Exercises for Real Zeros

Problem 1 *For the polynomial* $f(x) = x^3 - 2x^2 - 5x + 6$*:*

- *Use Cauchy's Bound to find an interval containing all of the real zeros.*

  $[-7, 7]$

- *Use the Rational Zeros Theorem to make a list of possible rational zeros.*

  Select All Correct Answers:

  1. ± 1 ✓
  2. ± 2 ✓
  3. ± 3 ✓
  4. ± 4
  5. ± 6 ✓
  6. ± 7

- *Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.*

  There are 2 or 0 positive real zeros; there is 1 negative real zero

Problem 2 *For the polynomial* $f(x) = x^4 + 2x^3 - 12x^2 - 40x - 32$*:*

- *Use Cauchy's Bound to find an interval containing all of the real zeros.*

  $[-41, 41]$

- *Use the Rational Zeros Theorem to make a list of possible rational zeros.*

  Select All Correct Answers:

  1. ± 1 ✓
  2. ± 2 ✓
  3. ± 3
  4. ± 4 ✓
  5. ± 6
  6. ± 8 ✓
  7. ± 16 ✓
  8. ± 32 ✓

- *Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.*

  There is 1 positive real zero; there are 3 or 1 negative real zeros

Problem 3 *For the polynomial* $p(z) = z^4 - 9z^2 - 4z + 12$*:*

- *Use Cauchy's Bound to find an interval containing all of the real zeros.*

  $[-13, 13]$

- *Use the Rational Zeros Theorem to select the possible rational zeros.*

  Select All Correct Answers:

1. ± 1 ✓
2. ± 2 ✓
3. ± 3 ✓
4. ± 4 ✓
5. ± 5
6. ± 6 ✓
7. ± 9
8. ± 12 ✓

- *Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.*

  There are 2 or 0 positive real zeros; there are 2 or 0 negative real zeros

Problem 4 *For the polynomial* $p(z) = z^3 + 4z^2 - 11z + 6$*:*

- *Use Cauchy's Bound to find an interval containing all of the real zeros.*

  $[-12, 12]$

- *Use the Rational Zeros Theorem to make a list of possible rational zeros.*

  Select All Correct Answers:

  1. ± 1 ✓
  2. ± 2 ✓
  3. ± 3 ✓
  4. ± 4
  5. ± 6 ✓
  6. ± 9

- *Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.*

  There are 2 or 0 positive real zeros; there is 1 negative real zero

Problem 5 *For the polynomial* $g(t) = t^3 - 7t^2 + t - 7$*:*

- *Use Cauchy's Bound to find an interval containing all of the real zeros.*

  $[-8, 8]$

- *Use the Rational Zeros Theorem to make a list of possible rational zeros.*

  Select All Correct Answers:

  1. ± 1 ✓
  2. ± 2
  3. ± 3
  4. ± 7 ✓

- *Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.*

  There are 3 or 1 positive real zeros; there are no negative real zeros

Problem 6 *For the polynomial $g(t) = -2t^3 + 19t^2 - 49t + 20$:*

- *Use Cauchy's Bound to find an interval containing all of the real zeros.*
  $\left[-\frac{51}{2}, \frac{51}{2}\right]$
- *Use the Rational Zeros Theorem to select the possible rational zeros.*

  Select All Correct Answers:

  1. ± 1 ✓
  2. ± 2 ✓
  3. ± 3
  4. ± 4 ✓
  5. ± 5 ✓
  6. ± 10 ✓
  7. ± 20 ✓
  8. $\pm\frac{1}{2}$ ✓
  9. $\pm\frac{3}{2}$
  10. $\pm\frac{5}{2}$ ✓

- *Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.*
  There are 3 or 1 positive real zeros; there are no negative real zeros

Problem 7 *For the polynomial $f(x) = -17x^3 + 5x^2 + 34x - 10$:*

- *Use Cauchy's Bound to find an interval containing all of the real zeros.*
  $[-3, 3]$
- *Use the Rational Zeros Theorem to make a list of possible rational zeros.*

  Select All Correct Answers:

