EXERCISES FOR LIN SYSTEMS

Question 1 *In Exercises* **??** *-* **??**, *put each system of linear equations into triangular form and solve the system if possible. Classify each system as consistent independent, consistent dependent, or inconsistent.*

NOTE: Because triangular form is not unique, we give only one possible answer to that part of the question. Yours may be different and still be correct.

Problem 1.1 $\begin{cases} -5x+y &= 17 \\ x+y &= 5 \end{cases}$

$\begin{cases} x+y &= 5 \\ y &= 7 \end{cases}$

Consistent independent
Solution $(-2,7)$

Problem 1.2 $\begin{cases} x+y+z &= 3 \\ 2x-y+z &= 0 \\ -3x+5y+7z &= 7 \end{cases}$

$\begin{cases} x-\frac{5}{3}y-\frac{7}{3}z &= -\frac{7}{3} \\ y+\frac{5}{4}z &= 2 \\ z &= 0 \end{cases}$

Consistent independent
Solution $(1,2,0)$

Problem 1.3 $\begin{cases} 4x-y+z &= 5 \\ 2y+6z &= 30 \\ x+z &= 5 \end{cases}$

$\begin{cases} x-\frac{1}{4}y+\frac{1}{4}z &= \frac{5}{4} \\ y+3z &= 15 \\ 0 &= 0 \end{cases}$

Consistent dependent
Solution $(-t+5,-3t+15,t)$
for all real numbers t

Problem 1.4 $\begin{cases} 4x-y+z &= 5 \\ 2y+6z &= 30 \\ x+z &= 6 \end{cases}$

$\begin{cases} x-\frac{1}{4}y+\frac{1}{4}z &= \frac{5}{4} \\ y+3z &= 15 \\ 0 &= 1 \end{cases}$

Inconsistent
No solution

Problem 1.5 $\begin{cases} x+y+z &= -17 \\ y-3z &= 0 \end{cases}$

$\begin{cases} x+y+z &= -17 \\ y-3z &= 0 \end{cases}$

Consistent dependent
Solution $(-4t-17, 3t, t)$
for all real numbers t

Problem 1.6
$$\begin{cases} x-2y+3z &= 7 \\ -3x+y+2z &= -5 \\ 2x+2y+z &= 3 \end{cases}$$

$$\begin{cases} x-2y+3z &= 7 \\ y-\frac{11}{5}z &= -\frac{16}{5} \\ z &= 1 \end{cases}$$

Consistent independent
Solution $(2, -1, 1)$

Problem 1.7
$$\begin{cases} 3x-2y+z &= -5 \\ x+3y-z &= 12 \\ x+y+2z &= 0 \end{cases}$$

$$\begin{cases} x+y+2z &= 0 \\ y-\frac{3}{2}z &= 6 \\ z &= -2 \end{cases}$$

Consistent independent
Solution $(1, 3, -2)$

Problem 1.8
$$\begin{cases} 2x-y+z &= -1 \\ 4x+3y+5z &= 1 \\ 5y+3z &= 4 \end{cases}$$

$$\begin{cases} x-\frac{1}{2}y+\frac{1}{2}z &= -\frac{1}{2} \\ y+\frac{3}{5}z &= \frac{3}{5} \\ 0 &= 1 \end{cases}$$

Inconsistent
no solution

Problem 1.9
$$\begin{cases} x-y+z &= -4 \\ -3x+2y+4z &= -5 \\ x-5y+2z &= -18 \end{cases}$$

$$\begin{cases} x-y+z &= -4 \\ y-7z &= 17 \\ z &= -2 \end{cases}$$

Consistent independent
Solution $(1, 3, -2)$

Problem 1.10
$$\begin{cases} 2x-4y+z &= -7 \\ x-2y+2z &= -2 \\ -x+4y-2z &= 3 \end{cases}$$

$$\begin{cases} x-2y+2z &= -2 \\ y &= \frac{1}{2} \\ z &= 1 \end{cases}$$

Consistent independent
Solution $\left(-3, \frac{1}{2}, 1\right)$

Problem 1.11 $\begin{cases} 2x-y+z &= 1 \\ 2x+2y-z &= 1 \\ 3x+6y+4z &= 9 \end{cases}$

$$\begin{cases} x-\frac{1}{2}y+\frac{1}{2}z &= \frac{1}{2} \\ y-\frac{2}{3}z &= 0 \\ z &= 1 \end{cases}$$

Consistent independent
Solution $\left(\frac{1}{3}, \frac{2}{3}, 1\right)$

Problem 1.12 $\begin{cases} x-3y-4z &= 3 \\ 3x+4y-z &= 13 \\ 2x-19y-19z &= 2 \end{cases}$

$$\begin{cases} x-3y-4z &= 3 \\ y+\frac{11}{13}z &= \frac{4}{13} \\ 0 &= 0 \end{cases}$$

