Exercises for Quadratic Functions

Question 1 *In Exercises **??** - **??**, graph the quadratic function. Find the vertex and axis intercepts of each graph, if they exist. State the domain and range, identify the maximum or minimum, and list the intervals over which the function is increasing or decreasing. If the function is given in general form, convert it into vertex form; if it is given in vertex form, convert it into general form.*

Problem 1.1 $f(x) = x^2 + 2$

$f(x) = x^2 + 2$ *(this is both forms!)*
No x-intercepts
y-intercept $(0, 2)$
Domain: $(-\infty, \infty)$
Range: $[2, \infty)$
Decreasing on $(-\infty, 0]$
Increasing on $[0, \infty)$
Vertex $(0, 2)$ *is a minimum*
Axis of symmetry $x = 0$

Graph of $f(x) = x^2 + 2$

Problem 1.2 $f(x) = -(x + 2)^2$

x-intercept $(-2, 0)$
y-intercept $(0, -4)$
Vertex $(-2, 0)$ *is a (maximum ✓/ minimum)*
Axis of symmetry $x = -2$

$f(x) = -(x + 2)^2 = -x^2 - 4x - 4$
Domain: $(-\infty, \infty)$
Range: $(-\infty, 0]$
Increasing on $(-\infty, -2]$
Decreasing on $[-2, \infty)$

Problem 1.3 $f(x) = x^2 - 2x - 8$

x-intercepts $(-2, 0)$ *and* $(4, 0)$
y-intercept $(0, -8)$
Vertex $(1, -9)$ *is a (maximum/ minimum ✓)*
Axis of symmetry $x = 1$

$f(x) = x^2 - 2x - 8 = (x - 1)^2 - 9$
Domain: $(-\infty, \infty)$
Range: $[-9, \infty)$
Decreasing on $(-\infty, 1]$
Increasing on $[1, \infty)$

Graph of $f(x) = x^2 - 2x - 8$

Problem 1.4 $g(t) = -2(t + 1)^2 + 4$

t-intercepts $(-1 - \sqrt{2}, 0)$ *and* $(-1 + \sqrt{2}, 0)$
y-intercept $(0, 2)$

Vertex $(-1, 4)$ *is a (maximum ✓/ minimum)*
Axis of symmetry $t = -1$

$g(t) = -2(t+1)^2 + 4 = -2t^2 - 4t + 2$
Domain: $(-\infty, \infty)$
Range: $(-\infty, 4]$
Increasing on $(-\infty, -1]$
Decreasing on $[-1, \infty)$

Problem 1.5 $g(t) = 2t^2 - 4t - 1$

Vertex $(1, -3)$ *is a (maximum/ minimum ✓)*
Axis of symmetry $t = 1$

$g(t) = 2t^2 - tx - 1 = 2(t-1)^2 - 3$
t*-intercepts* $\left(\frac{2-\sqrt{6}}{2}, 0\right)$ *and* $\left(\frac{2+\sqrt{6}}{2}, 0\right)$
y*-intercept* $(0, -1)$
Domain: $(-\infty, \infty)$
Range: $[-3, \infty)$
Increasing on $[1, \infty)$
Decreasing on $(-\infty, 1]$

Problem 1.6 $g(t) = -3t^2 + 4t - 7$

Vertex $(\frac{2}{3}, -\frac{17}{3})$ *is a (maximum ✓/ minimum)*
Axis of symmetry $t = \frac{2}{3}$

$g(t) = -3t^2 + 4t - 7 = -3\left(t - \frac{2}{3}\right)^2 - \frac{17}{3}$
No t*-intercepts*
y*-intercept* $(0, -7)$
Domain: $(-\infty, \infty)$
Range: $\left(-\infty, -\frac{17}{3}\right]$
Increasing on $\left(-\infty, \frac{2}{3}\right]$
Decreasing on $\left[\frac{2}{3}, \infty\right)$

Problem 1.7 $h(s) = s^2 + s + 1$

Vertex $(-\frac{1}{2}, \frac{3}{4})$ *is a (maximum/ minimum ✓)*

Axis of symmetry $s = -\frac{1}{2}$

$h(s) = s^2 + s + 1 = \left(s + \frac{1}{2}\right)^2 + \frac{3}{4}$
No s*-intercepts*
y*-intercept* $(0, 1)$

Domain: $(-\infty, \infty)$
Range: $\left[\frac{3}{4}, \infty\right)$
Increasing on $\left[-\frac{1}{2}, \infty\right)$
Decreasing on $\left(-\infty, -\frac{1}{2}\right]$

Problem 1.8 $h(s) = -3s^2 + 5s + 4$
Vertex $(\frac{5}{6}, \frac{73}{12})$ *is a (maximum ✓/ minimum)*
Axis of symmetry $s = \frac{5}{6}$

