Exercises for Graphs of Polynomials

Problem 1 *Starting with the graph of $f(x) = x^3$, use Theorem* **??** *to sketch the graph of $F(x) = (x+2)^3 + 1$. Track at least three points of your choice through the transformations. State the domain and range of F.*

Use the Desmos graph below with the settings $f(x) = x^3,\ h = -2,\ k = 1,\ a = 1$

Desmos link: https://www.desmos.com/calculator/180d988fc8

domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

Problem 2 *Starting with the graph of $f(x) = x^4$, use Theorem* **??** *to sketch the graph of $F(x) = (x+2)^4 + 1$. Track at least three points of your choice through the transformations. State the domain and range of F.*

Use the Desmos graph below with the settings $f(x) = x^4,\ h = -2,\ k = 1,\ a = 1$

Desmos link: https://www.desmos.com/calculator/180d988fc8

domain: $(-\infty, \infty)$

range: $[1, \infty)$

Problem 3 *Starting with the graph of $f(x) = x^4$, use Theorem* **??** *to sketch the graph of $F(x) = 2-3(x-1)^4$. Track at least three points of your choice through the transformations. State the domain and range of F.*

Use the Desmos graph below with the settings $f(x) = x^4,\ h = 1,\ k = 2,\ a = -3$

Desmos link: https://www.desmos.com/calculator/180d988fc8

domain: $(-\infty, \infty)$

range: $(-\infty, 2]$

Problem 4 *Starting with the graph of $f(x) = x^5$, use Theorem* **??** *to sketch the graph of $F(x) = -x^5 - 3$. Track at least three points of your choice through the transformations. State the domain and range of F.*

Use the Desmos graph below with the settings $f(x) = x^5,\ h = 0,\ k = -3,\ a = -1$

Desmos link: https://www.desmos.com/calculator/180d988fc8

domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

Problem 5 *Starting with the graph of $f(x) = x^5$, use Theorem* **??** *to sketch the graph of $F(x) = (x+1)^5 + 10$. Track at least three points of your choice through the transformations. State the domain and range of F.*

Use the Desmos graph below with the settings $f(x) = x^5,\ h = -1,\ k = 10,\ a = 1$

Desmos link: https://www.desmos.com/calculator/180d988fc8

domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

Problem 6 *Starting with the graph of $f(x) = x^6$, use Theorem* **??** *to sketch the graph of $F(x) = 8 - x^6$. Track at least three points of your choice through the transformations. State the domain and range of F.*

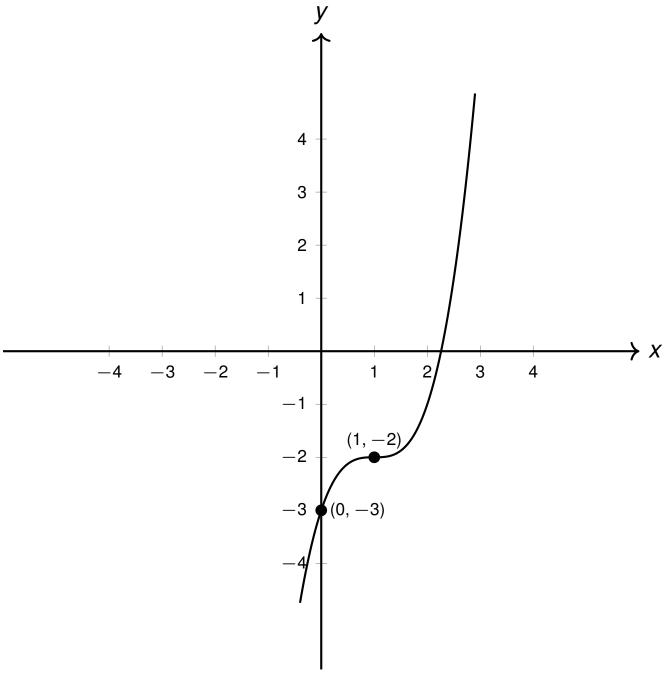
Use the Desmos graph below with the settings $f(x) = x^6,\ h = 0,\ k = 8,\ a = -1$

