Exercises for Polydivision

Problem 1 *Use synthetic division to compute* $\left(3x^2 - 2x + 1\right) \div (x - 1)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(x) = d(x)\, q(x) + r(x)$, *, as in Theorem* **??**.

$3x^2 - 2x + 1 = (x - 1)\,(3x + 1) + 2$

Problem 2 *Use synthetic division to compute* $\left(x^2 - 5\right) \div (x - 5)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(x) = d(x)\, q(x) + r(x)$, *, as in Theorem* **??**.

$x^2 - 5 = (x - 5)\,(x + 5) + 20$

Problem 3 *Use synthetic division to compute* $\left(3 - 4t - 2t^2\right) \div (t + 1)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(t) = d(t)\, q(t) + r(t)$, *, as in Theorem* **??**.

$3 - 4t - 2t^2 = (t + 1)\,(-2t - 2) + 5$

Problem 4 *Use synthetic division to compute* $\left(4t^2 - 5t + 3\right) \div (t + 3)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(t) = d(t)\, q(t) + r(t)$, *, as in Theorem* **??**.

$4t^2 - 5t + 3 = (t + 3)\,(4t - 17) + 54$

Problem 5 *Use synthetic division to compute* $\left(z^3 + 8\right) \div (z + 2)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(z) = d(z)\, q(z) + r(z)$, *, as in Theorem* **??**.

$z^3 + 8 = (z + 2)\,\left(z^2 - 2z + 4\right) + 0$

Problem 6 *Use synthetic division to compute* $\left(4z^3 + 2z - 3\right) \div (z - 3)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(z) = d(z)\, q(z) + r(z)$, *, as in Theorem* **??**.

$4z^3 + 2z - 3 = (z - 3)\,\left(4z^2 + 12z + 38\right) + 111$

Problem 7 *Use synthetic division to compute* $\left(18x^2 - 15x - 25\right) \div \left(x - \frac{5}{3}\right)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(x) = d(x)\, q(x) + r(x)$, *, as in Theorem* **??**.

$18x^2 - 15x - 25 = \left(x - \frac{5}{3}\right)(18x + 15) + 0$

Problem 8 *Use synthetic division to compute* $\left(4x^2 - 1\right) \div \left(x - \frac{1}{2}\right)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(x) = d(x)\, q(x) + r(x)$, *, as in Theorem* **??**.

$4x^2 - 1 = \left(x - \frac{1}{2}\right)(4x + 2) + 0$

Problem 9 *Use synthetic division to compute* $\left(2t^3 + t^2 + 2t + 1\right) \div \left(t + \frac{1}{2}\right)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(t) = d(t)\, q(t) + r(t)$, *, as in Theorem* **??**.

$2t^3 + t^2 + 2t + 1 = \left(t + \frac{1}{2}\right)\left(2t^2 + 2\right) + 0$

Problem 10 *Use synthetic division to compute* $\left(3t^3 - t + 4\right) \div \left(t - \frac{2}{3}\right)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(t) = d(t)\, q(t) + r(t)$, *, as in Theorem* **??**.

$3t^3 - t + 4 = \left(t - \frac{2}{3}\right)\left(3t^2 + 2t + \frac{1}{3}\right) + \frac{38}{9}$

Problem 11 *Use synthetic division to compute* $\left(2z^3 - 3z + 1\right) \div \left(z - \frac{1}{2}\right)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(z) = d(z)\,q(z) + r(z),$ *, as in Theorem* **??**.

$2z^3 - 3z + 1 = \left(z - \frac{1}{2}\right)\left(2z^2 + z - \frac{5}{2}\right) + -\frac{1}{4}$

Problem 12 *Use synthetic division to compute* $\left(4z^4 - 12z^3 + 13z^2 - 12z + 9\right) \div \left(z - \frac{3}{2}\right)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(z) = d(z)\,q(z) + r(z),$ *, as in Theorem* **??**.

$4z^4 - 12z^3 + 13z^2 - 12z + 9 = \left(z - \frac{3}{2}\right)\left(4z^3 - 6z^2 + 4z - 6\right) + 0$

Problem 13 *Use synthetic division to compute* $\left(x^4 - 6x^2 + 9\right) \div \left(x - \sqrt{3}\right)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(x) = d(x)\,q(x) + r(x),$ *, as in Theorem* **??**.

$x^4 - 6x^2 + 9 = \left(x - \sqrt{3}\right)\left(x^3 + \sqrt{3}\,x^2 - 3x - 3\sqrt{3}\right) + 0$

Problem 14 *Use synthetic division to compute* $\left(x^6 - 6x^4 + 12x^2 - 8\right) \div \left(x + \sqrt{2}\right)$. *Identify the quotient and remainder. Write the divisor, quotient and remainder in the form* $p(x) = d(x)\,q(x) + r(x),$ *, as in Theorem* **??**.