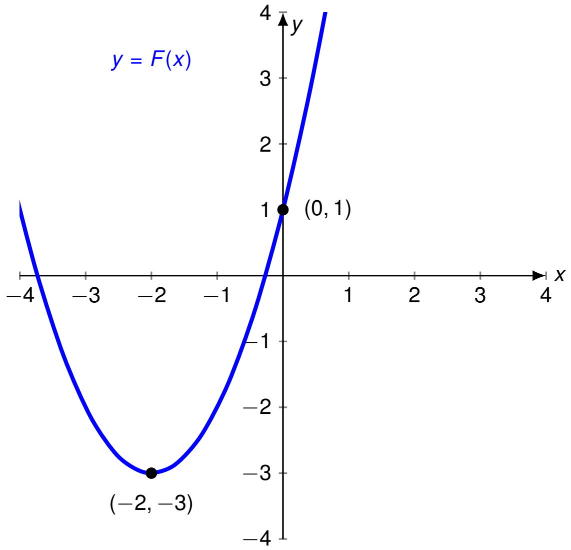
$h(s) = -3s^2 + 5s + 4 = -3\left(s - \frac{5}{6}\right)^2 + \frac{73}{12}$
s*-intercepts* $\left(\frac{5-\sqrt{73}}{6}, 0\right)$ *and* $\left(\frac{5+\sqrt{73}}{6}, 0\right)$
y*-intercept* $(0, 4)$
Domain: $(-\infty, \infty)$
Range: $\left(-\infty, \frac{73}{12}\right]$
Increasing on $\left(-\infty, \frac{5}{6}\right]$
Decreasing on $\left[\frac{5}{6}, \infty\right)$

Problem 1.9 $h(s) = s^2 - \frac{1}{100}s - 1$

$h(s) = s^2 - \frac{1}{100}s - 1 = \left(s - \frac{1}{200}\right)^2 - \frac{40001}{40000}$
s*-intercepts* $\left(\frac{1+\sqrt{40001}}{200}\right)$ *and* $\left(\frac{1-\sqrt{40001}}{200}\right)$
y*-intercept* $(0, -1)$
Domain: $(-\infty, \infty)$
Range: $\left[-\frac{40001}{40000}, \infty\right)$
Decreasing on $\left(-\infty, \frac{1}{200}\right]$
Increasing on $\left[\frac{1}{200}, \infty\right)$
Vertex $\left(\frac{1}{200}, -\frac{40001}{40000}\right)$ *is a minimum*[1]
Axis of symmetry $s = \frac{1}{200}$

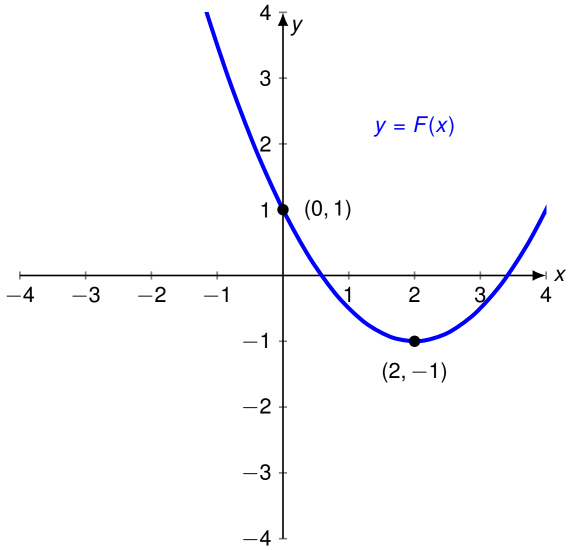
Question 2 *In Exercises* **??** - **??**, *find a formula for each function below in the form* $F(x) = a(x-h)^2 + k$.

[1] You'll need to use your calculator to zoom in far enough to see that the vertex is not the y-intercept.