Desmos link: https://www.desmos.com/calculator/180d988fc8

domain: $(-\infty, \infty)$

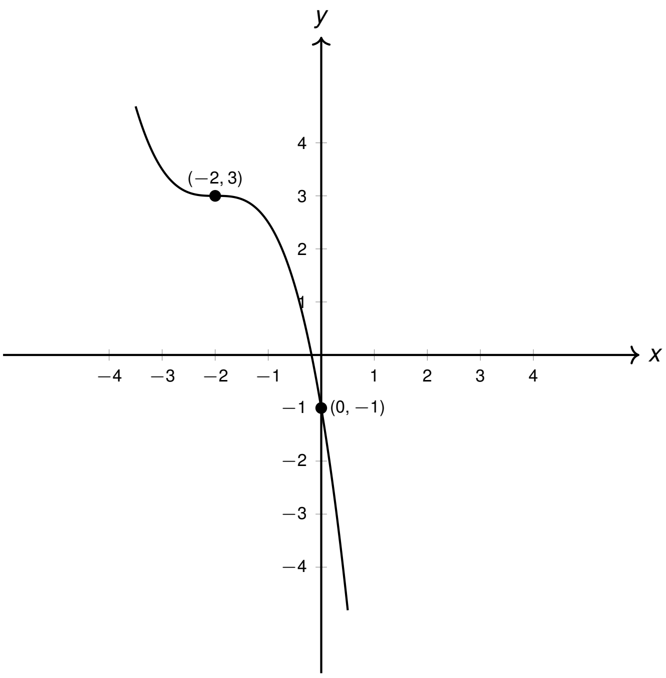
range: $(-\infty, 8]$

Problem 7 *Find a formula for the function below in the form* $F(x) = a(x-h)^3 + k$.



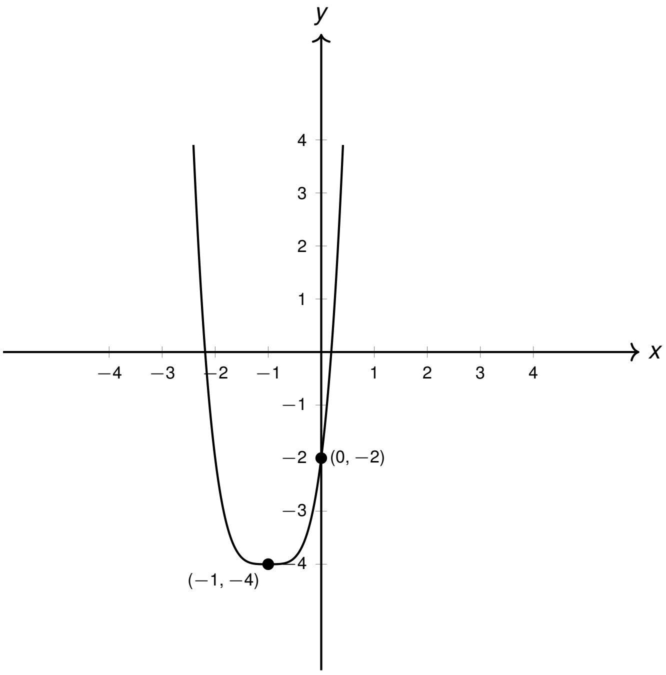
$F(x) = (x-1)^3 - 2$

Problem 8 *Find a formula for the function below in the form* $F(x) = a(x-h)^3 + k$.



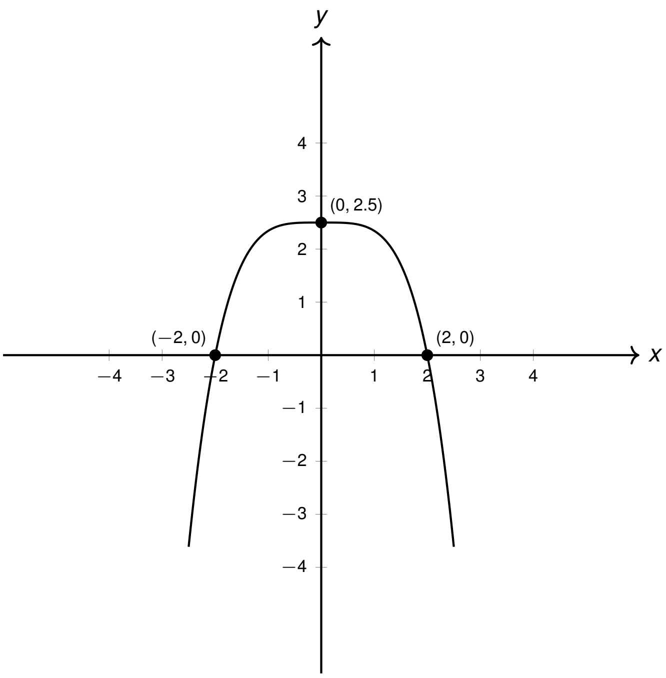
$F(x) = -\frac{1}{2}(x+2)^3 + 3$

Problem 9 *Find a formula for the function below in the form* $F(x) = a(x - h)^4 + k$.



$F(x) = 2(x + 1)^4 - 4$

Problem 10 *Find a formula for the function below in the form* $F(x) = a(x - h)^4 + k$.



$F(x) = -0.15625x^4 + 2.5$

Problem 11 *Find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the polynomial function* $f(x) = 4 - x - 3x^2$.

Degree = 2

Leading term = $-3x^2$

Leading coefficient = -3

Constant term = 4

$\lim_{x\to-\infty} f(x)$ = ($-\infty$ ✓, / ∞)

$\lim_{x\to\infty} f(x)$ = ($-\infty$ ✓, / ∞)

Problem 12 *Find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the polynomial function* $g(x) = 3x^5 - 2x^2 + x + 1$.