$\left(x^6 - 6x^4 + 12x^2 - 8\right) = \left(x + \sqrt{2}\right)\left(x^5 - \sqrt{2}\,x^4 - 4x^3 + 4\sqrt{2}\,x^2 + 4x - 4\sqrt{2}\right) + 0$

Problem 15 *Find* $p(c)$ *using the Remainder Theorem. If* $p(c) = 0$, *use the Factor Theorem to partially factor the polynomial function.*

$p(x) = 2x^2 - x + 1,\ c = 4$

$p(4) = 29$

Problem 16 *Find* $p(c)$ *using the Remainder Theorem. If* $p(c) = 0$, *use the Factor Theorem to partially factor the polynomial function.*

$p(x) = 4x^2 - 33x - 180,\ c = 12$

$p(12) = 0,\ p(x) = (x - 12)(4x + 15)$

Problem 17 *Find* $p(c)$ *using the Remainder Theorem. If* $p(c) = 0$, *use the Factor Theorem to partially factor the polynomial function.*

$p(t) = 2t^3 - t + 6,\ c = -3$

$p(-3) = -45$

Problem 18 *Find* $p(c)$ *using the Remainder Theorem. If* $p(c) = 0$, *use the Factor Theorem to partially factor the polynomial function.*

$p(t) = t^3 + 2t^2 + 3t + 4,\ c = -1$

$p(-1) = 2$

Problem 19 *Find* $p(c)$ *using the Remainder Theorem. If* $p(c) = 0$, *use the Factor Theorem to partially factor the polynomial function.*

$p(z) = 3z^3 - 6z^2 + 4z - 8,\ c = 2$

$p(2) = 0,\ p(z) = (z - 2)\left(3z^2 + 4\right)$

Problem 20 *Find $p(c)$ using the Remainder Theorem. If $p(c) = 0$, use the Factor Theorem to partially factor the polynomial function.*

$p(z) = 8z^3 + 12z^2 + 6z + 1$, $c = -\frac{1}{2}$

$p\left(-\frac{1}{2}\right) = 0$, $p(z) = \left(z + \frac{1}{2}\right)\left(8z^2 + 8z + 2\right)$

Problem 21 *Find $p(c)$ using the Remainder Theorem. If $p(c) = 0$, use the Factor Theorem to partially factor the polynomial function.*

$p(x) = x^4 - 2x^2 + 4$, $c = \frac{3}{2}$

$p\left(\frac{3}{2}\right) = \frac{73}{16}$

Problem 22 *Find $p(c)$ using the Remainder Theorem. If $p(c) = 0$, use the Factor Theorem to partially factor the polynomial function.*

$p(x) = 6x^4 - x^2 + 2$, $c = -\frac{2}{3}$

$p\left(-\frac{2}{3}\right) = \frac{74}{27}$

Problem 23 *Find $p(c)$ using the Remainder Theorem. If $p(c) = 0$, use the Factor Theorem to partially factor the polynomial function.*

$p(t) = t^4 + t^3 - 6t^2 - 7t - 7$, $c = -\sqrt{7}$

$p(-\sqrt{7}) = 0$, $p(t) = (t + \sqrt{7})\left(t^3 + (1 - \sqrt{7})t^2 + (1 - \sqrt{7})t - \sqrt{7}\right)$

Problem 24 *Find $p(c)$ using the Remainder Theorem. If $p(c) = 0$, use the Factor Theorem to partially factor the polynomial function.*

$p(t) = t^2 - 4t + 1$, $c = 2 - \sqrt{3}$

$p(2 - \sqrt{3}) = 0$, $p(t) = (t - (2 - \sqrt{3}))(t - (2 + \sqrt{3}))$

Problem 25 *In this exercise you are given a polynomial function and one of its zeros. Find the remaining real zeros and factor the polynomial.*

$x^3 - 6x^2 + 11x - 6, \quad c = 1$

$x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$

Problem 26 *In this exercise you are given a polynomial function and one of its zeros. Find the remaining real zeros and factor the polynomial.*

$x^3 - 24x^2 + 192x - 512, \quad c = 8$

When we factor the polynomial we get $x^3 - 24x^2 + 192x - 512 = (x - 8)^3$

Problem 27 *In this exercise you are given a polynomial function and one of its zeros. Find the remaining real zeros and factor the polynomial.*

$3t^3 + 4t^2 - t - 2, \quad c = \frac{2}{3}$

$3t^3 + 4t^2 - t - 2 = 3\left(t - \frac{2}{3}\right)(t + 1)^2$

Problem 28 *In this exercise you are given a polynomial function and one of its zeros. Find the remaining real zeros and factor the polynomial.*

$2t^3 - 3t^2 - 11t + 6, \quad c = \frac{1}{2}$

$2t^3 - 3t^2 - 11t + 6 = 2\left(t - \frac{1}{2}\right)(t+2)(t-3)$

Problem 29 *In this exercise you are given a polynomial function and one of its zeros. Find the remaining real zeros and factor the polynomial.*

$z^3 + 2z^2 - 3z - 6, \quad c = -2$

$z^3 + 2z^2 - 3z - 6 = (z+2)(z+\sqrt{3})(z-\sqrt{3})$

Problem 30 *In this exercise you are given a polynomial function and one of its zeros. Find the remaining real zeros and factor the polynomial.*