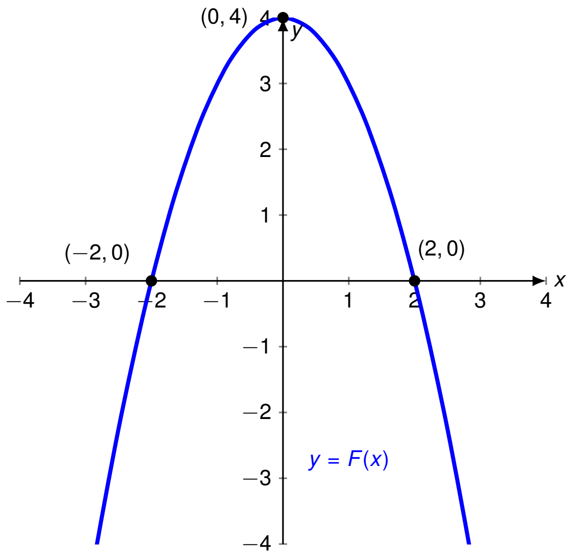
Problem 2.1

$F(x) = (x+2)^2 - 3$



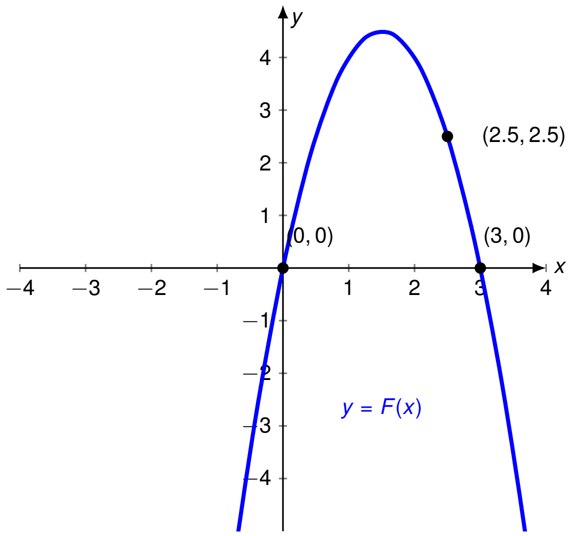
Problem 2.2

$F(x) = \frac{1}{2}(x-2)^2 - 1$



Problem 2.3

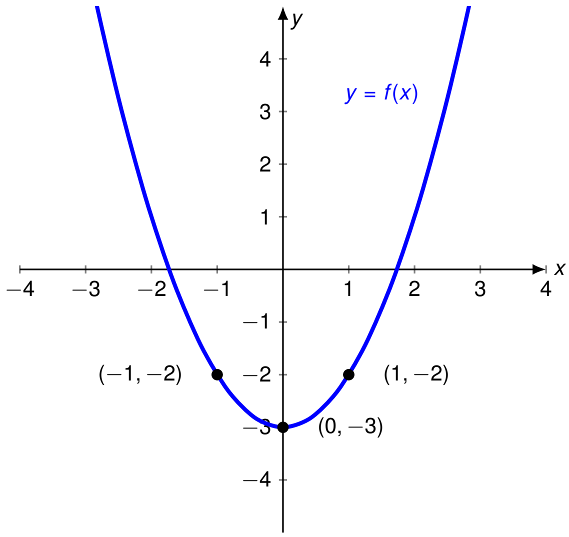
$F(x) = -x^2 + 4$



Problem 2.4

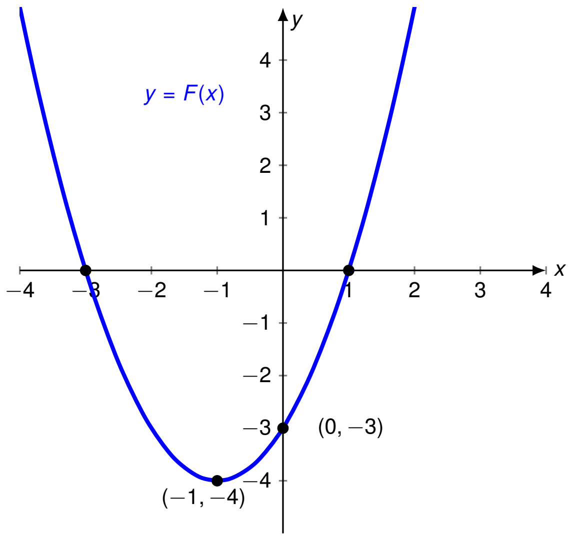
$F(x) = -2(x-1.5)^2 + 4.5$

Question 3 *In Exercises* **??** - **??** *Find both the vertex form and the general form of the quadratic functions below.*