  1. ± 1 ✓
  2. ± 2 ✓
  3. ± 3
  4. ± 5 ✓
  5. ± 10 ✓
  6. $\pm\frac{1}{17}$ ✓
  7. $\pm\frac{2}{17}$ ✓
  8. $\pm\frac{3}{17}$
  9. $\pm\frac{5}{17}$ ✓

10. $\pm\frac{10}{17}$ ✓

- *Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.*

  There are 2 or 0 positive real zeros; there is 1 negative real zero

Problem 8 *For the polynomial $f(x) = 36x^4 - 12x^3 - 11x^2 + 2x + 1$:*

- *Use Cauchy's Bound to find an interval containing all of the real zeros.*
- *Use the Rational Zeros Theorem to make a list of possible rational zeros.*

  Select All Correct Answers:

  1. ± 1 ✓
  2. ± 2
  3. $\pm\frac{1}{36}$ ✓
  4. $\pm\frac{1}{18}$ ✓
  5. $\pm\frac{1}{12}$ ✓
  6. $\pm\frac{1}{10}$
  7. $\pm\frac{1}{9}$ ✓
  8. $\pm\frac{1}{8}$
  9. $\pm\frac{1}{6}$ ✓
  10. $\pm\frac{1}{4}$ ✓
  11. $\pm\frac{1}{3}$ ✓
  12. $\pm\frac{1}{2}$ ✓

- *Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.*

  There are 2 or 0 positive real zeros; there are 2 or 0 negative real zeros

Problem 9 *For the polynomial $p(z) = 3z^3 + 3z^2 - 11z - 10$:*

- *Use Cauchy's Bound to find an interval containing all of the real zeros.*
- *Use the Rational Zeros Theorem to select the possible rational zeros.*

  Select All Correct Answers:

  1. ± 1 ✓
  2. ± 2 ✓
  3. ± 3

4. ± 4
5. ± 5 ✓
6. ± 10 ✓
7. $\pm \frac{1}{3}$ ✓
8. $\pm \frac{2}{3}$ ✓
9. $\pm \frac{4}{3}$
10. $\pm \frac{5}{3}$ ✓
11. $\pm \frac{5}{3}$ ✓

- *Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.*

  There is 1 positive real zero; there are 2 or 0 negative real zeros

Problem 10 *For the polynomial* $p(z) = 2z^4 + z^3 - 7z^2 - 3z + 3$*:*

- *Use Cauchy's Bound to find an interval containing all of the real zeros.*
- *Use the Rational Zeros Theorem to make a list of possible rational zeros.*

Select All Correct Answers:

1. ± 1 ✓
2. ± 2
3. ± 3 ✓
4. $\pm \frac{1}{2}$ ✓
5. $\pm \frac{2}{3}$
6. $\pm \frac{3}{2}$ ✓

- *Use Descartes' Rule of Signs to list the possible number of positive and negative real zeros, counting multiplicities.*

  There are 2 or 0 positive real zeros; there are 2 or 0 negative real zeros

Problem 11 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$f(x) = x^3 - 2x^2 - 5x + 6$

$x = -2$ *(mult. 1),* $x = 1$ *(mult. 1),* $x = 3$ *(mult. 1)*

Problem 12 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$f(x) = x^4 + 2x^3 - 12x^2 - 40x - 32$

$x = -2$ *(mult. 3),* $x = 4$ *(mult. 1)*

EXERCISES FOR REAL ZEROS

Problem 13 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$p(z) = z^4 - 9z^2 - 4z + 12$

$z = -2$ *(mult. 2),* $z = 1$ *(mult. 1),* $z = 3$ *(mult. 1)*

Problem 14 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$p(z) = z^3 + 4z^2 - 11z + 6$

$z = -6$ *(mult. 1),* $z = 1$ *(mult. 2)*

Problem 15 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$g(t) = t^3 - 7t^2 + t - 7$

$t = 7$ *(mult. 1)*

Problem 16 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$g(t) = -2t^3 + 19t^2 - 49t + 20$

