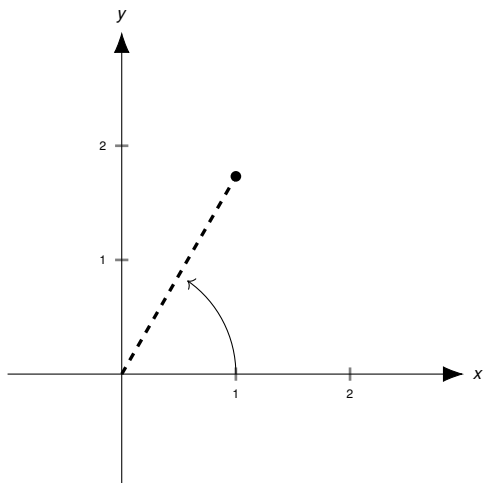


EXERCISES FOR POLAR COORDINATES

Question 1 In Exercises ?? - ??, plot the point given in polar coordinates in the xy -plane and then give three different expressions for the point such that (a) $r < 0$ and $0 \leq \theta \leq 2\pi$, (b) $r > 0$ and $\theta \leq 0$ (c) $r > 0$ and $\theta \geq 2\pi$

Problem 1.1 $\left(2, \frac{\pi}{3}\right)$

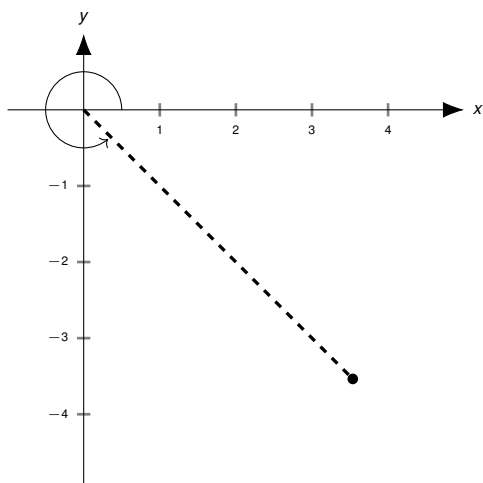


(a) $\left(-2, \frac{4\pi}{3}\right)$

(b) $\left(2, -\frac{5\pi}{3}\right)$

(c) $\left(2, \frac{7\pi}{3}\right)$

Problem 1.2 $\left(5, \frac{7\pi}{4}\right)$



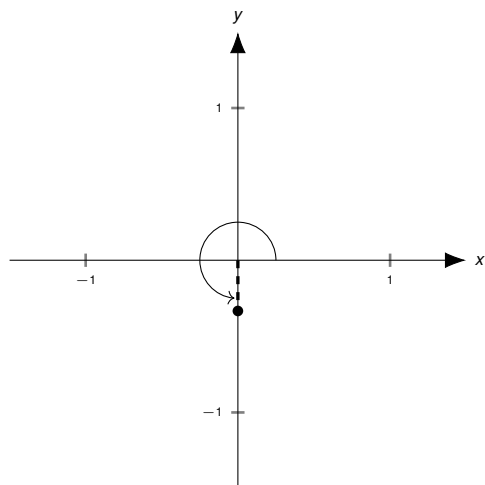
(a) $\left(-5, \frac{3\pi}{4}\right)$

(b) $\left(5, -\frac{\pi}{4}\right)$

(c) $\left(5, \frac{15\pi}{4}\right)$

EXERCISES FOR POLAR COORDINATES

Problem 1.3 $\left(\frac{1}{3}, \frac{3\pi}{2}\right)$

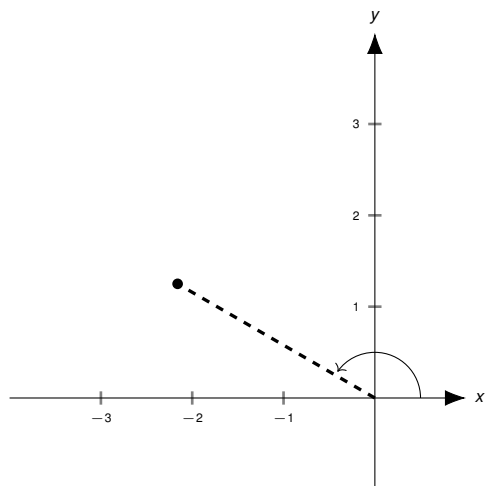


(a) $\left(-\frac{1}{3}, \frac{\pi}{2}\right)$

(b) $\left(\frac{1}{3}, -\frac{\pi}{2}\right)$

(c) $\left(\frac{1}{3}, \frac{7\pi}{2}\right)$

Problem 1.4 $\left(\frac{5}{2}, \frac{5\pi}{6}\right)$



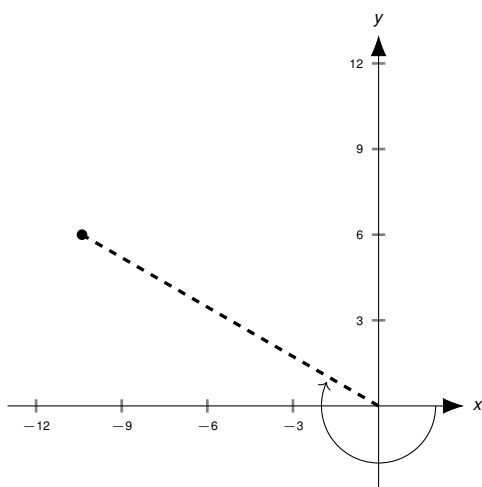
(a) $\left(-\frac{5}{2}, \frac{11\pi}{6}\right)$

