Exercises for Power (In)Equalities

Question 1 *In Exercises* **??** - **??**, *solve the equation or inequality.*

Problem 1.1 $x + 1 = (3x + 7)^{\frac{1}{2}}$

$x = 3$

Problem 1.2 $2x + 1 = (3 - 3x)^{\frac{1}{2}}$

$x = \frac{1}{4}$

Problem 1.3 $t + (3t + 10)^{0.5} = -2$

$t = -3$

Problem 1.4 $3t + (6 - 9t)^{0.5} = 2$

Select All Correct Answers:

1. $t = -\frac{2}{3}$
2. $t = -\frac{1}{3}$ ✓
3. $x = 0$
4. $t = \frac{1}{3}$
5. $t = \frac{2}{3}$ ✓
6. $x = 3$

Problem 1.5 $x^{-1.5} = 8$

$x = \frac{1}{4}$

Problem 1.6 $2x - 1 = (x + 1)^{-0.5}$

$x = \frac{\sqrt{3}}{2}$

Problem 1.7 $t^{\frac{2}{3}} = 4$

$t = \pm 8$

Problem 1.8 $(t - 2)^{\frac{1}{2}} + (t - 5)^{\frac{1}{2}} = 3$

$t = 6$

Problem 1.9 $(2x + 1)^{\frac{1}{2}} = 3 + (4 - x)^{\frac{1}{2}}$

$x = 4$

Problem 1.10 $5 - (4 - 2x)^{\frac{2}{3}} = 1$

$x = -2, 6$

Problem 1.11 $2t^{\frac{2}{3}} = 6 - t^{\frac{1}{3}}$

$t = -8, \dfrac{27}{8}$

Problem 1.12 $2t^{\frac{1}{3}} = 1 - 3t^{\frac{2}{3}}$

$t = -1, \dfrac{1}{27}$

Problem 1.13 $2x^{1.5} = 15x^{0.75} + 8$

$x = 16, \dfrac{1}{16}$

Problem 1.14 $35x^{-0.75} = x^{-1.5} + 216$

$x = \dfrac{1}{81}, \dfrac{1}{16}$

Problem 1.15 $10 - \sqrt{t-2} \leq 11$

$[2, \infty)$

Problem 1.16 $t^{\frac{2}{3}} \leq 4$

$[-8, 8]$

Problem 1.17 $\sqrt[3]{x} \leq x$

$[-1, 0] \cup [1, \infty)$

Problem 1.18 $(2 - 3x)^{\frac{1}{3}} > 3x$

$\left(-\infty, \dfrac{1}{3}\right)$

Problem 1.19 $(t^2 - 1)^{-\frac{1}{2}} \geq 2$

$\left[-\dfrac{\sqrt{5}}{2}, -1\right) \cup \left(1, \dfrac{\sqrt{5}}{2}\right]$

Problem 1.20 $(t^2 - 1)^{-\frac{1}{3}} \leq 2$

$\left(-\infty, -\dfrac{3\sqrt{2}}{4}\right] \cup (-1, 1) \cup \left[\dfrac{3\sqrt{2}}{4}, \infty\right)$

Problem 1.21 $3(x-1)^{\frac{1}{3}} + x(x-1)^{-\frac{2}{3}} \geq 0$

$\left[\dfrac{3}{4}, 1\right) \cup (1, \infty)$

Problem 1.22 $3(x-1)^{\frac{2}{3}} + 2x(x-1)^{-\frac{1}{3}} \geq 0$

$\left(-\infty, \dfrac{3}{5}\right] \cup (1, \infty)$

Problem 1.23 $2(t-2)^{-\frac{1}{3}} - \dfrac{2}{3}t(t-2)^{-\frac{4}{3}} \leq 0$

$(-\infty, 2) \cup (2, 3]$

Problem 1.24 $-\frac{4}{3}(t-2)^{-\frac{4}{3}}+\frac{8}{9}t(t-2)^{-\frac{7}{3}}\geq 0$

$(2,6]$

Problem 1.25 $2x^{-\frac{1}{3}}(x-3)^{\frac{1}{3}}+x^{\frac{2}{3}}(x-3)^{-\frac{2}{3}}\geq 0$

$(-\infty,0)\cup[2,3)\cup(3,\infty)$

Problem 1.26 $\sqrt[3]{x^3+3x^2-6x-8}>x+1$

$(-\infty,-1)$

Problem 1.27 $4(7-t)^{0.75}-3t(7-t)^{-0.25}\leq 0$

$[4,7)$

Problem 1.28 $4t^{0.75}(t-3)^{-\frac{2}{3}}+9t^{-0.25}(t-3)^{\frac{1}{3}}<0$

$\left(0,\frac{27}{13}\right)$

Problem 1.29 $x^{-\frac{1}{3}}(x-3)^{-\frac{2}{3}}-x^{-\frac{4}{3}}(x-3)^{-\frac{5}{3}}(x^2-3x+2)\geq 0$

$(-\infty,0)\cup(0,3)$

Problem 1.30 $\frac{2}{3}(t+4)^{\frac{3}{5}}(t-2)^{-\frac{1}{3}}+\frac{3}{5}(t+4)^{-\frac{2}{5}}(t-2)^{\frac{2}{3}}\geq 0$

$(-\infty,-4)\cup\left(-4,-\frac{22}{19}\right]\cup(2,\infty)$

Problem 2 *The Cobb-Douglas production model*[1] *for the country of Sasquatchia is* $P=1.25L^{0.4}K^{0.6}$*. Here, P represents the country's production (measured in thousands of Bigfoot Bullion), L represents the total labor (measured in thousands of hours) and K represents the total investment in capital (measured in Bigfoot Bullion.)*

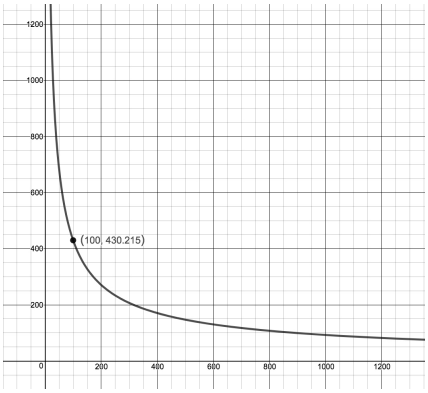
- *Let* $P=300$ *and solve for K as a function of L. If* $L=100$*, what is K? Interpret each of the quantities in this case.*

  $K=f(L)=(240)^{\frac{5}{3}}L^{-\frac{2}{3}}$*.* $f(100)\approx 430.2148$*. This means in order for the production level of Sasquatchia to reach 300,000 Bigfoot Bullion with a labor investment of 100,000 hours, the country needs to invest approximately 430 Bigfoot Bullion into capital.*

- *Graph your answer to* **??** *using a graphing utility. What information does an ordered pair* (L,K) *on this graph represent?*

  If a point (L,K) *is on the graph of this function, it means a combination of L thousand hours of labor with an investment of K Bigfoot Bullion into the Sasquatian Economy will result in a production level of 300,000 Bigfoot Bullion.*

[1] See Example **??** for more details on these sorts of models.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