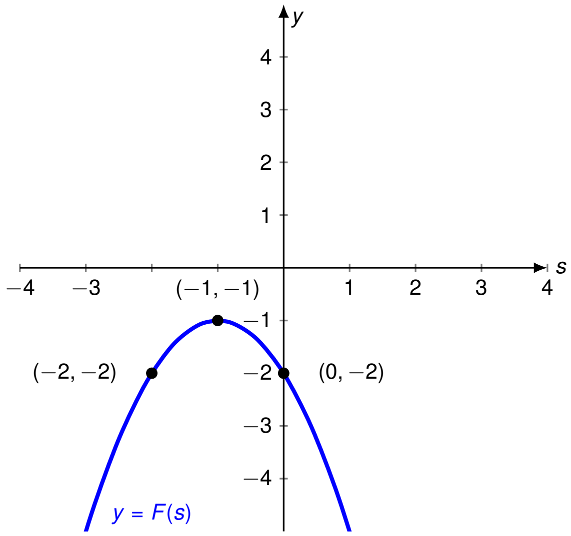
Problem 3.1

$f(x) = x^2 - 3$



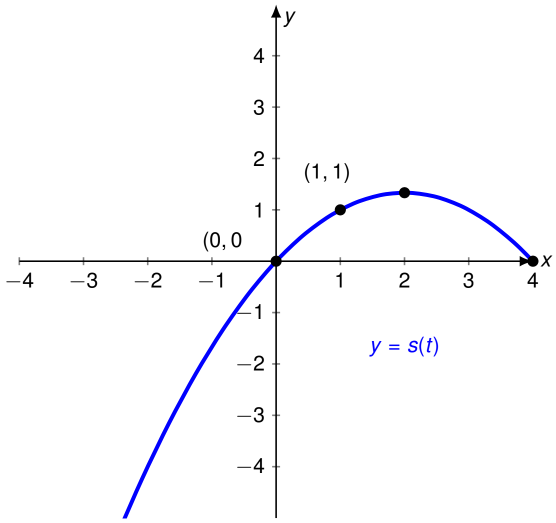
Problem 3.2

$F(x) = (x+1)^2 - 4 = x^2 + 2x - 3$



Problem 3.3

$F(s) = -(s+1)^2 - 1 = -s^2 - 2s - 2$



Problem 3.4

$s(t) = -\frac{1}{3}(t-2)^2 + \frac{4}{3} = -\frac{1}{3}t^2 + \frac{4}{3}t$

Question 4 *In Exercises* **??** *-* **??***, solve the inequality. Write your answer using interval notation.*

Problem 4.1 $x^2 + 2x - 3 \geq 0$

$(-\infty, -3] \cup [1, \infty)$

Problem 4.2 $16x^2 + 8x + 1 > 0$

$\left(-\infty, -\frac{1}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$

Problem 4.3 $t^2 + 9 < 6t$

No solution

Problem 4.4 $9t^2 + 16 \geq 24t$

$(-\infty, \infty)$

Problem 4.5 $u^2 + 4 \leq 4u$

$\{2\}$

Problem 4.6 $u^2 + 1 < 0$

No solution

Problem 4.7 $3x^2 \leq 11x + 4$

$\left[-\frac{1}{3}, 4\right]$

Problem 4.8 $x > x^2$

$(0, 1)$

Problem 4.9 $2t^2 - 4t - 1 > 0$

$\left(-\infty*, 1 - \frac{\sqrt{6}}{2}\right) \cup \left(1 + \frac{\sqrt{6}}{2}, \infty\right)$

Problem 4.10 $5t + 4 \leq 3t^2$

$\left(-\infty, \frac{5 - \sqrt{73}}{6}\right] \cup \left[\frac{5 + \sqrt{73}}{6}, \infty\right)$

Problem 4.11 $2 \leq |x^2 - 9| < 9$

$\left(-3\sqrt{2}, -\sqrt{11}\right] \cup \left[-\sqrt{7}, 0\right) \cup \left(0, \sqrt{7}\right] \cup \left[\sqrt{11}, 3\sqrt{2}\right)$

Problem 4.12 $x^2 \leq |4x - 3|$

$\left[-2 - \sqrt{7}, -2 + \sqrt{7}\right] \cup [1, 3]$

Problem 4.13 $t^2 + t + 1 \geq 0$

$(-\infty, \infty)$

Problem 4.14 $t^2 \geq |t|$

$(-\infty, -1] \cup \{0\} \cup [1, \infty)$

Problem 4.15 $x|x + 5| \geq -6$

$[-6, -3] \cup [-2, \infty)$

Problem 4.16 $x|x - 3| < 2$

$(-\infty, 1) \cup \left(2, \frac{3 + \sqrt{17}}{2}\right)$

Question 5 *In Exercises* **??** - **??***, cost and price-demand functions are given. For each scenario,*

- *Find the profit function $P(x)$.*
- *Find the number of items which need to be sold in order to maximize profit.*

- *Find the maximum profit.*
- *Find the price to charge per item in order to maximize profit.*
- *Find and interpret break-even points.*

Problem 5.1 *The cost, in dollars, to produce x "I'd rather be a Sasquatch" T-Shirts is* $C(x) = 2x + 26$, $x \geq 0$ *and the price-demand function, in dollars per shirt, is* $p(x) = 30 - 2x$, *for* $0 \leq x \leq 15$.

The profit function is $P(x) = -2x^2 + 28x - 26$, *for* $0 \leq x \leq 15$.

7 *T-shirts should be made and sold to maximize profit.*

The maximum profit is \$72.

The price per T-shirt should be set at \$16 *to maximize profit.*

The break even points are $x = 1$ *and* $x = 13$ ... *So to make a profit, between 1 and 13 T-shirts need to be made and sold.*

Problem 5.2 *The cost, in dollars, to produce x bottles of* 100% *All-Natural Certified Free-Trade Organic Sasquatch Tonic is* $C(x) = 10x + 100$, $x \geq 0$ *and the price-demand function, in dollars per bottle, is* $p(x) = 35 - x$, *for* $0 \leq x \leq 35$.