(b) $\left(\frac{5}{2}, -\frac{7\pi}{6}\right)$

(c) $\left(\frac{5}{2}, \frac{17\pi}{6}\right)$

Problem 1.5 $\left(12, -\frac{7\pi}{6}\right)$

EXERCISES FOR POLAR COORDINATES

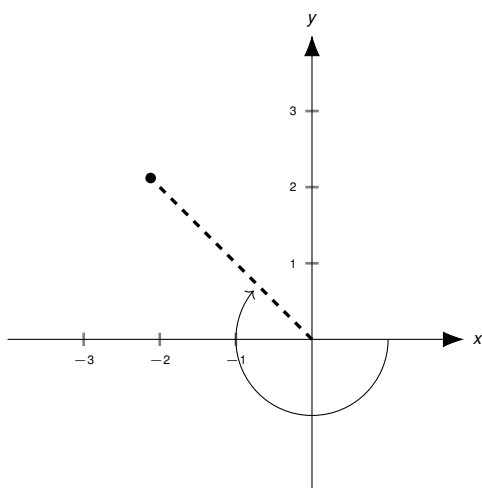


(a) $\left(-12, \frac{11\pi}{6}\right)$

(b) $\left(12, -\frac{19\pi}{6}\right)$

(c) $\left(12, \frac{17\pi}{6}\right)$

Problem 1.6 $\left(3, -\frac{5\pi}{4}\right)$



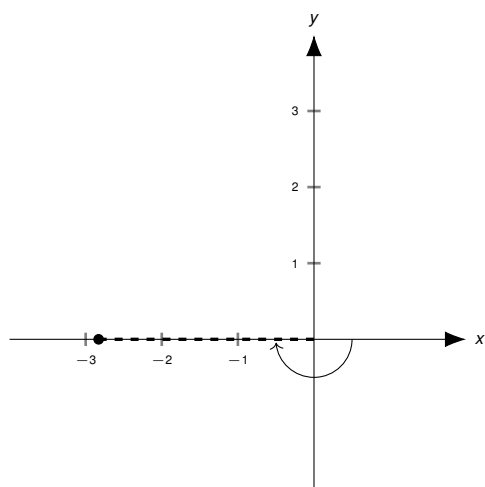
(a) $\left(-3, \frac{7\pi}{4}\right)$

(b) $\left(3, -\frac{13\pi}{4}\right)$

(c) $\left(3, \frac{11\pi}{4}\right)$

Problem 1.7 $\left(2\sqrt{2}, -\pi\right)$

EXERCISES FOR POLAR COORDINATES

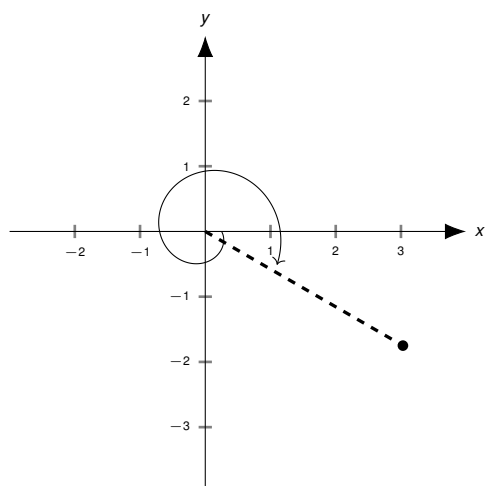


(a) $(-2\sqrt{2}, 0)$

(b) $(2\sqrt{2}, -3\pi)$

(c) $(2\sqrt{2}, 3\pi)$

Problem 1.8 $\left(\frac{7}{2}, -\frac{13\pi}{6}\right)$



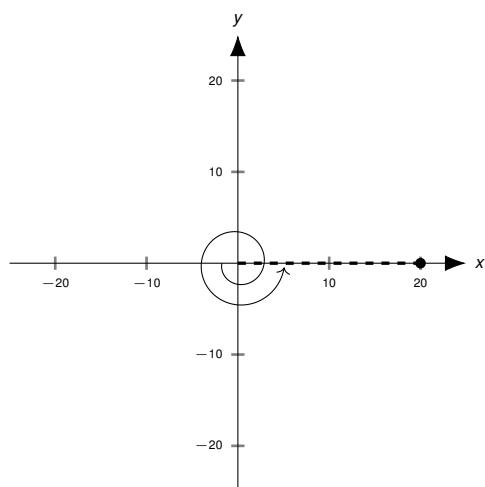
(a) $\left(-\frac{7}{2}, \frac{5\pi}{6}\right)$