Degree = 5

Leading term = $3x^5$

Leading coefficient = 3

Constant term = 1

$\lim_{x\to-\infty} g(x)$ = ($-\infty$ ✓, / ∞)

$\lim_{x\to\infty} g(x)$ = ($-\infty$, / ∞ ✓)

Problem 13 *Find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the polynomial function* $q(r) = 1 - 16r^4$.

Degree = 2

Leading term = $-16r^4$

Leading coefficient = -16

Constant term = 1

$\lim_{r\to-\infty} q(r)$ = ($-\infty$ ✓, / ∞)

$\lim_{r\to\infty} q(r)$ = ($-\infty$ ✓, / ∞)

Problem 14 *Find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the polynomial function* $Z(b) = 42b - b^3$.

Degree = 3

Leading term = $-b^3$

Leading coefficient = -1

Constant term = 0

$\lim_{b\to-\infty} Z(b)$ = ($-\infty$, / ∞ ✓)

$\lim_{b\to\infty} Z(b)$ = ($-\infty$ ✓, / ∞)

Problem 15 *Find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the polynomial function* $f(x) = \sqrt{3}x^{17} + 22.5x^{10} - \pi x^7 + \dfrac{1}{3}$.

Degree = 17

Leading term = $\sqrt{3}x^{17}$

Leading coefficient = $\sqrt{3}$

Constant term = $1/3$

$\lim_{x\to-\infty} f(x)$ = ($-\infty$ ✓, / ∞)

$\lim_{x\to\infty} f(x)$ = ($-\infty$, / ∞ ✓)

Problem 16 *Find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the polynomial function* $s(t) = -4.9t^2 + v_0 t + s_0$.

Degree = 2

Leading term = $-4.9t^2$

Leading coefficient = -4.9

Constant term = s_0

$\lim_{t\to-\infty} s(t)$ = ($-\infty$ ✓, / ∞)

$\lim_{t\to\infty} s(t)$ = ($-\infty$ ✓, / ∞)

Problem 17 *Find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the polynomial function* $P(x) = (x-1)(x-2)(x-3)(x-4)$.

Degree = 4

Leading term = x^4

Leading coefficient = 1

Constant term = 12

$\lim_{x\to-\infty} f(x)$ = ($-\infty$, / ∞ ✓)

$\lim_{x\to\infty} f(x)$ = ($-\infty$, / ∞ ✓)

Problem 18 *Find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the polynomial function* $p(t) = -t^2(3-5t)(t^2+t+4)$.

Degree = 5

Leading term = $5t^5$

Leading coefficient = 5

Constant term = 0

$\lim_{t\to-\infty} p(t)$ = ($-\infty$ ✓, / ∞)

$\lim_{t\to\infty} p(t)$ = ($-\infty$, / ∞ ✓)

Problem 19 *Find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the polynomial function* $f(x) = -2x^3(x+1)(x+2)^2$.

Degree = 6

Leading term = $-2x^6$

Leading coefficient = -2

Constant term = 0

$\lim_{x\to-\infty} f(x)$ = ($-\infty$ ✓, / ∞)

$\lim_{x\to\infty} f(x)$ = ($-\infty$ ✓, / ∞)

Problem 20 *Find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the polynomial function* $G(t) = 4(t-2)^2\left(t+\frac{1}{2}\right)$.

Degree = 3

Leading term = $4t^3$

Leading coefficient = 4

Constant term = 8

$\lim_{t\to-\infty} G(t) = (-\infty \checkmark, /\infty)$

$\lim_{t\to\infty} G(t) = (-\infty, /\infty \checkmark)$

Problem 21 *Find the real zeros of the polynomial $a(x) = x(x+2)^2$ and their corresponding multiplicities. Use this information along with end behavior to provide a rough sketch of the graph of the polynomial function. Compare your answer with the result from a graphing utility.*

$x = 0$ *multiplicity* 1

$x = -2$ *multiplicity* 2

Problem 22 *Find the real zeros of the polynomial $g(t) = t(t+2)^3$ and their corresponding multiplicities. Use this information along with end behavior to provide a rough sketch of the graph of the polynomial function. Compare your answer with the result from a graphing utility.*

$t = 0$ *multiplicity* 1

$t = -2$ *multiplicity* 3

Problem 23 *Find the real zeros of the polynomial $f(z) = -2(z-2)^2(z+1)$ and their corresponding multiplicities. Use this information along with end behavior to provide a rough sketch of the graph of the polynomial function. Compare your answer with the result from a graphing utility.*

$z = 2$ *multiplicity* 2

$z = -1$ *multiplicity* 1

Problem 24 *Find the real zeros of the polynomial $g(x) = (2x+1)^2(x-3)$ and their corresponding multiplicities. Use this information along with end behavior to provide a rough sketch of the graph of the polynomial function. Compare your answer with the result from a graphing utility.*