$t = \frac{1}{2}$ *(mult. 1),* $t = 4$ *(mult. 1),* $t = 5$ *(mult. 1)*

Problem 17 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$f(x) = -17x^3 + 5x^2 + 34x - 10$

$x = \frac{5}{17}$ *(mult. 1),* $x = \pm\sqrt{2}$ *(mult. 1)*

Problem 18 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$f(x) = 36x^4 - 12x^3 - 11x^2 + 2x + 1$

$x = \frac{1}{2}$ *(mult. 2),* $x = -\frac{1}{3}$ *(mult. 2)*

Problem 19 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$p(z) = 3z^3 + 3z^2 - 11z - 10$

$z = -2$ *(mult. 1),* $z = \frac{3 \pm \sqrt{69}}{6}$ *(mult. 1)*

Problem 20 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$p(z) = 2z^4 + z^3 - 7z^2 - 3z + 3$

$z = -1$ *(mult. 1),* $z = \frac{1}{2}$ *(mult. 1),* $z = \pm\sqrt{3}$ *(mult. 1)*

Problem 21 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$g(t) = 9t^3 - 5t^2 - t$

$t = 0$, $t = \frac{5 \pm \sqrt{61}}{18}$ *(each has mult. 1)*

Problem 22 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$g(t) = 6t^4 - 5t^3 - 9t^2$

$t = 0$ *(mult. 2)*, $t = \dfrac{5 \pm \sqrt{241}}{12}$ *(each has mult. 1)*

Problem 23 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$f(x) = x^4 + 2x^2 - 15$

$x = \pm\sqrt{3}$ *(each has mult. 1)*

Problem 24 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$f(x) = x^4 - 9x^2 + 14$

$x = \pm\sqrt{2}$, $x = \pm\sqrt{7}$ *(each has mult. 1)*

Problem 25 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$p(z) = 3z^4 - 14z^2 - 5$

$z = \pm\sqrt{5}$ *(each has mult. 1)*

Problem 26 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$p(z) = 2z^4 - 7z^2 + 6$

$z = \pm\dfrac{\sqrt{6}}{2}$, $z = \pm\sqrt{2}$ *(each has mult. 1)*

Problem 27 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$g(t) = t^6 - 3t^3 - 10$

$t = \sqrt[3]{-2} = -\sqrt[3]{2}$, $t = \sqrt[3]{5}$ *(each has mult. 1)*

Problem 28 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$g(t) = 2t^6 - 9t^3 + 10$

$t = \dfrac{\sqrt[3]{20}}{2}$, $t = \sqrt[3]{2}$ *(each has mult. 1)*

Problem 29 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$f(x) = x^5 - 2x^4 - 4x + 8$

$x = 2$, $x = \pm\sqrt{2}$ *(each has mult. 1)*

Problem 30 *Find the real zeros of the polynomial using the techniques specified by your instructor. State the multiplicity of each real zero.*

$f(x) = 2x^5 + 3x^4 - 18x - 27$

$x = -\dfrac{3}{2}$, $x = \pm\sqrt{3}$ *(each has mult. 1)*

EXERCISES FOR REAL ZEROS

Problem 31 *Use your calculator* [1] *to help you find the real zeros of the polynomial. State the multiplicity of each real zero.*

$f(x) = x^5 - 60x^3 - 80x^2 + 960x + 2304$

$x = -4$ *(mult. 3)*, $x = 6$ *(mult. 2)*

Problem 32 *Use your calculator* [2] *to help you find the real zeros of the polynomial. State the multiplicity of each real zero.*

$f(x) = 25x^5 - 105x^4 + 174x^3 - 142x^2 + 57x - 9$

$x = \frac{3}{5}$ *(mult. 2)*, $x = 1$ *(mult. 3)*

Problem 33 *Use your calculator* [3] *to help you find the real zeros of the polynomial. State the multiplicity of each real zero.*

$f(x) = 90x^4 - 399x^3 + 622x^2 - 399x + 90$

$x = \frac{2}{3}, x = \frac{3}{2}, x = \frac{5}{3}, x = \frac{3}{5}$ *(each has mult. 1)*