(b) $\left(\frac{7}{2}, -\frac{\pi}{6}\right)$

(c) $\left(\frac{7}{2}, \frac{23\pi}{6}\right)$

Problem 1.9 $(-20, 3\pi)$

EXERCISES FOR POLAR COORDINATES

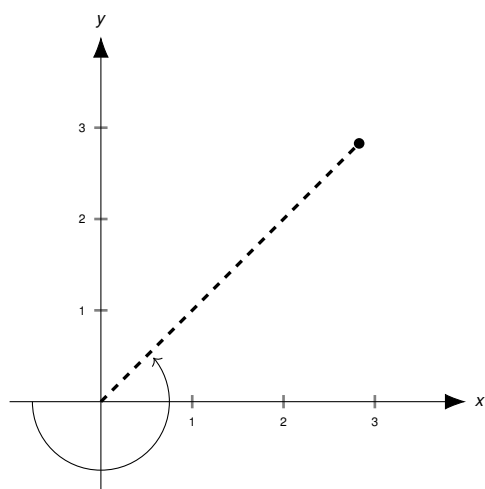


(a) $(-20, \pi)$

(b) $(20, -2\pi)$

(c) $(20, 4\pi)$

Problem 1.10 $\left(-4, \frac{5\pi}{4}\right)$



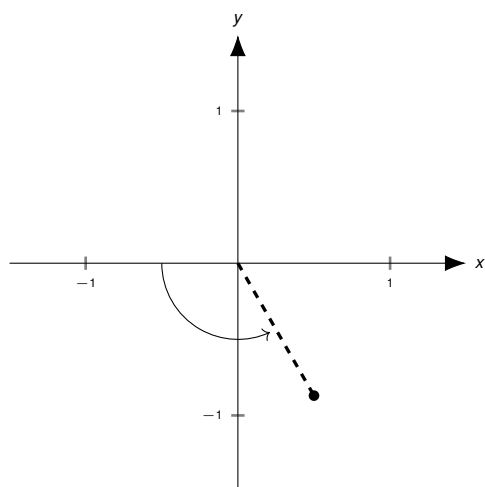
(a) $\left(-4, \frac{5\pi}{4}\right)$

(b) $\left(4, -\frac{7\pi}{4}\right)$

(c) $\left(4, \frac{9\pi}{4}\right)$

Problem 1.11 $\left(-1, \frac{2\pi}{3}\right)$

EXERCISES FOR POLAR COORDINATES

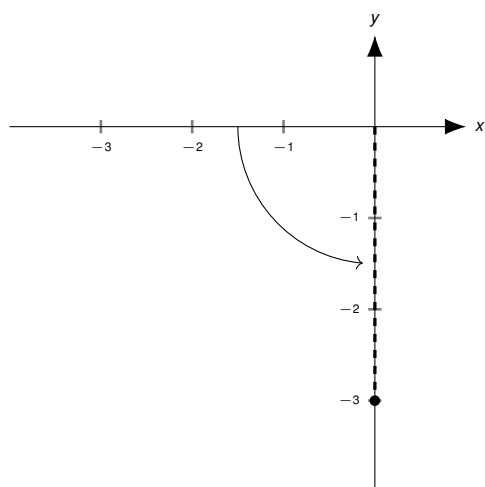


(a) $\left(-1, \frac{2\pi}{3}\right)$

(b) $\left(1, -\frac{\pi}{3}\right)$

(c) $\left(1, \frac{11\pi}{3}\right)$

Problem 1.12 $\left(-3, \frac{\pi}{2}\right)$



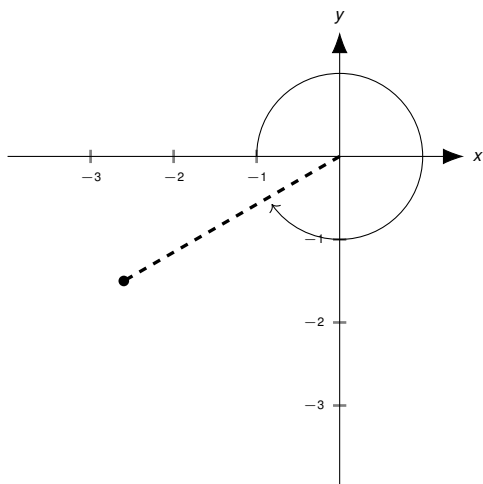
(a) $\left(-3, \frac{\pi}{2}\right)$

(b) $\left(3, -\frac{\pi}{2}\right)$

(c) $\left(3, \frac{7\pi}{2}\right)$

Problem 1.13 $\left(-3, -\frac{11\pi}{6}\right)$

EXERCISES FOR POLAR COORDINATES

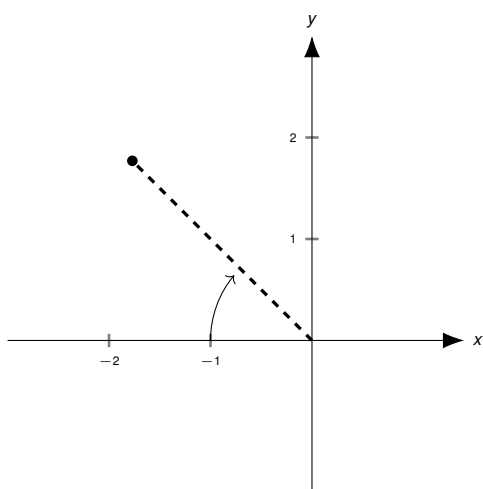


(a) $\left(-3, \frac{\pi}{6}\right)$

(b) $\left(3, -\frac{5\pi}{6}\right)$

(c) $\left(3, \frac{19\pi}{6}\right)$

Problem 1.14 $\left(-2.5, -\frac{\pi}{4}\right)$



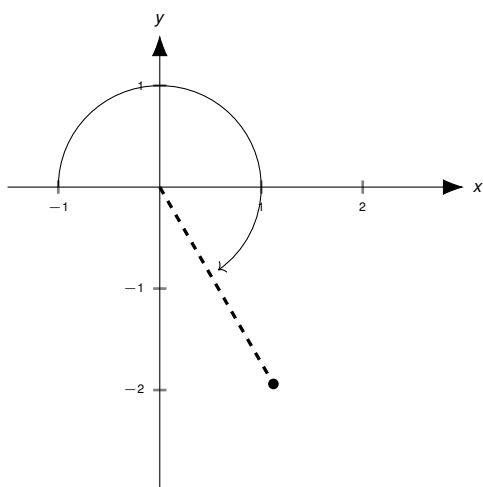
(a) $\left(-2.5, \frac{7\pi}{4}\right)$

(b) $\left(2.5, -\frac{5\pi}{4}\right)$