$x = -\frac{1}{2}$ *multiplicity* 2

$x = 3$ *multiplicity* 1

Problem 25 *Find the real zeros of the polynomial $F(t) = t^3(t+2)^2$ and their corresponding multiplicities. Use this information along with end behavior to provide a rough sketch of the graph of the polynomial function. Compare your answer with the result from a graphing utility.*

$t = 0$ *multiplicity* 3

$t = -2$ *multiplicity* 2

Problem 26 *Find the real zeros of the polynomial $P(z) = (z-1)(z-2)(z-3)(z-4)$ and their corresponding multiplicities. Use this information along with end behavior to provide a rough sketch of the graph of the polynomial function. Compare your answer with the result from a graphing utility.*

$z = 1$ *multiplicity* 1

$z = 2$ *multiplicity* 1

$z = 3$ *multiplicity* 1

$z = 4$ *multiplicity* 1

Problem 27 *Find the real zeros of the polynomial $Q(x) = (x+5)^2(x-3)^4$ and their corresponding multiplicities. Use this information along with end behavior to provide a rough sketch of the graph of the polynomial function. Compare your answer with the result from a graphing utility.*

$x = -5$ *multiplicity* 2

$x = 3$ *multiplicity* 4

Problem 28 *Find the real zeros of the polynomial $h(t) = t^2(t-2)^2(t+2)^2$ and their corresponding multiplicities. Use this information along with end behavior to provide a rough sketch of the graph of the polynomial function. Compare your answer with the result from a graphing utility.*

$t = -2$ *multiplicity* 2

$t = 0$ *multiplicity* 2

$t = 2$ *multiplicity* 2

Problem 29 *Find the real zeros of the polynomial $H(z) = (3-z)(z^2+1)$ and their corresponding multiplicities. Use this information along with end behavior to provide a rough sketch of the graph of the polynomial function. Compare your answer with the result from a graphing utility.*

$z = 3$ *multiplicity* 1

Problem 30 *Find the real zeros of the polynomial $Z(x) = x(42 - x^2)$ and their corresponding multiplicities. Use this information along with end behavior to provide a rough sketch of the graph of the polynomial function. Compare your answer with the result from a graphing utility.*

$x = -\sqrt{42}$ *multiplicity* 1

$x = 0$ *multiplicity* 1

$x = \sqrt{42}$ *multiplicity* 1

Problem 31 *Determine analytically if the function $f(x) = 7x$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even,/ odd ✓, / neither)

Problem 32 *Determine analytically if the function $g(t) = 7t+2$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even,/ odd, / neither ✓)

Problem 33 *Determine analytically if the function $p(z) = 7$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even ✓,/ odd, / neither)

Problem 34 *Determine analytically if the function $F(s) = 3s^2 - 4$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even ✓,/ odd, / neither)

Problem 35 *Determine analytically if the function $h(t) = 4 - t^2$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even ✓,/ odd, / neither)

Problem 36 *Determine analytically if the function $g(x) = x^2 - x - 6$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even,/ odd, / neither ✓)

Problem 37 *Determine analytically if the function $f(x) = 2x^3 - x$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even,/ odd ✓, / neither)

Problem 38 *Determine analytically if the function $p(z) = -z^5 + 2z^3 - z$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even,/ odd ✓, / neither)

Problem 39 *Determine analytically if the function $G(t) = t^6 - t^4 + t^2 + 9$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even ✓,/ odd, / neither)

Problem 40 *Determine analytically if the function $G(s) = s(s^2 - 1)$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even,/ odd ✓, / neither)

Problem 41 *Determine analytically if the function $f(x) = (x^2 + 1)(x - 1)$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even,/ odd, / neither ✓)

Problem 42 *Determine analytically if the function $H(t) = (t^2 - 1)(t^4 + t^2 + 3)$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even ✓,/ odd, / neither)

Problem 43 *Determine analytically if the function $g(t) = t(t - 2)(t + 2)$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even,/ odd ✓, / neither)

Problem 44 *Determine analytically if the function $P(z) = (2z^5 - 3z)(5z^3 + z)$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even ✓,/ odd, / neither)

Problem 45 *Determine analytically if the function $f(x) = 0$ is even, odd, or neither. Confirm your answer using a graphing utility.*

(even ✓,/ odd ✓, / neither)

Problem 46 *Suppose $p(x)$ is a polynomial function written in the form of Definition* **??**.

1. *If the nonzero terms of $p(x)$ consist of even powers of x (or a constant), explain why p is even.*
2. *If the nonzero terms of $p(x)$ consist of odd powers of x, explain why p is odd.*
3. *If $p(x)$ the nonzero terms of $p(x)$ contain at least one odd power of x and one even power of x (or a constant term), then p is neither even nor odd.*

Problem 47 *Use the results of Exercise* **??** *to determine whether the following functions are even, odd, or neither.*

1. $p(x) = 3x^4 + x^2 - 1$

   even

2. $F(s) = s^3 - 14s$
   odd
3. $f(t) = 2t^5 - t^2 + 1$
   neither
4. $g(x) = x^3(x^2 + 1)$
   odd[1]

Problem 48 *Show $f(x) = |x|$ is an even function.*

For $f(x) = |x|$, $f(-x) = |-x| = |(-1)x| = |-1||x| = (1)|x| = |x|$. Hence, $f(-x) = f(x)$.

