $x = 0$
5. $x = 4$

Problem 27.8 *Solving $g(t) = 0$ gives us*

Select All Correct Answers:

1. $t = -4$ ✓
2. $t = -2$
3. $t = 0$ ✓
4. $t = 2$

CC BY-NC-SA 4.0

5. $t = 4$ ✓

Problem 27.9 *State the domain and range of f .*

Domain: $[-5, 3]$*, Range:* $[-5, 4]$*.*

Problem 27.10 *State the domain and range of g .*

Domain: $[-4, 4]$*, Range:* $[-5, 5)$*.*

Question 28 *In Exercises* **??** *-* **??***, graph each function by making a table, plotting points, and using a graphing utility (if needed.) Use the independent variable as the horizontal axis label and the default 'y' label for the vertical axis label. State the domain and range of each function.*

Problem 28.1 $f(x) = 2 - x$

Domain: $(-\infty, \infty)$ *Range:* $(-\infty, \infty)$

Graph of $f(x) = 2 - x$

Problem 28.2 $g(t) = \dfrac{t-2}{3}$

Domain: $(-\infty, \infty)$ *Range:* $(-\infty, \infty)$

Graph of $g(t) = \dfrac{t-2}{3}$

Problem 28.3 $h(s) = s^2 + 1$

Domain: $(-\infty, \infty)$ *Range:* $[1, \infty)$

Graph of $h(s) = s^2 + 1$

Problem 28.4 $f(x) = 4 - x^2$

Domain: $(-\infty, \infty)$ *Range:* $(-\infty, 4]$

Graph of $f(x) = 4 - x^2$

Problem 28.5 $g(t) = 2$

Domain: $(-\infty, \infty)$ *Range:* 2

Graph of $g(t) = 2$

Problem 28.6 $h(s) = s^3$

Domain: $(-\infty, \infty)$ *Range:* $(-\infty, \infty)$

Graph of $h(s) = s^3$

Problem 28.7 $f(x) = x(x-1)(x+2)$

Domain: $(-\infty, \infty)$ *Range:* $(-\infty, \infty)$

Graph of $f(x) = x(x-1)(x+2)$

Problem 28.8 $g(t) = \sqrt{t-2}$

Domain: $[2, \infty)$ *Range:* $[0, \infty)$

Graph of $g(t) = \sqrt{t-2}$

Problem 28.9 $h(s) = \sqrt{5-s}$