(c) $\left(2.5, \frac{11\pi}{4}\right)$

Problem 1.15 $\left(-\sqrt{5}, -\frac{4\pi}{3}\right)$

EXERCISES FOR POLAR COORDINATES

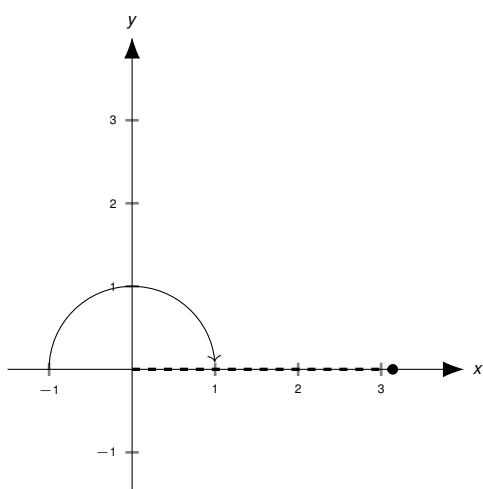


(a) $\left(-\sqrt{5}, \frac{2\pi}{3}\right)$

(b) $\left(\sqrt{5}, -\frac{\pi}{3}\right)$

(c) $\left(\sqrt{5}, \frac{11\pi}{3}\right)$

Problem 1.16 $(-\pi, -\pi)$



(a) $(-\pi, \pi)$

(b) $(\pi, -2\pi)$

(c) $(\pi, 2\pi)$

Question 2 In Exercises ?? - ??, convert the point from polar coordinates into rectangular coordinates.

Problem 2.1 $\left(5, \frac{7\pi}{4}\right)$

$\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$

Problem 2.2 $\left(2, \frac{\pi}{3}\right)$

EXERCISES FOR POLAR COORDINATES

$$(1, \sqrt{3})$$

Problem 2.3 $\left(11, -\frac{7\pi}{6}\right)$

$$\left(-\frac{11\sqrt{3}}{2}, \frac{11}{2}\right)$$

Problem 2.4 $(-20, 3\pi)$

$$(20, 0)$$

Problem 2.5 $\left(\frac{3}{5}, \frac{\pi}{2}\right)$

$$\left(0, \frac{3}{5}\right)$$

Problem 2.6 $\left(-4, \frac{5\pi}{6}\right)$

$$(2\sqrt{3}, -2)$$

Problem 2.7 $\left(9, \frac{7\pi}{2}\right)$

$$(0, -9)$$

Problem 2.8 $\left(-5, -\frac{9\pi}{4}\right)$

$$\left(-\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$$

Problem 2.9 $\left(42, \frac{13\pi}{6}\right)$

$$(21\sqrt{3}, 21)$$

Problem 2.10 $(-117, 117\pi)$