Problem 49 *Rework Example **??** assuming the box is to be made from an 8.5 inch by 11 inch sheet of paper. Using scissors and tape, construct the box. Are you surprised?*[2]

$V(x) = x(8.5 - 2x)(11 - 2x) = 4x^3 - 39x^2 + 93.5x$, $0 < x < 4.25$. Volume is maximized when $x \approx 1.58$, so we get the dimensions of the box with maximum volume are: height $\approx$ 1.58 inches, width $\approx$ 5.34 inches, and depth $\approx$ 7.84 inches. The maximum volume is $\approx$ 66.15 cubic inches.

Problem 50 *For each function $f(x)$ listed below, compute the average rate of change over the indicated interval.*[3] *What trends do you observe? How do your answers manifest themselves graphically?*

$f(x)$	$[-0.1, 0]$	$[0, 0.1]$	$[0.9, 1]$	$[1, 1.1]$	$[1.9, 2]$	$[2, 2.1]$
1						
x						
x^2						
x^3						
x^4						
x^5						

Each of these average rates of change indicate slope of the curve over the given interval. Smaller slopes correspond to 'flatter' curves and higher slopes correspond to 'steeper' curves.

$f(x)$	$[-0.1, 0]$	$[0, 0.1]$	$[0.9, 1]$	$[1, 1.1]$	$[1.9, 2]$	$[2, 2.1]$
1	0	0	0	0	0	0
x	1	1	1	1	1	1
x^2	-0.1	0.1	1.9	2.1	3.9	4.1
x^3	0.01	0.01	2.71	3.31	11.41	12.61
x^4	-0.001	0.001	3.439	4.641	29.679	34.481
x^5	0.0001	0.0001	4.0951	6.1051	72.3901	88.4101

Problem 51 *For each function $f(x)$ listed below, compute the average rate of change over the indicated interval.*[4] *What trends do you observe? How do your answers manifest themselves graphically?*

[1] You need to first multiply out the expression for $g(x)$ so it is in the form prescribed by Definition **??**.

[2] Consider decorating the box and presenting it to your instructor. If done well enough, maybe your instructor will issue you some bonus points. Or maybe not.

[3] See Definition **??** in Section **??** for a review of this concept, as needed.

[4] See Definition **??** in Section **??** for a review of this concept, as needed.

$f(x)$	$[0.9, 1.1]$	$[0.99, 1.01]$	$[0.999, 1.001]$	$[0.9999, 1.0001]$
1				
x				
x^2				
x^3				
x^4				
x^5				

As we sample points closer to $x = 1$, the slope of the curve approaches the exponent on x.

$f(x)$	$[0.9, 1.1]$	$[0.99, 1.01]$	$[0.999, 1.001]$	$[0.9999, 1.0001]$
1	0	0	0	0
x	1	1	1	1
x^2	2	2	2	2
x^3	3.01	3.0001	≈ 3	≈ 3
x^4	4.04	4.0004	≈ 4	≈ 4
x^5	5.1001	≈ 5.001	≈ 5	≈ 5

Problem 52 *Suppose the revenue R, in thousands of dollars, from producing and selling x hundred LCD TVs is given by $R(x) = -5x^3 + 35x^2 + 155x$ for $0 \leq x \leq 10.07$. Use a graphing utility to graph $y = R(x)$ and determine the number of TVs which should be sold to maximize revenue. What is the maximum revenue?*

The calculator gives the location of the absolute maximum (rounded to three decimal places) as $x \approx 6.305$ and $y \approx 1115.417$. Since x represents the number of TVs sold in hundreds, $x = 6.305$ corresponds to 630.5 TVs. Since we can't sell half of a TV, we compare $R(6.30) \approx 1115.415$ and $R(6.31) \approx 1115.416$, so selling 631 TVs results in a (slightly) higher revenue. Since y represents the revenue in thousands of dollars, the maximum revenue is \$1,115,416.

Problem 53 *Suppose the revenue R, in thousands of dollars, from producing and selling x hundred LCD TVs is given by $R(x) = -5x^3 + 35x^2 + 155x$ for $0 \leq x \leq 10.07$. Assume the cost, in thousands of dollars, to produce x hundred LCD TVs is given by the function $C(x) = 200x + 25$ for $x \geq 0$. Find and simplify an expression for the profit function $P(x)$.*

(Remember: Profit = Revenue - Cost.)

$P(x) = R(x) - C(x) = -5x^3 + 35x^2 - 45x - 25$, $0 \leq x \leq 10.07$.