$2z^3 - z^2 - 10z + 5, \quad c = \frac{1}{2}$

$2z^3 - z^2 - 10z + 5 = 2\left(z - \frac{1}{2}\right)(z+\sqrt{5})(z-\sqrt{5})$

Problem 31 *In this exercise you are given a polynomial function and one of its zeros. Find the remaining real zeros and factor the polynomial.*

$4x^4 - 28x^3 + 61x^2 - 42x + 9$, $c = \frac{1}{2}$ *is a zero of multiplicity 2*

$4x^4 - 28x^3 + 61x^2 - 42x + 9 = 4\left(x - \frac{1}{2}\right)^2 (x-3)^2$

Problem 32 *In this exercise you are given a polynomial function and one of its zeros. Find the remaining real zeros and factor the polynomial.*

$t^5 + 2t^4 - 12t^3 - 38t^2 - 37t - 12$, $c = -1$ *is a zero of multiplicity 3*

When we factor the polynomial we get $t^5 + 2t^4 - 12t^3 - 38t^2 - 37t - 12 = (t+1)^3(t+3)(t-4)$

Problem 33 *In this exercise you are given a polynomial function and one of its zeros. Find the remaining real zeros and factor the polynomial.*

$125z^5 - 275z^4 - 2265z^3 - 3213z^2 - 1728z - 324$, $c = -\frac{3}{5}$ *is a zero of multiplicity 3*

$125z^5 - 275z^4 - 2265z^3 - 3213z^2 - 1728z - 324 = 125\left(z + \frac{3}{5}\right)^3 (z+2)(z-6)$

Problem 34 *In this exercise you are given a polynomial function and one of its zeros. Find the remaining real zeros and factor the polynomial.*

$x^2 - 2x - 2, \quad c = 1 - \sqrt{3}$

$x^2 - 2x - 2 = (x - (1-\sqrt{3}))(x - (1+\sqrt{3}))$

Problem 35 *Find a quadratic polynomial with integer coefficients which has* $x = \frac{3}{5} \pm \frac{\sqrt{29}}{5}$ *as its real zeros.*

$p(x) = 5x^2 - 6x - 4$

Problem 36 *For* $f(x) = x^3 + 4x^2 - 5x - 14$, *show* $f(-3-\sqrt{2}) = 0$ *and* $f(-3+\sqrt{2}) = 0$ *two ways:*

1. *By direct substitution.*
2. *Using synthetic division and the Factor Theorem*

Problem 37 *Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$ be a polynomial function with the property that $a_n + a_{n-1} + ... + a_1 + a_0 = 0$. (That is, the sum of the coefficients and the constant term is 0.)*

Prove that $(x - 1)$ is a factor of $f(x)$.

Hint: *Show $f(1) = 0$ and invoke the Factor Theorem.*

Problem 38 *Verify the result in number **??** with the functions: $f(x) = x^3 - 2x + 1$ and $f(x) = 3x^4 - x - 2$.*

- *For $f(x) = x^3 - 2x + 1$, the coefficients $1 + (-2) + 1 = 0$ and $f(x) = (x - 1)(x^2 + x - 1)$.*
- *For $f(x) = 3x^4 - x - 2$ the coefficients $3 + (-1) + (-2) = 0$ and $f(x) = (x - 1)(3x^3 + 3x^2 + 3x + 2)$.*

Problem 39 *Suppose a is a nonzero real number. Find the quotients below, using synthetic division as required.*

- $\dfrac{x - a}{x - a} = 1$
- $\dfrac{x^2 - a^2}{x - a} = x + a$
- $\dfrac{x^3 - a^3}{x - a} = x^2 + ax + a^2$
- $\dfrac{x^4 - a^4}{x - a} = x^3 + ax^2 + a^2 x + a^3$
- $\dfrac{x^5 - a^5}{x - a} = x^4 + ax^3 + a^2 x^2 + a^3 x + a^4$

Based on the pattern that evolves, find the quotient: $\dfrac{x^{10} - a^{10}}{x - a}$. What about $\dfrac{x^n - a^n}{x - a}$?

Following the pattern:

- $\dfrac{x^{10} - a^{10}}{x - a} = x^9 + ax^8 + a^2x^7 + a^3x^6 + a^4x^5 + a^5x^4 + a^6x^3 + a^7x^2 + a^8x + a^9$
- $\dfrac{x^n - a^n}{x - a} = x^{n-1} + ax^{n-2} + a^2x^{n-3} + ... + a^{n-2}x + a^{n-1}$

Problem 40 *Use your result from number **??** to rewrite the sum: $1 + r + r^2 + \cdots + r^{n-2} + r^{n-1}$ as a quotient. What assumptions need to be made about r?*

Put $x = 1$ and $a = r$ so that $1 + r + r^2 + \cdots + r^{n-2} + r^{n-1} = \dfrac{1 - r^n}{1 - r}$. Here, $r \neq 1$ as otherwise we'd be dividing by 0.