$$(117, 0)$$

Problem 2.11 $(6, \arctan(2))$

$$\left(\frac{6\sqrt{5}}{5}, \frac{12\sqrt{5}}{5}\right)$$

Problem 2.12 $(10, \arctan(3))$

$$(\sqrt{10}, 3\sqrt{10})$$

Problem 2.13 $\left(-3, \arctan\left(\frac{4}{3}\right)\right)$

$$\left(-\frac{9}{5}, -\frac{12}{5}\right)$$

EXERCISES FOR POLAR COORDINATES

Problem 2.14 $\left(5, \arctan\left(-\frac{4}{3}\right)\right)$

$(3, -4)$

Problem 2.15 $\left(2, \pi - \arctan\left(\frac{1}{2}\right)\right)$

$\left(-\frac{4\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right)$

Problem 2.16 $\left(-\frac{1}{2}, \pi - \arctan(5)\right)$

$\left(\frac{\sqrt{26}}{52}, -\frac{5\sqrt{26}}{52}\right)$

Problem 2.17 $\left(-1, \pi + \arctan\left(\frac{3}{4}\right)\right)$

$\left(\frac{4}{5}, \frac{3}{5}\right)$

Problem 2.18 $\left(\frac{2}{3}, \pi + \arctan(2\sqrt{2})\right)$

$\left(-\frac{2}{9}, -\frac{4\sqrt{2}}{9}\right)$

Problem 2.19 $(\pi, \arctan(\pi))$

$\left(\frac{\pi}{\sqrt{1+\pi^2}}, \frac{\pi^2}{\sqrt{1+\pi^2}}\right)$

Problem 2.20 $\left(13, \arctan\left(\frac{12}{5}\right)\right)$

$(5, 12)$

Question 3 In Exercises ?? - ??, convert the given point from rectangular coordinates to polar coordinates. Choose $r \geq 0$ and $0 \leq \theta < 2\pi$. Remember it helps to plot the point first to help choose the correct value for θ .