The profit function is $P(x) = -x^2 + 25x - 100$, *for* $0 \leq x \leq 35$

The vertex occurs at $x = 12.5$

Since the vertex occurs at $x = 12.5$, *and it is impossible to make or sell that many bottles of tonic, maximum profit occurs when either* 12 *or* 13 *bottles of tonic are made and sold.*

The maximum profit is \$56.

The price per bottle can be either \$23 *(to sell 12 bottles) or* \$22 *(to sell 13 bottles.) Both will result in the maximum profit.*

The break even points are $x = 5$ *and* $x = 20$ ... *So to make a profit, between 5 and 20 bottles of tonic need to be made and sold.*

Problem 5.3 *The cost, in cents, to produce x cups of Mountain Thunder Lemonade at Junior's Lemonade Stand is* $C(x) = 18x + 240$, $x \geq 0$ *and the price-demand function, in cents per cup, is* $p(x) = 90 - 3x$, *for* $0 \leq x \leq 30$.

The profit function is $P(x) = -3x^2 + 72x - 240$, *for* $0 \leq x \leq 30$

12 *cups of lemonade need to be made and sold to maximize profit.*

The maximum profit is 192¢ *or* \$1.92.

The price per cup should be set at 54¢ *per cup to maximize profit.*

The break even points are $x = 4$ *and* $x = 20$ ... *So to make a profit, between 4 and 20 cups of lemonade need to be made and sold.*

Problem 5.4 *The daily cost, in dollars, to produce x Sasquatch Berry Pies is* $C(x) = 3x + 36$, $x \geq 0$ *and the price-demand function, in dollars per pie, is* $p(x) = 12 - 0.5x$, *for* $0 \leq x \leq 24$.

The profit function is $P(x) = -0.5x^2 + 9x - 36$, *for* $0 \leq x \leq 24$

9 *pies should be made and sold to maximize the daily profit.*

The maximum daily profit is \$4.50.

The price per pie should be set at \$7.50 *to maximize profit.*

The break even points are $x = 6$ *and* $x = 12$ ... *So to make a profit, between 6 and 12 pies need to be made and sold daily.*

Problem 5.5 *The monthly cost, in* hundreds *of dollars, to produce x custom built electric scooters is* $C(x) = 20x + 1000$, $x \geq 0$ *and the price-demand function, in* hundreds *of dollars per scooter, is* $p(x) = 140 - 2x$, *for* $0 \leq x \leq 70$.

The profit function is $P(x) = -2x^2 + 120x - 1000$, *for* $0 \leq x \leq 70$

30 *scooters need to be made and sold to maximize profit.*

The maximum monthly profit is 800 *hundred dollars, or* \$80000.

The price per scooter should be set at 80 *hundred dollars, or* \$8000 *per scooter.*

The break even points are $x = 10$ *and* $x = 50$ ... *So to make a profit, between 10 and 50 scooters need to be made and sold monthly.*

Problem 6 *The International Silver Strings Submarine Band holds a bake sale each year to fund their trip to the National Sasquatch Convention. It has been determined that the cost in dollars of baking x cookies is* $C(x) = 0.1x + 25$ *and that the demand function for their cookies is* $p = 10 - .01x$ *for* $0 \leq x \leq 1000$. *How many cookies should they bake in order to maximize their profit?*

495 *cookies*

Problem 7 *Using data from Bureau of Transportation Statistics, the average fuel economy $F(t)$ in miles per gallon for passenger cars in the US t years after 1980 can be modeled by* $F(t) = -0.0076t^2 + 0.45t + 16$, $0 \leq t \leq 28$. *Find and interpret the coordinates of the vertex of the graph of* $y = F(t)$.

The vertex is (approximately) $(29.60, 22.66)$, *which corresponds to a maximum fuel economy of 22.66 miles per gallon, reached sometime between 2009 and 2010 (29 – 30 years after 1980.) Unfortunately, the model is only valid up until 2008 (28 years after 1908.) So, at this point, we are using the model to predict the maximum fuel economy.*

Problem 8 *The temperature T, in degrees Fahrenheit, t hours after 6 AM is given by:*

$$T(t) = -\frac{1}{2}t^2 + 8t + 32, \quad 0 \leq t \leq 12$$

What is the warmest temperature of the day? When does this happen?