Problem 34 *Find the real zeros of* $f(x) = x^3 - \frac{1}{12}x^2 - \frac{7}{72}x + \frac{1}{72}$ *by first finding a polynomial* $q(x)$ *with integer coefficients such that* $q(x) = N \cdot f(x)$ *for some integer* N*. (Recall that the Rational Zeros Theorem required the polynomial in question to have integer coefficients.) Show that* f *and* q *have the same real zeros.*

We choose $q(x) = 72x^3 - 6x^2 - 7x + 1 = 72 \cdot f(x)$*. Clearly* $f(x) = 0$ *if and only if* $q(x) = 0$ *so they have the same real zeros. In this case,* $x = -\frac{1}{3}$, $x = \frac{1}{6}$ *and* $x = \frac{1}{4}$ *are the real zeros of both* f *and* q*.*

Problem 35 *Find the real solutions of the polynomial equation. (See Example* **??***.)*

$9x^3 = 5x^2 + x$

$x = 0, \frac{5 \pm \sqrt{61}}{18}$

Problem 36 *Find the real solutions of the polynomial equation. (See Example* **??***.)*

$9x^2 + 5x^3 = 6x^4$

$x = 0, \frac{5 \pm \sqrt{241}}{12}$

Problem 37 *Find the real solutions of the polynomial equation. (See Example* **??***.)*

$z^3 + 6 = 2z^2 + 5z$

$z = -2, 1, 3$

Problem 38 *Find the real solutions of the polynomial equation. (See Example* **??***.)*

$z^4 + 2z^3 = 12z^2 + 40z + 32$

$z = -2, 4$

Problem 39 *Find the real solutions of the polynomial equation. (See Example* **??***.)*

$t^3 - 7t^2 = 7 - t$

$t = 7$

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[1] You *can* do these without your calculator, but it may test your mettle!
[2] You *can* do these without your calculator, but it may test your mettle!
[3] You *can* do these without your calculator, but it may test your mettle!

Problem 40 *Find the real solutions of the polynomial equation. (See Example **??**.)*

$2t^3 = 19t^2 - 49t + 20$

$t = \frac{1}{2}, 4, 5$

Problem 41 *Find the real solutions of the polynomial equation. (See Example **??**.)*

$x^3 + x^2 = \frac{11x + 10}{3}$

$x = -2, \frac{3 \pm \sqrt{69}}{6}$

Problem 42 *Find the real solutions of the polynomial equation. (See Example **??**.)*

$x^4 + 2x^2 = 15$

$z = \pm\sqrt{3}$

Problem 43 *Find the real solutions of the polynomial equation. (See Example **??**.)*

$14z^2 + 5 = 3z^4$

$z = \pm\sqrt{5}$

Problem 44 *Find the real solutions of the polynomial equation. (See Example **??**.)*

$2z^5 + 3z^4 = 18z + 27$

$z = -\frac{3}{2}, \pm\sqrt{3}$

Problem 45 *Solve the polynomial inequality and state your answer using interval notation. (Hint: type the word "infinity" to get the ∞ symbol.)*

$-2x^3 + 19x^2 - 49x + 20 > 0$

$(-\infty, \frac{1}{2}) \cup (4, 5)$

Problem 46 *Solve the polynomial inequality and state your answer using interval notation.*

$x^4 - 9x^2 \leq 4x - 12$

$\{-2\} \cup [1, 3]$

Problem 47 *Solve the polynomial inequality and state your answer using interval notation.*

$(z - 1)^2 \geq 4$ $(-\infty, -1] \cup [3, \infty)$

Problem 48 *Solve the polynomial inequality and state your answer using interval notation.*

$4z^3 \geq 3z + 1$

$\left\{-\frac{1}{2}\right\} \cup [1, \infty)$

Problem 49 *Solve the polynomial inequality and state your answer using interval notation.*

$t^4 \leq 16 + 4t - t^3$ $[-2, 2]$

Problem 50 *Solve the polynomial inequality and state your answer using interval notation.*