Domain: $(-\infty, 5]$ *Range:* $[0, \infty)$

Graph of $h(s) = \sqrt{5-s}$

CC BY-NC-SA 4.0

Problem 28.10 $f(x) = 3 - 2\sqrt{x+2}$

Domain: $[-2, \infty)$ *Range:* $(-\infty, 3]$

Graph of $f(x) = 3 - 2\sqrt{x+2}$

Problem 28.11 $g(t) = \sqrt[3]{t}$

Domain: $(-\infty, \infty)$ *Range:* $(-\infty, \infty)$

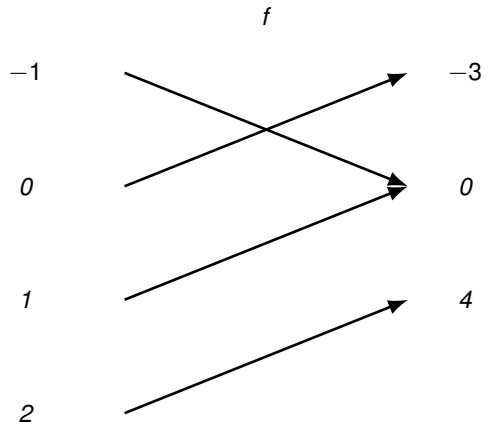
Graph of $g(t) = \sqrt[3]{t}$

Problem 28.12 $h(s) = \dfrac{1}{s^2+1}$

Domain: $(-\infty, \infty)$ *Range:* $(0, 1]$

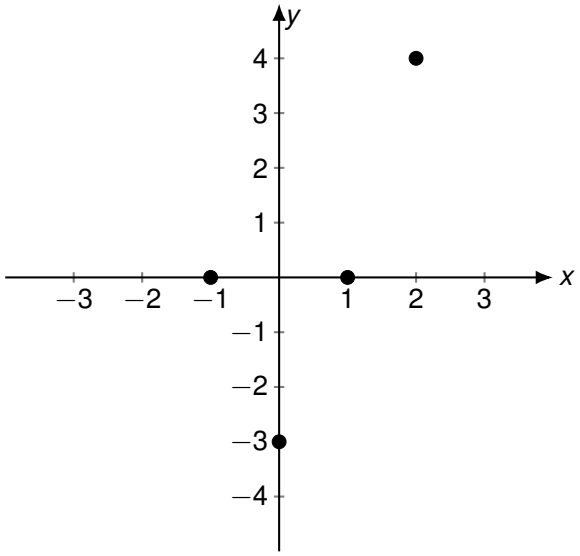
Graph of $h(s) = \dfrac{1}{s^2+1}$

Problem 29 *Consider the function f described below:*



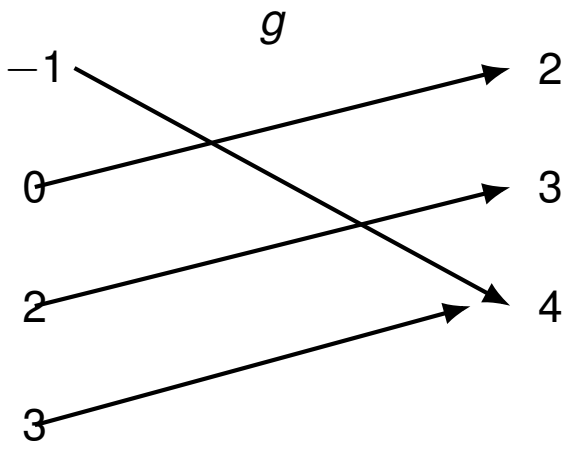
1. *State the domain and range of f.*
   domain $= \{-1, 0, 1, 2\}$, *range* $= \{-3, 0, 4\}$
2. *Find* $f(0)$ *and solve* $f(x) = 0$.
   $f(0) = -3$, $f(x) = 0$ *for* $x = -1, 1$.
3. *Write f as a set of ordered pairs.*
   $f = \{(-1, 0), (0, -3), (1, 0), (2, 4)\}$
4. *Graph f.*

CC BY-NC-SA 4.0



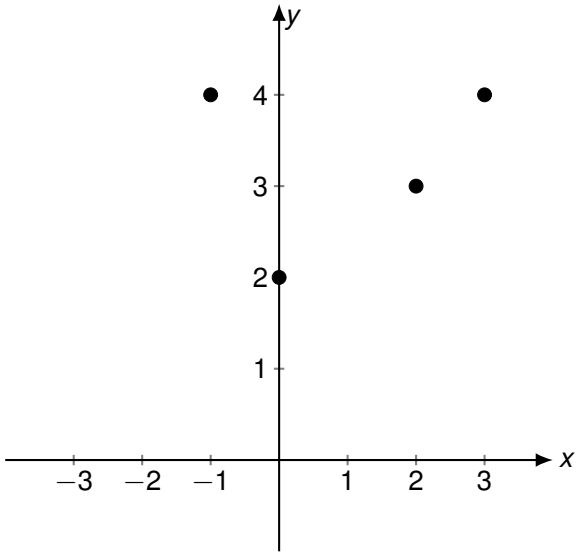
Problem 30 *Let* $g = \{(-1, 4), (0, 2), (2, 3), (3, 4)\}$

1. *State the domain and range of g.*
   domain $= \{-1, 0, 2, 3\}$*, range* $= \{2, 3, 4\}$
2. *Create a mapping diagram for g.*



3. *Find* $g(0)$ *and solve* $g(x) = 0$*.*
   $g(0) = 2$ *but* $g(x) = 0$ *has no solutions.*
4. *Graph g.*

CC BY-NC-SA 4.0



Problem 31 *Let $F = \{(t, t^2) \mid t \text{ is a real number}\}$. Find $F(4)$ and solve $F(x) = 4$.*

Hint: *Elements of F are of the form $(x, F(x))$.*

$F(4) = 4^2 = 16$ (when $t = 4$), the solutions to $F(x) = 4$ are $x = \pm 2$ (when $t = \pm 2$).