64° *at* 2 *PM or* 8 *hours after 6 AM.*

Problem 9 *Suppose* $C(x) = x^2 - 10x + 27$ *represents the costs, in hundreds, to produce x thousand pens. How many pens should be produced to minimize the cost? What is this minimum cost?*

5000 *pens should be produced for a cost of* 200 *dollars.*

Problem 10 *Skippy wishes to plant a vegetable garden along one side of his house. In his garage, he found 32 linear feet of fencing. Since one side of the garden will border the house, Skippy doesn't need fencing along that side. What are the dimensions of the garden which will maximize the area of the garden? What is the maximum area of the garden?*

8 *feet by* 16 *feet; maximum area is* 128 *square feet.*

Problem 11 *In the situation of Example* **??**, *Donnie has a nightmare that one of his alpaca fell into the river. To avoid this, he wants to move his rectangular pasture away from the river so that all four sides of the pasture require fencing. If the total amount of fencing available is still 200 linear feet, what dimensions maximize the area of the pasture now? What is the maximum area? Assuming an average alpaca requires 25 square feet of pasture, how many alpaca can he raise now?*

50 *feet by* 50 *feet; maximum area is* 2500 *feet; he can raise* 100 *average alpacas.*

Problem 12 *What is the largest rectangular area one can enclose with 14 inches of string?*

The largest rectangle has area 12.25 *square inches.*

Problem 13 *The height of an object dropped from the roof of an eight story building is modeled by by the function* $h(t) = -16t^2 + 64$, $0 \le t \le 2$. *Here,* $h(t)$ *is the height of the object off the ground, in feet,* t *seconds after the object is dropped. How long before the object hits the ground?*

2 *seconds.*

Problem 14 *The height* $h(t)$ *in feet of a model rocket above the ground* t *seconds after lift-off is given by the function* $h(t) = -5t^2 + 100t$, *for* $0 \le t \le 20$. *When does the rocket reach its maximum height above the ground? What is its maximum height?*

The rocket reaches its maximum height of 500 *feet* 10 *seconds after lift-off.*

Problem 15 *Carl's friend Jason participates in the Highland Games. In one event, the hammer throw, the height* $h(t)$ *in feet of the hammer above the ground* t *seconds after Jason lets it go is modeled by the function* $h(t) = -16t^2 + 22.08t + 6$. *What is the hammer's maximum height? What is the hammer's total time in the air? Round your answers to two decimal places.*

The hammer reaches a maximum height of approximately 13.62 *feet. The hammer is in the air approximately* 1.61 *seconds.*

Problem 16 *Assuming no air resistance or forces other than the Earth's gravity, the height above the ground at time* t *of a falling object is given by* $s(t) = -4.9t^2 + v_0 t + s_0$ *where* s *is in meters,* t *is in seconds,* v_0 *is the object's initial velocity in meters per second and* s_0 *is its initial position in meters.*

1. *What is the applied domain of this function?*

   The applied domain is $[0, \infty)$

2. *Discuss with your classmates what each of* $v_0 > 0$, $v_0 = 0$ *and* $v_0 < 0$ *would mean.*

3. *Come up with a scenario in which* $s_0 < 0$.

4. *Let's say a slingshot is used to shoot a marble straight up from the ground* ($s_0 = 0$) *with an initial velocity of 15 meters per second. What is the marble's maximum height above the ground? At what time will it hit the ground?*

   The height function is this case is $s(t) = -4.9t^2 + 15t + 0$. *The vertex of this parabola is approximately* $(1.53, 11.48)$ *so the maximum height reached by the marble is* 11.48 *meters. It hits the ground again when* $t \approx 3.06$ *seconds.*

5. *If the marble is shot from the top of a 25 meter tall tower, when does it hit the ground?*

   The revised height function is $s(t) = -4.9t^2 + 15t + 25$ *which has zeros at* $t \approx -1.20$ *and* $t \approx 4.26$. *We ignore the negative value and claim that the marble will hit the ground after* 4.26 *seconds.*

6. *What would the height function be if instead of shooting the marble up off of the tower, you were to shoot it straight DOWN from the top of the tower?*

   Shooting down means the initial velocity is negative so the height functions becomes $s(t) = -4.9t^2 + -15t + 25$.

Problem 17 *The two towers of a suspension bridge are 400 feet apart. The parabolic cable*[2] *attached to the tops of the towers is 10 feet above the point on the bridge deck that is midway between the towers. If the*

[2] The weight of the bridge deck forces the bridge cable into a parabola and a free hanging cable such as a power line does not form a parabola. We shall see in Exercise **??** in Section **??** what shape a free hanging cable makes.

towers are 100 feet tall, find the height of the cable directly above a point of the bridge deck that is 50 feet to the right of the left-hand tower.