$3t^2 + 2t < t^4$

$(-\infty, -1) \cup (-1, 0) \cup (2, \infty)$

Problem 51 *Solve the polynomial inequality and state your answer using interval notation.*

$\frac{x^3 + 2x^2}{2} < x + 2$

$(-\infty, -2) \cup \left(-\sqrt{2}, \sqrt{2}\right)$

Problem 52 *Solve the polynomial inequality and state your answer using interval notation.*

$\frac{x^3 + 20x}{8} \geq x^2 + 2$

$\{2\} \cup [4, \infty)$

Problem 53 *Solve the polynomial inequality and state your answer using interval notation.*

$2z^4 > 5z^2 + 3$

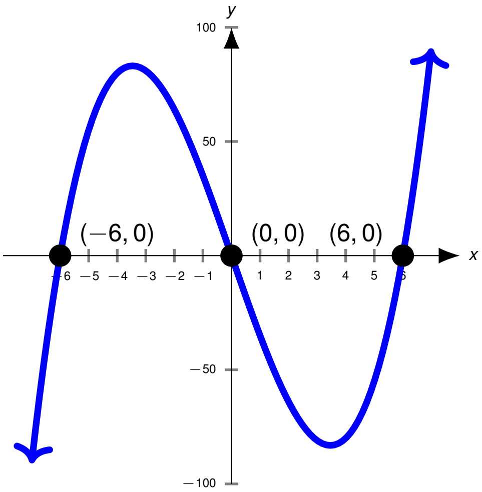
$(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

Problem 54 *Solve the polynomial inequality and state your answer using interval notation.*

$z^6 + z^3 \geq 6$

$(-\infty, -\sqrt[3]{3}) \cup (\sqrt[3]{2}, \infty)$

Problem 55 *Use the the graph of the given polynomial function to solve the stated inequality.*

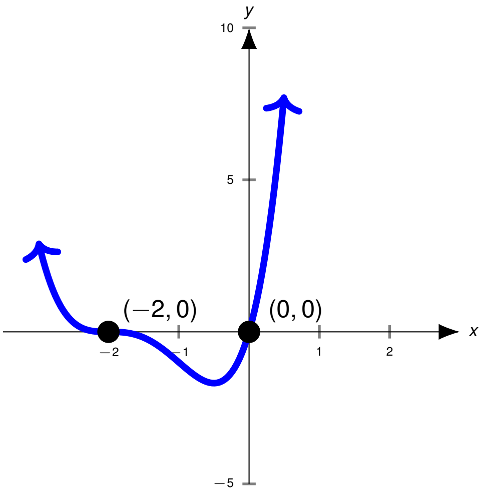


The graph of $y = f(x)$.

Solve $f(x) < 0$.

$f(x) < 0$ *on* $(-\infty, -6) \cup (0, 6)$

Problem 56 *Use the the graph of the given polynomial function to solve the stated inequality.*

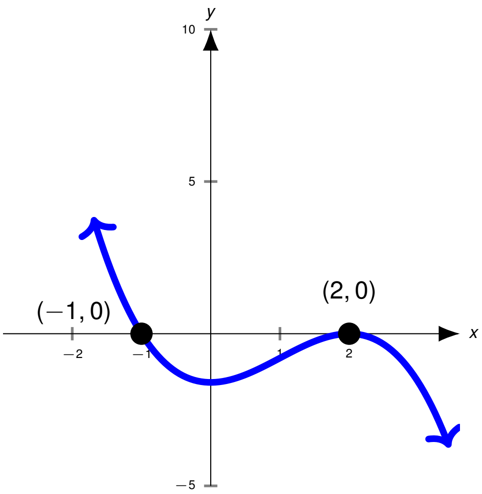


The graph of $y = g(t)$.

Solve $g(t) > 0$.