Problem 54 *Suppose the revenue R, in thousands of dollars, from producing and selling x hundred LCD TVs is given by $R(x) = -5x^3 + 35x^2 + 155x$ for $0 \leq x \leq 10.07$. Use a graphing utility to graph $y = P(x)$ and determine the number of TVs which should be sold to maximize profit. What is the maximum profit?*

The calculator gives the location of the absolute maximum (rounded to three decimal places) as $x \approx 3.897$ and $y \approx 35.255$. Since x represents the number of TVs sold in hundreds, $x = 3.897$ corresponds to 389.7 TVs. Since we can't sell 0.7 of a TV, we compare $P(3.89) \approx 35.254$ and $P(3.90) \approx 35.255$, so selling 390 TVs results in a (slightly) higher revenue. Since y represents the revenue in thousands of dollars, the maximum revenue is \$35,255.

Problem 55 *While developing their newest game, Sasquatch Attack!, the makers of the PortaBoy (from Example **??**) revised their cost function and now use $C(x) = .03x^3 - 4.5x^2 + 225x + 250$, for $x \geq 0$. As before, $C(x)$ is the cost to make x PortaBoy Game Systems. Market research indicates that the demand function $p(x) = -1.5x + 250$ remains unchanged. Use a graphing utility to find the production level x that maximizes the profit made by producing and selling x PortaBoy game systems.*

Making and selling 71 PortaBoys yields a maximized profit of \$5910.67.

Problem 56 *According to US Postal regulations, a rectangular shipping box must satisfy the following inequality: "Length + Girth $\leq$ 130 inches" for Parcel Post and "Length + Girth $\leq$ 108 inches" for other services.*

Let's assume we have a closed rectangular box with a square face of side length x as drawn below. The length is the longest side and is clearly labeled. The girth is the distance around the box in the other two dimensions so in our case it is the sum of the four sides of the square, $4x$.

1. *Assuming that we'll be mailing a box via Parcel Post where Length + Girth = 130 inches, express the length of the box in terms of x and then express the volume V of the box in terms of x.*

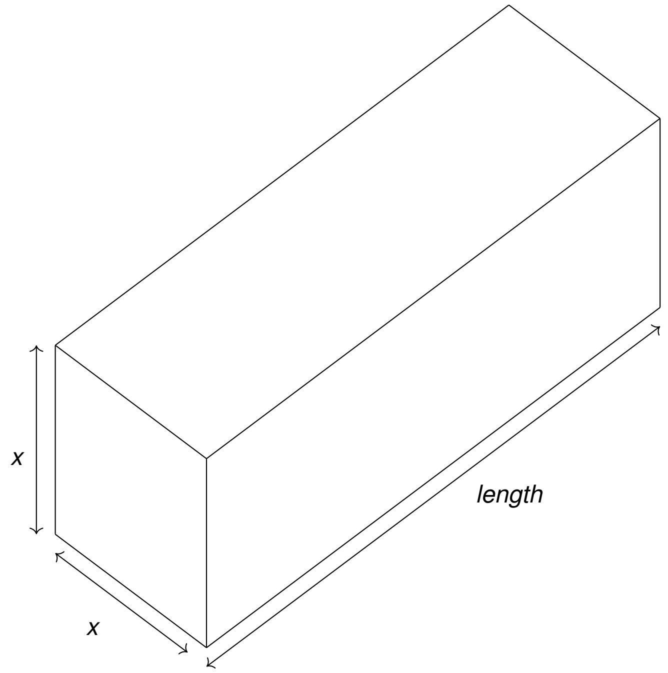
   To maximize the volume, we assume we start with the maximum Length + Girth of 130, so the length is $130-4x$. The volume of a rectangular box is 'length $\times$ width $\times$ height' so we get $V(x) = x^2(130-4x) = -4x^3 + 130x^2$.

2. *Find the dimensions of the box of maximum volume that can be shipped via Parcel Post.*

   Using a graphing utility, we get a (local) maximum of $y = V(x)$ at $(21.67, 20342.59)$. Hence, the maximum volume is $20342.59 in.^3$ using a box with dimensions $21.67 in. \times 21.67 in. \times 43.32 in.$.

3. *Repeat parts **??** and **??** if the box is shipped using "other services".*

   If we start with Length + Girth = 108 then the length is $108 - 4x$ so $V(x) = -4x^3 + 108x^2$. Graphing $y = V(x)$ shows a (local) maximum at $(18.00, 11664.00)$ so the dimensions of the box with maximum volume are $18.00 in. \times 18.00 in. \times 36 in.$ for a volume of $11664.00 in.^3$. (Calculus will confirm that the measurements which maximize the volume are exactly 18in. by 18in. by 36in., however, as I'm sure you are aware by now, we treat all numerical results as approximations and list them as such.)