Make the vertex of the parabola $(0, 10)$ *so that the point on the top of the left-hand tower where the cable connects is* $(-200, 100)$ *and the point on the top of the right-hand tower is* $(200, 100)$*. Then the parabola is given by* $p(x) = \frac{9}{4000}x^2 + 0x + 10$*. Standing* 50 *feet to the right of the left-hand tower means you're standing at* $x = -150$ *and* $p(-150) = 60.625$*. So the cable is* 60.625 *feet above the bridge deck there.*

Problem 18 *On New Year's Day, Jeff started weighing himself every morning in order to have an interesting data set for this section of the book. (Discuss with your classmates if that makes him a nerd or a geek. Also, the professionals in the field of weight management strongly discourage weighing yourself every day. When you focus on the number and not your overall health, you tend to lose sight of your objectives. Jeff was making a noble sacrifice for science, but you should not try this at home.) The whole chart would be too big to put into the book neatly, so we've decided to give only a small portion of the data to you. This then becomes a Civics lesson in honesty, as you shall soon see. There are two charts given below. One has Jeff's weight for the first eight Thursdays of the year (January 1, 2009 was a Thursday and we'll count it as Day 1.) and the other has Jeff's weight for the first 10 Saturdays of the year.*

Day # (Thursday)	1	8	15	22	29	36	43	50
My weight in pounds	238.2	237.0	235.6	234.4	233.0	233.8	232.8	232.0

Day # (Saturday)	3	10	17	24	31	38	45	52	59	66
My weight in pounds	238.4	235.8	235.0	234.2	236.2	236.2	235.2	233.2	236.8	238.2

1. *Find the least squares line for the Thursday data and comment on its goodness of fit.*

   The line for the Thursday data is $y = -.12x + 237.69$*. We have* $r = -.9568$ *and* $r^2 = .9155$ *so this is a really good fit.*

2. *Find the least squares line for the Saturday data and comment on its goodness of fit.*

   The line for the Saturday data is $y = -0.000693x + 235.94$*. We have* $r = -0.008986$ *and* $r^2 = 0.0000807$ *which is horrible. This data is not even close to linear.*

3. *Use Quadratic Regression to find a parabola which models the Saturday data and comment on its goodness of fit.*

   The parabola for the Saturday data is $y = 0.003x^2 - 0.21x + 238.30$*. We have* $R^2 = .47497$ *which isn't good. Thus the data isn't modeled well by a quadratic function, either.*

4. *Compare and contrast the predictions the three models make for Jeff's weight on January 1, 2010 (Day #366). Can any of these models be used to make a prediction of Jeff's weight 20 years from now? Explain your answer.*

   The Thursday linear model had my weight on January 1, 2010 at 193.77 pounds. The Saturday models give 235.69 and 563.31 pounds, respectively. The Thursday line has my weight going below 0 pounds in about five and a half years, so that's no good. The quadratic has a positive leading coefficient which would mean unbounded weight gain for the rest of my life. The Saturday line, which mathematically does not fit the data at all, yields a plausible weight prediction in the end. I think this is why grown-ups talk about "Lies, Damned Lies and Statistics."

5. *Why is this a Civics lesson in honesty? Well, compare the two linear models you obtained above. One was a good fit and the other was not, yet both came from careful selections of real data. In*

presenting the tables to you, we've not lied about Jeff's weight, nor have you used any bad math to falsify the predictions. The word we're looking for here is 'disingenuous'. Look it up and then discuss the implications this type of data manipulation could have in a larger, more complex, politically motivated setting.

Problem 19 *(Data that is neither linear nor quadratic.) We'll close this exercise set with two data sets that, for reasons presented later in the book, cannot be modeled correctly by lines or parabolas. It is a good exercise, though, to see what happens when you attempt to use a linear or quadratic model when it's not appropriate.*

1. *This first data set came from a Summer 2003 publication of the Portage County Animal Protective League called "Tattle Tails". They make the following statement and then have a chart of data that supports it. "It doesn't take long for two cats to turn into 80 million. If two cats and their surviving offspring reproduced for ten years, you'd end up with 80,399,780 cats." We assume $N(0) = 2$.*

Year x	1	2	3	4	5	6	7	8	9	10
Number of Cats $N(x)$	12	66	382	2201	12680	73041	420715	2423316	13968290	80399780

Use Quadratic Regression to find a parabola which models this data and comment on its goodness of fit. (Spoiler Alert: Does anyone know what type of function we need here?)

The quadratic model for the cats in Portage county is $y = 1917803.54x^2 - 16036408.29x + 24094857.7$. Although $R^2 = .70888$ this is not a good model because it's so far off for small values of x. The model gives us 24,094,858 cats when $x = 0$ but we know $N(0) = 2$.

2. *This next data set comes from the U.S. Naval Observatory. That site has loads of awesome stuff on it, but for this exercise I used the sunrise/sunset times in Fairbanks, Alaska for 2009 to give you a chart of the number of hours of daylight they get on the 21^{st} of each month. We'll let $x = 1$ represent January 21, 2009, $x = 2$ represent February 21, 2009, and so on.*

Month Number	1	2	3	4	5	6	7	8	9	10	11	12
Hours of Daylight	5.8	9.3	12.4	15.9	19.4	21.8	19.4	15.6	12.4	9.1	5.6	3.3

Use Quadratic Regression to find a parabola which models this data and comment on its goodness of fit. (Spoiler Alert: Does anyone know what type of function we need here?)