Problem 3.1 $(0, 5)$

$\left(5, \frac{\pi}{2}\right)$

Problem 3.2 $(3, \sqrt{3})$

$\left(2\sqrt{3}, \frac{\pi}{6}\right)$

Problem 3.3 $(7, -7)$

$\left(7\sqrt{2}, \frac{7\pi}{4}\right)$

EXERCISES FOR POLAR COORDINATES

Problem 3.4 $(-3, -\sqrt{3})$

$$\left(2\sqrt{3}, \frac{7\pi}{6}\right)$$

Problem 3.5 $(-3, 0)$

$$(3, \pi)$$

Problem 3.6 $(-\sqrt{2}, \sqrt{2})$

$$\left(2, \frac{3\pi}{4}\right)$$

Problem 3.7 $(-4, -4\sqrt{3})$

$$\left(8, \frac{4\pi}{3}\right)$$

Problem 3.8 $\left(\frac{\sqrt{3}}{4}, -\frac{1}{4}\right)$

$$\left(\frac{1}{2}, \frac{11\pi}{6}\right)$$

Problem 3.9 $\left(-\frac{3}{10}, -\frac{3\sqrt{3}}{10}\right)$

$$\left(\frac{3}{5}, \frac{4\pi}{3}\right)$$

Problem 3.10 $(-\sqrt{5}, -\sqrt{5})$

$$\left(\sqrt{10}, \frac{5\pi}{4}\right)$$

Problem 3.11 $(6, 8)$

$$\left(10, \arctan\left(\frac{4}{3}\right)\right)$$

Problem 3.12 $(\sqrt{5}, 2\sqrt{5})$

$$(5, \arctan(2))$$

Problem 3.13 $(-8, 1)$

$$\left(\sqrt{65}, \pi - \arctan\left(\frac{1}{8}\right)\right)$$

Problem 3.14 $(-2\sqrt{10}, 6\sqrt{10})$

$$(20, \pi - \arctan(3))$$

Problem 3.15 $(-5, -12)$

$$\left(13, \pi + \arctan\left(\frac{12}{5}\right)\right)$$

EXERCISES FOR POLAR COORDINATES

Problem 3.16 $\left(-\frac{\sqrt{5}}{15}, -\frac{2\sqrt{5}}{15}\right)$

$\left(\frac{1}{3}, \pi + \arctan(2)\right)$

Problem 3.17 $(24, -7)$

$\left(25, 2\pi - \arctan\left(\frac{7}{24}\right)\right)$

Problem 3.18 $(12, -9)$

$\left(15, 2\pi - \arctan\left(\frac{3}{4}\right)\right)$

Problem 3.19 $\left(\frac{\sqrt{2}}{4}, \frac{\sqrt{6}}{4}\right)$

$\left(\frac{\sqrt{2}}{2}, \frac{\pi}{3}\right)$

Problem 3.20 $\left(-\frac{\sqrt{65}}{5}, \frac{2\sqrt{65}}{5}\right)$

$\left(\sqrt{13}, \pi - \arctan(2)\right)$

Question 4 In Exercises ?? - ??, convert the equation from rectangular coordinates into polar coordinates. Solve for r in all but ?? through ??. In Exercises ?? - ??, solve for θ