Problem 57 *This exercise revisits the data set from Exercise **??** in Section **??**. In that exercise, you were given a chart of the number of hours of daylight they get on the 21^{st} of each month in Fairbanks, Alaska based on the 2009 sunrise and sunset data found on the U.S. Naval Observatory website. Here $x = 1$ represents January 21, 2009, $x = 2$ represents February 21, 2009, and so on.*

Month Number	1	2	3	4	5	6	7	8	9	10	11	12
Hours of Daylight	5.8	9.3	12.4	15.9	19.4	21.8	19.4	15.6	12.4	9.1	5.6	3.3

1. *Find cubic (third degree) and quartic (fourth degree) polynomials which model this data and comment on the goodness of fit for each. What can we say about using either model to make predictions about the year 2020? (Hint: Think about the end behavior of polynomials.)*

   The cubic regression model is $p_3(x) = 0.0226x^3 - 0.9508x^2 + 8.615x - 3.446$. *It has* $R^2 = 0.9377$ *which isn't bad. The graph of* $y = p_3(x)$ *along with the data is shown below on the left. Note* p_3 *hits the* x*-axis at about* $x = 12.45$ *making this a bad model for future predictions.*

   The quartic regression model is $p_4(x) = 0.0144x^4 - 0.3507x^3 + 2.259x^2 - 1.571x + 5.513$. *It has* $R^2 = 0.9859$ *which is good. The graph of* $y = p_4(x)$ *along with data is shown below on the right. Note* $p_4(15)$ *is above* 24 *making this a bad model as well for future predictions.*

2. *Use the models to see how many hours of daylight they got on your birthday and then check the website to see how accurate the models are.*

   To use the model to approximate the number of hours of sunlight on your birthday, you'll have to figure out what decimal value of x *is close enough to your birthday and then plug it into the model. Jeff's birthday is July 31 which is 10 days after July 21 (*$x = 7$*). Assuming 30 days in a month, I think* $x = 7.33$ *should work for my birthday and* $p_3(7.33) \approx 17.5$. *The website says there will be about* 18.25 *hours of daylight that day.*

   Here, $p_4(7.33) \approx 18.71$ *so this model more accurately predicts the number of hours of daylight on Jeff's birthday.*

3. *Sasquatch are largely nocturnal, so what days of the year according to your models allow for at least 14 hours of darkness for field research on the elusive creatures?*

   To have 14 hours of darkness we need 10 hours of daylight. We see that $p_3(1.96) \approx 10$ *and* $p_3(10.05) \approx 10$ *so it seems reasonable to say that we'll have at least 14 hours of darkness from December 21, 2008 (*$x = 0$*) to February 21, 2009 (*$x = 2$*) and then again from October 21,2009 (*$x = 10$*) to December 21, 2009 (*$x = 12$*).*

   This model says we'll have at least 14 hours of darkness from December 21, 2008 ($x = 0$*) to about March 1, 2009 (*$x = 2.30$*) and then again from October 10, 2009 (*$x = 9.667$*) to December 21, 2009 (*$x = 12$*).*

Problem 58 *An electric circuit is built with a variable resistor installed. For each of the following resistance values (measured in kilo-ohms, kΩ), the corresponding power to the load (measured in milliwatts, mW) is given in the table below.* [5]

Resistance: (kΩ)	1.012	2.199	3.275	4.676	6.805	9.975
Power: (mW)	1.063	1.496	1.610	1.613	1.505	1.314

1. *Make a scatter diagram of the data using the Resistance as the independent variable and Power as the dependent variable.*

2. *Use your calculator to find quadratic (2nd degree), cubic (3rd degree) and quartic (4th degree) regression models for the data and judge the reasonableness of each.*

   The quadratic model is $P_2(x) = -0.021x^2 + 0.241x + 0.956$, $R^2 = 0.7771$.
   The cubic model is $P_3(x) = 0.005x^3 - 0.103x^2 + 0.602x + 0.573$, $R^2 = 0.9815$.
   The quartic model is $P_4(x) = -0.000969x^4 + 0.0253x^3 - 0.240x^2 + 0.944x + 0.330$, $R^2 = 0.9993$.

3. *For each of the models found above, find the predicted maximum power that can be delivered to the load. What is the corresponding resistance value?*

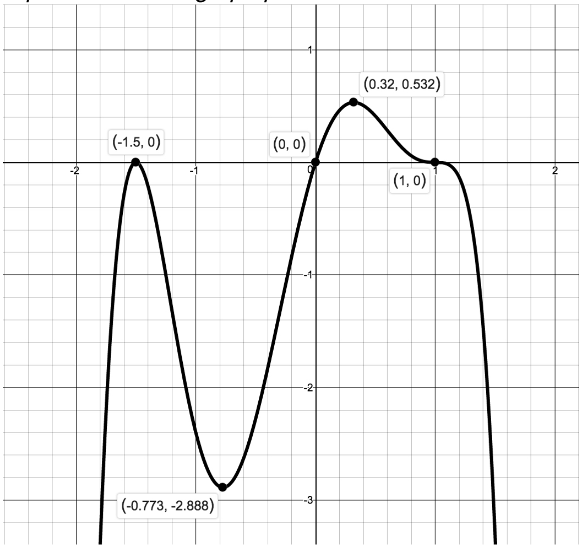
   The models give maximums: $P_2(5.737) \approx 1.648$, $P_3(4.232) \approx 1.657$ *and* $P_4(3.784) \approx 1.630$.