3. *The quadratic model for the hours of daylight in Fairbanks, Alaska is $y = .51x^2 + 6.23x - .36$. Even with $R^2 = .92295$ we should be wary of making predictions beyond the data. Case in point, the model gives -4.84 hours of daylight when $x = 13$. So January 21, 2010 will be "extra dark"? Obviously a parabola pointing down isn't telling us the whole story.*

Problem 20 *Redraw the three scenarios discussed in the discriminant box for $a < 0$.*

Problem 21 *Graph $f(x) = |1 - x^2|$*

Problem 22 *Find all of the points on the line $y = 1 - x$ which are 2 units from $(1, -1)$.* $\left(\frac{3-\sqrt{7}}{2}, \frac{-1+\sqrt{7}}{2}\right)$, $\left(\frac{3+\sqrt{7}}{2}, \frac{-1-\sqrt{7}}{2}\right)$

Problem 23 *Let L be the line $y = 2x + 1$. Find a function $D(x)$ which measures the distance squared from a point on L to $(0, 0)$. Use this to find the point on L closest to $(0, 0)$.*

$D(x) = x^2 + (2x + 1)^2 = 5x^2 + 4x + 1$ is minimized when $x = -\frac{2}{5}$. Hence to find the point on $y = 2x + 1$ closest to $(0, 0)$ we substitute $x = -\frac{2}{5}$ into $y = 2x + 1$ to get $\left(-\frac{2}{5}, \frac{1}{5}\right)$.

Problem 24 *With the help of your classmates, show that if a quadratic function $f(x) = ax^2 + bx + c$ has two real zeros then the x-coordinate of the vertex is the midpoint of the zeros.*

Problem 25 *On page **??**, we argued that any quadratic function in vertex form $f(x) = a(x - h)^2 + k$ can be converted to a quadratic function in general form $f(x) = ax^2 + bx + c$ by making the identifications $b = -2ah$ and $c = ah^2 + k$. In this exercise, we use same identifications to show every parabola given in general form can be converted to vertex form without completing the square.*

Solve $b = -2ah$ for h and substitute the result into the equation $c = ah^2 + k$ and then solve for k. Show $h = -\frac{b}{2a}$ and $k = \frac{4ac - b^2}{4a}$ so that

$$f(x) = ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}.$$

Question 26 *In Exercises **??** - **??**, solve the quadratic equation for the indicated variable.*

Problem 26.1 $x^2 - 10y^2 = 0$ *for x*

$x = \pm y\sqrt{10}$

Problem 26.2 $y^2 - 4y = x^2 - 4$ *for x*

$x = \pm(y - 2)$

Problem 26.3 $x^2 - mx = 1$ *for x*

$x = \dfrac{m \pm \sqrt{m^2 + 4}}{2}$

Problem 26.4 $y^2 - 3y = 4x$ *for y*

$y = \dfrac{3 \pm \sqrt{16x + 9}}{2}$

Problem 26.5 $y^2 - 4y = x^2 - 4$ *for y*

$y = 2 \pm x$

Problem 26.6 $-gt^2 + v_0 t + s_0 = 0$ *for t (Assume $g \neq 0$.)*

$t = \dfrac{v_0 \pm \sqrt{v_0^2 + 4gs_0}}{2g}$

Problem 27 *(This is a follow-up to Exercise **??** in Section **??**.) The Lagrange Interpolate function L for three points (x_0, y_0), (x_1, y_1), and (x_2, y_2) where x_0, x_1, and x_2 are three distinct real numbers is given by:*

$$L(x) = y_0\frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + y_1\frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} + y_2\frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

1. *For each of the following sets of points, find $L(x)$ using the formula above and verify each of the points lies on the graph of $y = L(x)$.*

(i) $(-1, 1)$, $(1, 1)$, $(2, 4)$
$L(x) = x^2$

(ii) $(1, 3)$, $(2, 10)$, $(3, 21)$
$L(x) = 2x^2 + x$

(iii) $(0, 1)$, $(1, 5)$, $(2, 7)$
$L(x) = -x^2 + 5x + 1$

2. *Verify that, in general,* $L(x_0) = y_0$, $L(x_1) = y_1$, *and* $L(x_2) = y_2$.

3. *Find* $L(x)$ *for the points* $(-1, 6)$, $(1, 4)$ *and* $(3, 2)$. *What happens?*

   The three points lie on the same line and we get $L(x) = -x + 5$.

4. *Under what conditions will* $L(x)$ *produce a quadratic function? Make a conjecture, test some cases, and prove your answer.*

   To obtain a quadratic function, we require that the points are not collinear (i.e., they do not all lie on the same line.)