Problem 4.1 $x = 6$

$r = 6 \sec(\theta)$

Problem 4.2 $x = -3$

$r = -3 \sec(\theta)$

Problem 4.3 $y = 7$

$r = 7 \csc(\theta)$

Problem 4.4 $y = 0$

$\theta = 0$

Problem 4.5 $y = -x$

$\theta = \frac{3\pi}{4}$

Problem 4.6 $y = x\sqrt{3}$

$\theta = \frac{\pi}{3}$

EXERCISES FOR POLAR COORDINATES

Problem 4.7 $y = 2x$

$\theta = \arctan(2)$

Problem 4.8 $x^2 + y^2 = 25$

$r = 5$

Problem 4.9 $x^2 + y^2 = 117$

$r = \sqrt{117}$

Problem 4.10 $y = 4x - 19$

$$r = \frac{19}{4 \cos(\theta) - \sin(\theta)}$$

Problem 4.11 $x = 3y + 1$

$$x = \frac{1}{\cos(\theta) - 3 \sin(\theta)}$$

Problem 4.12 $y = -3x^2$

$$r = \frac{-\sec(\theta) \tan(\theta)}{3}$$

Problem 4.13 $4x = y^2$

$r = 4 \csc(\theta) \cot(\theta)$

Problem 4.14 $x^2 + y^2 - 2y = 0$

$r = 2 \sin(\theta)$

Problem 4.15 $x^2 - 4x + y^2 = 0$

$r = 4 \cos(\theta)$

Problem 4.16 $x^2 + y^2 = x$

$r = \cos(\theta)$

Problem 4.17 $y^2 = 7y - x^2$

$r = 7 \sin(\theta)$

Problem 4.18 $(x + 2)^2 + y^2 = 4$

$r = -4 \cos(\theta)$

Problem 4.19 $x^2 + (y - 3)^2 = 9$

$r = 6 \sin(\theta)$

Problem 4.20 $4x^2 + 4 \left(y - \frac{1}{2}\right)^2 = 1$

$r = \sin(\theta)$

Question 5 In Exercises ?? - ??, convert the equation from polar coordinates into rectangular coordinates.

EXERCISES FOR POLAR COORDINATES

Problem 5.1 $r = 7$

$$x^2 + y^2 = 49$$

Problem 5.2 $r = -3$

$$x^2 + y^2 = 9$$

Problem 5.3 $r = \sqrt{2}$

$$x^2 + y^2 = 2$$

Problem 5.4 $\theta = \frac{\pi}{4}$

$$y = x$$

Problem 5.5 $\theta = \frac{2\pi}{3}$

$$y = -\sqrt{3}x$$

Problem 5.6 $\theta = \pi$

$$y = 0$$

Problem 5.7 $\theta = \frac{3\pi}{2}$

$$x = 0$$

Problem 5.8 $r = 4 \cos(\theta)$

$$x^2 + y^2 = 4x \text{ or } (x - 2)^2 + y^2 = 4$$

Problem 5.9 $5r = \cos(\theta)$

$$5x^2 + 5y^2 = x \text{ or } \left(x - \frac{1}{10}\right)^2 + y^2 = \frac{1}{100}$$

Problem 5.10 $r = 3 \sin(\theta)$

$$x^2 + y^2 = 3y \text{ or } x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$$

Problem 5.11 $r = -2 \sin(\theta)$

$$x^2 + y^2 = -2y \text{ or } x^2 + (y + 1)^2 = 1$$

Problem 5.12 $r = 7 \sec(\theta)$

$$x = 7$$

Problem 5.13 $12r = \csc(\theta)$

$$y = \frac{1}{12}$$

Problem 5.14 $r = -2 \sec(\theta)$

$$x = -2$$

EXERCISES FOR POLAR COORDINATES

Problem 5.15 $r = -\sqrt{5} \csc(\theta)$

$$y = -\sqrt{5}$$

Problem 5.16 $r = 2 \sec(\theta) \tan(\theta)$

$$x^2 = 2y$$

Problem 5.17 $r = -\csc(\theta) \cot(\theta)$

$$y^2 = -x$$

Problem 5.18 $r^2 = \sin(2\theta)$

$$(x^2 + y^2)^2 = 2xy$$

Problem 5.19 $r = 1 - 2 \cos(\theta)$

$$(x^2 + 2x + y^2)^2 = x^2 + y^2$$

Problem 5.20 $r = 1 + \sin(\theta)$

$$(x^2 + y^2 - y)^2 = x^2 + y^2$$

Problem 6 Convert the origin $(0, 0)$ into polar coordinates in four different ways.

Any point of the form $(0, \theta)$ will work, e.g. $(0, \pi)$, $(0, -117)$, $\left(0, \frac{23\pi}{4}\right)$ and $(0, 0)$.

Problem 7 With the help of your classmates, use the Law of Cosines to develop a formula for the distance between two points in polar coordinates.