Consistent dependent
Solution $\left(\frac{19}{13}t+\frac{51}{13}, -\frac{11}{13}t+\frac{4}{13}, t\right)$
for all real numbers t

Problem 1.13 $\begin{cases} x+y+z &= 4 \\ 2x-4y-z &= -1 \\ x-y &= 2 \end{cases}$

$$\begin{cases} x+y+z &= 4 \\ y+\frac{1}{2}z &= \frac{3}{2} \\ 0 &= 1 \end{cases}$$

Inconsistent
no solution

Problem 1.14 $\begin{cases} x-y+z &= 8 \\ 3x+3y-9z &= -6 \\ 7x-2y+5z &= 39 \end{cases}$

$$\begin{cases} x-y+z &= 8 \\ y-2z &= -5 \\ z &= 1 \end{cases}$$

Consistent independent
Solution $(4, -3, 1)$

Problem 1.15
$$\begin{cases} 2x - 3y + z &= -1 \\ 4x - 4y + 4z &= -13 \\ 6x - 5y + 7z &= -25 \end{cases}$$

$$\begin{cases} x - \frac{3}{2}y + \frac{1}{2}z &= -\frac{1}{2} \\ y + z &= -\frac{11}{2} \\ 0 &= 0 \end{cases}$$

Consistent dependent
Solution $\left(-2t - \frac{35}{4}, -t - \frac{11}{2}, t\right)$
for all real numbers t

Problem 1.16
$$\begin{cases} 2x_1 + x_2 - 12x_3 - x_4 &= 16 \\ -x_1 + x_2 + 12x_3 - 4x_4 &= -5 \\ 3x_1 + 2x_2 - 16x_3 - 3x_4 &= 25 \\ x_1 + 2x_2 - 5x_4 &= 11 \end{cases}$$

$$\begin{cases} x_1 + \frac{2}{3}x_2 - \frac{16}{3}x_3 - x_4 &= \frac{25}{3} \\ x_2 + 4x_3 - 3x_4 &= 2 \\ 0 &= 0 \\ 0 &= 0 \end{cases}$$

Consistent dependent
Solution $(8s - t + 7, -4s + 3t + 2, s, t)$
for all real numbers s and t

Problem 1.17
$$\begin{cases} x_1 - x_3 &= -2 \\ 2x_2 - x_4 &= 0 \\ x_1 - 2x_2 + x_3 &= 0 \\ -x_3 + x_4 &= 1 \end{cases}$$

$$\begin{cases} x_1 - x_3 &= -2 \\ x_2 - \frac{1}{2}x_4 &= 0 \\ x_3 - \frac{1}{2}x_4 &= 1 \\ x_4 &= 4 \end{cases}$$

Consistent independent
Solution $(1, 2, 3, 4)$

Problem 1.18
$$\begin{cases} x_1 - x_2 - 5x_3 + 3x_4 &= -1 \\ x_1 + x_2 + 5x_3 - 3x_4 &= 0 \\ x_2 + 5x_3 - 3x_4 &= 1 \\ x_1 - 2x_2 - 10x_3 + 6x_4 &= -1 \end{cases}$$

$$\begin{cases} x_1 - x_2 - 5x_3 + 3x_4 &= -1 \\ x_2 + 5x_3 - 3x_4 &= \frac{1}{2} \\ 0 &= 1 \\ 0 &= 0 \end{cases}$$

Inconsistent
No solution

Problem 2 *Find two other forms of the parametric solution to Exercise* **??** *above by reorganizing the equations so that x or y can be the free variable.*

If x is the free variable then the solution is $(t, 3t, -t+5)$ and if y is the free variable then the solution is $\left(\frac{1}{3}t, t, -\frac{1}{3}t+5\right)$.

Problem 3 *At The Crispy Critter's Head Shop and Patchouli Emporium along with their dried up weeds, sunflower seeds and astrological postcards they sell an herbal tea blend. By weight, Type I herbal tea is 30% peppermint, 40% rose hips and 30% chamomile, Type II has percents 40%, 20% and 40%, respectively, and Type III has percents 35%, 30% and 35%, respectively. How much of each Type of tea is needed to make 2 pounds of a new blend of tea that is equal parts peppermint, rose hips and chamomile?*

$\frac{4}{3}-\frac{1}{2}t$ pounds of Type I, $\frac{2}{3}-\frac{1}{2}t$ pounds of Type II and t pounds of Type III where $0 \leq t \leq \frac{4}{3}$.

Problem 4 *Discuss with your classmates how you would approach Exercise* **??** *above if they needed to use up a pound of Type I tea to make room on the shelf for a new canister.*

Problem 5 *If you were to try to make 100 mL of a 60% acid solution using stock solutions at 20% and 40%, respectively, what would the triangular form of the resulting system look like? Explain.*