$g(t) > 0$ *on* $(-\infty, -2) \cup (0, \infty)$

Problem 57 *Use the the graph of the given polynomial function to solve the stated inequality.*

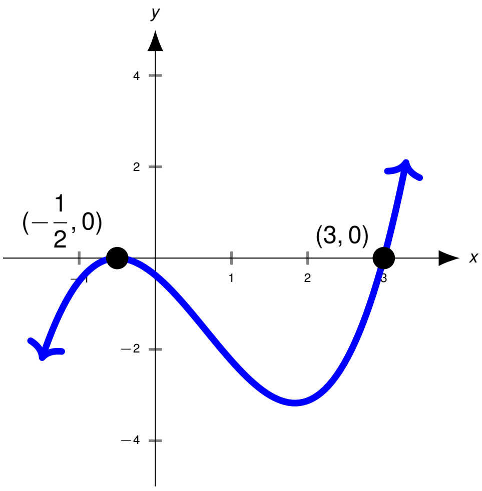


The graph of $y = p(z)$.

Solve $p(z) \geq 0$.

$p(z) \geq 0$ *on* $(-\infty, -1] \cup \{2\}$

Problem 58 *Use the the graph of the given polynomial function to solve the stated inequality.*

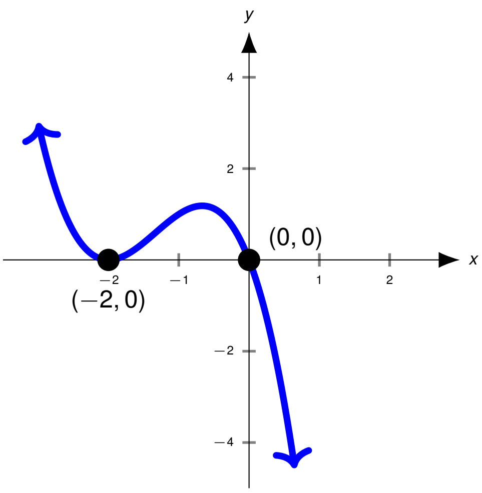


The graph of $y = f(x)$.

Solve $f(x) < 0$.

$f(x) < 0$ *on* $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 3\right)$

Problem 59 *Use the the graph of the given polynomial function to solve the stated inequality.*

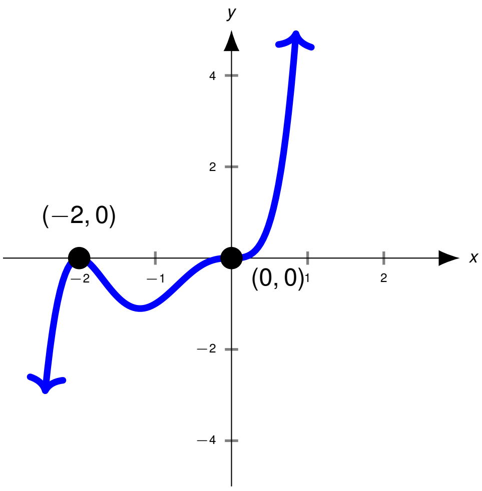


The graph of $y = F(s)$.

Solve $F(s) \leq 0$.

$F(s) \leq 0$ *on* $\{-2\} \cup [0, \infty)$

Problem 60 *Use the the graph of the given polynomial function to solve the stated inequality.*



The graph of $y = G(t)$.

Solve $G(t) \geq 0$.

$G(t) \geq 0$ *on* $\{-2\} \cup [0, \infty)$

Problem 61 *Use the Intermediate Value Theorem, Theorem **??**, to prove that $f(x) = x^3 - 9x + 5$ has a real zero in each of the following intervals: $[-4, -3]$, $[0, 1]$ and $[2, 3]$.*

Since $f(-4) = -23$, $f(-3) = 5$, $f(0) = 5$, $f(1) = -3$, $f(2) = -5$ and $f(3) = 5$ the Intermediate Value Theorem gives that $f(x) = x^3 - 9x + 5$ has real zeros in the intervals $[-4, -3]$, $[0, 1]$ and $[2, 3]$.

Problem 62 *Use the concepts of End Behavior and the Intermediate Value Theorem to prove any odd-degree polynomial function with real number coefficients has at least one real zero.*

An odd degree polynomial function f has 'mismatched' end behavior. That is, the end behavior of $f(x)$ is either: $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$ or $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -\infty$. This means at some point, $f(x) > 0$ and at some other point $f(x) < 0$. The Intermediate Value Theorem guarantees at least one place where $f(x) = 0$.