Problem 32 *Let $G = \{(2t, t + 5) \mid t \text{ is a real number}\}$. Find $G(4)$ and solve $G(x) = 4$.*

Hint: *Elements of G are of the form $(x, G(x))$.*

$G(4) = 7$ (when $t = 2$), the solution to $G(t) = 4$ is $x = -2$ (when $t = -1$)

Problem 33 *The area enclosed by a square, in square inches, is a function of the length of one of its sides ℓ, when measured in inches. This function is represented by the formula $A(\ell) = \ell^2$ for $\ell > 0$. Find $A(3)$ and solve $A(\ell) = 36$. Interpret your answers to each. Why is ℓ restricted to $\ell > 0$?*

$A(3) = 9$, so the area enclosed by a square with a side of length 3 inches is 9 square inches. The solutions to $A(\ell) = 36$ are $\ell = \pm 6$. Since ℓ is restricted to $\ell > 0$, we only keep $\ell = 6$. This means for the area enclosed by the square to be 36 square inches, the length of the side needs to be 6 inches. Since ℓ represents a length, $\ell > 0$.

Problem 34 *The area enclosed by a circle, in square meters, is a function of its radius r, when measured in meters. This function is represented by the formula $A(r) = \pi r^2$ for $r > 0$. Find $A(2)$ and solve $A(r) = 16\pi$. Interpret your answers to each. Why is r restricted to $r > 0$?*

$A(2) = 4\pi$, so the area enclosed by a circle with radius 2 meters is 4π square meters. The solutions to $A(r) = 16\pi$ are $r = \pm 4$. Since r is restricted to $r > 0$, we only keep $r = 4$. This means for the area enclosed by the circle to be 16π square meters, the radius needs to be 4 meters. Since r represents a radius (length), $r > 0$.

Problem 35 *The volume enclosed by a cube, in cubic centimeters, is a function of the length of one of its sides s, when measured in centimeters. This function is represented by the formula $V(s) = s^3$ for $s > 0$. Find $V(5)$ and solve $V(s) = 27$. Interpret your answers to each. Why is s restricted to $s > 0$? $V(5) = 125$, so the volume enclosed by a cube with a side of length 5 centimeters is 125 cubic centimeters. The solution to $V(s) = 27$ is $s = 3$. This means for the volume enclosed by the cube to be 27 cubic centimeters, the length of the side needs to 3 centimeters. Since x represents a length, $x > 0$.*

CC BY-NC-SA 4.0

Problem 36 *The volume enclosed by a sphere, in cubic feet, is a function of the radius of the sphere r, when measured in feet. This function is represented by the formula $V(r) = \frac{4\pi}{3}r^3$ for $r > 0$. Find $V(3)$ and solve $V(r) = \frac{32\pi}{3}$. Interpret your answers to each. Why is r restricted to $r > 0$?*

$V(3) = 36\pi$, so the volume enclosed by a sphere with radius 3 feet is 36π cubic feet. The solution to $V(r) = \frac{32\pi}{3}$ is $r = 2$. This means for the volume enclosed by the sphere to be $\frac{32\pi}{3}$ cubic feet, the radius needs to 2 feet. Since r represents a radius (length), $r > 0$.

Problem 37 *The height of an object dropped from the roof of an eight story building is modeled by the function: $h(t) = -16t^2 + 64$, $0 \leq t \leq 2$. Here, $h(t)$ is the height of the object off the ground, in feet, t seconds after the object is dropped. Find $h(0)$ and solve $h(t) = 0$. Interpret your answers to each. Why is t restricted to $0 \leq t \leq 2$?*

$h(0) = 64$, so at the moment the object is dropped off the building, the object is 64 feet off of the ground. The solutions to $h(t) = 0$ are $t = \pm 2$. Since we restrict $0 \leq t \leq 2$, we only keep $t = 2$. This means 2 seconds after the object is dropped off the building, it is 0 feet off the ground. Said differently, the object hits the ground after 2 seconds. The restriction $0 \leq t \leq 2$ restricts the time to be between the moment the object is released and the moment it hits the ground.

Problem 38 *The temperature in degrees Fahrenheit t hours after 6 AM is given by $T(t) = -\frac{1}{2}t^2 + 8t + 3$ for $0 \leq t \leq 12$. Find and interpret $T(0)$, $T(6)$ and $T(12)$.*

$T(0) = 3$, so at 6 AM (0 hours after 6 AM), it is 3° Fahrenheit. $T(6) = 33$, so at noon (6 hours after 6 AM), the temperature is 33° Fahrenheit. $T(12) = 27$, so at 6 PM (12 hours after 6 AM), it is 27° Fahrenheit.