4. *Discuss with your classmates the limitations of these models - in particular, discuss the end behavior of each.*

[5] The authors wish to thank Don Anthan and Ken White of Lakeland Community College for devising this problem and generating the accompanying data set.

EXERCISES FOR GRAPHS OF POLYNOMIALS

Problem 59 *Below is a graph of a polynomial function $y = p(x)$ as generated by a graphing utility. Answer the following questions about p based on the graph provided.*



1. *Describe the end behavior of $y = p(x)$.*

   $\lim_{x \to -\infty} p(x) = -\infty$ *and* $\lim_{x \to \infty} p(x) = -\infty$

2. *List the real zeros of p along with their respective multiplicities.*

   The zeros appear to be: $x = -1.5$, even multiplicity - probably 2 since it doesn't 'look like' the graph is very flat near $x = 2$; $x = 0$, odd multiplicity - probably 1 since the graph seems fairly linear as it passes through the origin; $x = 1$ odd multiplicity - probably 3 or higher since the graph seems fairly 'flat' near $x = 1$.

3. *List the local minimums and local maximums of the graph of $y = p(x)$.*

   local minimum: approximately $(-0.773, -2.888)$; local maximums: approximately $(-1.5, 0)$, and $(0.32, 0.532)$

4. *What can be said about the degree of and leading coefficient $p(x)$?*

   Based on the graph, even degree (at least 6 based on multiplicities) with a negative leading coefficient based on the end behavior.

5. *It turns out that $p(x)$ is a seventh degree polynomial.*[6] *How can this be?*

   We only have a portion of the graph represented here.

Problem 60 *(This Exercise is a follow up to Example* **??***.) Use a graphing utility to compare and contrast the graphs of $f(x) = (2x - 1)(x + 1)^2(1 - x)(x^2 + 1)$ and $g(x) = (2x - 1)(x + 1)^2(1 - x)$.*

Problem 61 *Use the graph of $y = p(x) = (2x - 1)(x + 1)(1 - x^4)$ on page* **??** *to estimate the largest open interval containing $x = -0.235$ which satisfies the the criteria for 'local minimum' in Definition* **??***.*

We are looking for the largest open interval containing $x = -0.235$ for which the graph of $y = p(x)$ is at or above $y = -1.121$. Since each of the gridlines on the x-axis correspond to 0.2 units, we approximate this interval as $(-1.25$ ish$, 1.1$ ish$)$.

Problem 62 *In light of Definition* **??***, explain why every point on the graph of a constant function is both a local maximum and a local minimum.*

[6] to be exact, $p(x) = -0.1\,(x + 1.5)^2\,(3x)\,(x - 1)^3\,(x + 5)$.

Problem 63 *This exercise involves the greatest integer function, $f(x) = \lfloor x \rfloor$, introduced in Example* **??**. *Explain why the points (k, k) for integers k are local maximums but not local minimums.*

Problem 64 *Use Theorems* **??** *and* **??** *prove Theorem* **??**.

Problem 65 *Here are a few other questions for you to discuss with your classmates.*

1. *How many and how few local extrema could a polynomial of degree n have?*
2. *Could a polynomial have two local maxima but no local minima?*
3. *If a polynomial has two local maxima and two local minima, can it be of odd degree? Can it be of even degree?*
4. *Can a polynomial have local extrema without having any real zeros?*
5. *Why must every polynomial of odd degree have at least one real zero?*
6. *Can a polynomial have two distinct real zeros and no local extrema?*
7. *Can an x-intercept yield a local extrema? Can it yield an absolute extrema?*
8. *If the y-intercept yields an absolute minimum, what can we say about the degree of the polynomial and the sign of the leading coefficient?*

Problem 66 *(This is a follow-up to Exercises* **??** *in Section* **??** *and* **??** *in Section* **??**.*) The Lagrange Interpolate function L for four points: (x_0, y_0), (x_1, y_1), (x_2, y_2), (x_3, y_3) where x_0, x_1, x_2, and x_3 are four distinct real numbers is given by the formula:*

$$\begin{aligned} L(x) &= y_0 \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + y_1 \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \\ &\quad + y_2 \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + y_3 \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \end{aligned}$$

1. *Choose four points with different x-values and construct the Lagrange Interpolate for those points. Verify each of the points lies on the polynomial.*
2. *Verify that, in general, $L(x_0) = y_0$, $L(x_1) = y_1$, $L(x_2) = y_2$, and $L(x_3) = y_3$.*
3. *Find $L(x)$ for the points $(-1, 1)$, $(0, 0)$, $(1, 1)$ and $(2, 4)$. What happens?*
   $L(x) = x^2$
4. *Find $L(x)$ for the points $(-1, 0)$, $(0, 1)$, $(1, 2)$ and $(2, 3)$. What happens?*
   $L(x) = x + 1$
5. *Generalize the formula for $L(x)$ to five points. What's the pattern?*