Problem 63 *Find an even-degree polynomial function with real number coefficients which has no real zeros.*

The function $f(x) = x^2 + 1$ has no real zeros.

Problem 64 *Continue the Bisection Method as introduced on **??** to approximate the real zero of $f(x) = x^5 - x - 1$ to three decimal places.*

$x \approx 1.167$.

Problem 65 *In this exercise, we prove $\sqrt{2}$ is an irrational number and approximate its value. Let $f(x) = x^2 - 2$.*

1. *Use Decartes' Rule of Signs to prove f has exactly one positive real zero.*

   $f(x)$ has only one variation in sign, so the result follows from Descartes' Rule of Signs.

2. *Use the Intermediate Value Theorem to prove f has a zero in $[1, 2]$.*

   $f(1) = -1 < 0$ and $f(2) = 2 > 0$ so the Intermediate Value Theorem promises a zero in $[1, 2]$.

3. *Use the Rational Zeros Theorem to prove f has no rational zeros.*

   The Rational Zeros Theorem gives the only possible rational zeros of f are ± 1 and ± 2. Since $f(\pm 1) = -1$ and $f(\pm 2) = 2$, f has no rational zeros.

4. *Use the Bisection Method (see* **??***) to approximate the zero of f on $[1, 2]$ to three decimal places.*
   The zero of f is $\sqrt{2} \approx 1.414$.

Problem 66 *Generalize the argument given in Exercise* **??** *to prove:*

1. *If N is not the perfect square of an integer, then $\sqrt{N}$ is irrational.*

   Hint: *Consider $f(x) = x^2 - N$.*

2. *For natural numbers $n \geq 2$, if N is not the perfect n^{th} power of an integer, then $\sqrt[n]{N}$ is irrational.*

   Hint: *Consider $f(x) = x^n - N$.*

Problem 67 *In Example* **??** *in Section* **??***, a box with no top is constructed from a 10 inch $\times$ 12 inch piece of cardboard by cutting out congruent squares from each corner of the cardboard and then folding the resulting tabs. We determined the volume of that box (in cubic inches) is given by the function $V(x) = 4x^3 - 44x^2 + 120x$, where x denotes the length of the side of the square which is removed from each corner (in inches), $0 < x < 5$. Solve the inequality $V(x) \geq 80$ analytically and interpret your answer in the context of that example.*

$V(x) \geq 80$ on $[1, 5 - \sqrt{5}] \cup [5 + \sqrt{5}, \infty)$. Only the portion $[1, 5 - \sqrt{5}]$ lies in the applied domain, however. In the context of the problem, this says for the volume of the box to be at least 80 cubic inches, the square removed from each corner needs to have a side length of at least 1 inch, but no more than $5 - \sqrt{5} \approx 2.76$ inches.

Problem 68 *From Exercise* **??** *in Section* **??***, $C(x) = .03x^3 - 4.5x^2 + 225x + 250$, for $x \geq 0$ models the cost, in dollars, to produce x PortaBoy game systems. If the production budget is \$5000, find the number of game systems which can be produced and still remain under budget.*

$C(x) \leq 5000$ on (approximately) $(-\infty, 82.18]$. The portion of this which lies in the applied domain is $(0, 82.18]$. Since x represents the number of game systems, we check $C(82) = 4983.04$ and $C(83) = 5078.11$, so to remain within the production budget, anywhere between 1 and 82 game systems can be produced.

Problem 69 *Let $f(x) = 5x^7 - 33x^6 + 3x^5 - 71x^4 - 597x^3 + 2097x^2 - 1971x + 567$. With the help of your classmates, find the x- and y- intercepts of the graph of f. Find the intervals on which the function is increasing, the intervals on which it is decreasing and the local extrema. Sketch the graph of f, using more than one picture if necessary to show all of the important features of the graph.*

Problem 70 *With the help of your classmates, create a list of five polynomials with different degrees whose real zeros cannot be found using any of the techniques in this section.*