Problem 39 *The function $C(x) = x^2 - 10x + 27$ models the cost, in hundreds of dollars, to produce x thousand pens. Find and interpret $C(0)$, $C(2)$ and $C(5)$.*

$C(0) = 27$, so to make 0 pens, it costs[1] \$2700. $C(2) = 11$, so to make 2000 pens, it costs \$1100. $C(5) = 2$, so to make 5000 pens, it costs \$2000.

Problem 40 *Using data from the Bureau of Transportation Statistics, the average fuel economy in miles per gallon for passenger cars in the US can be modeled by $E(t) = -0.0076t^2 + 0.45t + 16$, $0 \leq t \leq 28$, where t is the number of years since 1980. Use a calculator to find $E(0)$, $E(14)$ and $E(28)$. Round your answers to two decimal places and interpret your answers to each.*

$E(0) = 16.00$, so in 1980 (0 years after 1980), the average fuel economy of passenger cars in the US was 16.00 miles per gallon. $E(14) = 20.81$, so in 1994 (14 years after 1980), the average fuel economy of passenger cars in the US was 20.81 miles per gallon. $E(28) = 22.64$, so in 2008 (28 years after 1980), the average fuel economy of passenger cars in the US was 22.64 miles per gallon.

Problem 41 *The perimeter of a square, in centimeters, is four times the length of one if its sides, also measured in centimeters. Represent the function P which takes as its input the length of the side of a square in centimeters, s and returns the perimeter of the square$P(s) = 4s$, $s > 0$. in inches, $P(s)$ using a formula.*

$P(s) = 4s$, $s > 0$.

Problem 42 *The circumference of a circle, in feet, is π times the diameter of the circle, also measured in feet. Represent the function C which takes as its input the length of the diameter of a circle in feet, D and returns the circumference of a circle in inches, $C(D)$ using a formula.*

$C(D) = \pi D$, $D > 0$.

[1] This is called the 'fixed' or 'start-up' cost. We'll revisit this concept in Example **??** in Section **??**.

CC BY-NC-SA 4.0

Problem 43 *Suppose $A(P)$ gives the amount of money in a retirement account (in dollars) after 30 years as a function of the amount of the monthly payment (in dollars), P.*

1. *What does $A(50)$ mean?*

   The amount in the retirement account after 30 years if the monthly payment is \$50.

2. *What is the significance of the solution to the equation $A(P) = 250000$?*

   The solution to $A(P) = 250000$ is what the monthly payment needs to be in order to have \$250,000 *in the retirement account after 30 years.*

3. *Explain what each of the following expressions mean: $A(P + 50)$, $A(P) + 50$, and $A(P) + A(50)$.*

   $A(P + 50)$ is how much is in the retirement account in 30 years if \$50 *is added to the monthly payment P. $A(P) + 50$ represents the amount of money in the retirement account after 30 years if* \$$P$ *is invested each month plus an additional* \$50. *$A(P) + A(50)$ is the sum of money from two retirement accounts after 30 years: one with monthly payment* \$$P$ *and one with monthly payment* \$50.

Problem 44 *Suppose $P(t)$ gives the chance of precipitation (in percent) t hours after 8 AM.*

1. *Write an expression which gives the chance of precipitation at noon.*

   Since noon is 4 hours after 8 AM, $P(4)$ gives the chance of precipitation at noon.

2. *Write an inequality which determines when the chance of precipitation is more than 50%.*

   We would need to solve $P(t) \geq 50\%$ or $P(t) \geq 0.5$.

Problem 45 *Explain why the graph in Exercise* **??** *suggests that not only is v as a function of w but also w is a function of v. Suppose $w = f(v)$ and $v = g(w)$. That is, f is the name of the function which takes v values as inputs and returns w values as outputs and g is the name of the function which does vice-versa. Find the domain and range of g and compare these to the domain and range of f.*

The graph in question passes the horizontal line test meaning for each w there is only one v. The domain of g is $[0, \infty)$ (which is the range of f) and the range of g is $[2, \infty)$ which is the domain of f.

Problem 46 *Sketch the graph of a function with domain $(-\infty, 3) \cup [4, 5)$ with range $\{2\} \cup (5, \infty)$.*

CC BY-NC-SA 4.0