

EXERCISES FOR PAR FRAC

Question 1 In Exercises ?? - ??, find only the form needed to begin the process of partial fraction decomposition. Do not create the system of linear equations or attempt to find the actual decomposition.

Problem 1.1 $\frac{7}{(x-3)(x+5)}$

$$\frac{A}{x-3} + \frac{B}{x+5}$$

Problem 1.2 $\frac{5x+4}{x(x-2)(2-x)}$

$$\frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

Problem 1.3 $\frac{m}{(7x-6)(x^2+9)}$

$$\frac{A}{7x-6} + \frac{Bx+C}{x^2+9}$$

Problem 1.4 $\frac{ax^2+bx+c}{x^3(5x+9)(3x^2+7x+9)}$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{5x+9} + \frac{Ex+F}{3x^2+7x+9}$$

Problem 1.5 $\frac{\text{A polynomial of degree } < 9}{(x+4)^5(x^2+1)^2}$

$$\frac{A}{x+4} + \frac{B}{(x+4)^2} + \frac{C}{(x+4)^3} + \frac{D}{(x+4)^4} + \frac{E}{(x+4)^5} + \frac{Fx+G}{x^2+1} + \frac{Hx+I}{(x^2+1)^2}$$

Problem 1.6 $\frac{\text{A polynomial of degree } < 7}{x(4x-1)^2(x^2+5)(9x^2+16)}$

$$\frac{A}{x} + \frac{B}{4x-1} + \frac{C}{(4x-1)^2} + \frac{Dx+E}{x^2+5} + \frac{Fx+G}{9x^2+16}$$

Question 2 In Exercises ?? - ??, find the partial fraction decomposition of the following rational expressions.

Problem 2.1 $\frac{2x}{x^2-1} = \frac{1}{x+1} + \frac{1}{x-1}$

Problem 2.2 $\frac{-7x+43}{3x^2+19x-14}$

$$\frac{-7x+43}{3x^2+19x-14} = \frac{5}{3x-2} - \frac{4}{x+7}$$

Problem 2.3 $\frac{11z^2-5z-10}{5z^3-5z^2}$

$$\frac{11z^2-5z-10}{5z^3-5z^2} = \frac{3}{z} + \frac{2}{z^2} - \frac{4}{5(z-1)}$$

Problem 2.4 $\frac{-2z^2+20z-68}{z^3+4z^2+4z+16}$

$$\frac{-2z^2+20z-68}{z^3+4z^2+4z+16} = -\frac{9}{z+4} + \frac{7z-8}{z^2+4}$$

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Problem 2.5 $\frac{-s^2 + 15}{4s^4 + 40s^2 + 36}$

$$\frac{-s^2 + 15}{4s^4 + 40s^2 + 36} = \frac{1}{2(s^2 + 1)} - \frac{3}{4(s^2 + 9)}$$

Problem 2.6 $\frac{-21s^2 + s - 16}{3s^3 + 4s^2 - 3s + 2}$

$$\frac{-21s^2 + s - 16}{3s^3 + 4s^2 - 3s + 2} = -\frac{6}{s + 2} - \frac{3s + 5}{3s^2 - 2s + 1}$$

Problem 2.7 $\frac{5x^4 - 34x^3 + 70x^2 - 33x - 19}{(x - 3)^2}$

$$\frac{5x^4 - 34x^3 + 70x^2 - 33x - 19}{(x - 3)^2} = 5x^2 - 4x + 1 + \frac{9}{x - 3} - \frac{1}{(x - 3)^2}$$

Problem 2.8 $\frac{x^6 + 5x^5 + 16x^4 + 80x^3 - 2x^2 + 6x - 43}{x^3 + 5x^2 + 16x + 80}$

$$\frac{x^6 + 5x^5 + 16x^4 + 80x^3 - 2x^2 + 6x - 43}{x^3 + 5x^2 + 16x + 80} = x^3 + \frac{x + 1}{x^2 + 16} - \frac{3}{x + 5}$$

Problem 2.9 $\frac{-7z^2 - 76z - 208}{z^3 + 18z^2 + 108z + 216}$

$$\frac{-7z^2 - 76z - 208}{z^3 + 18z^2 + 108z + 216} = -\frac{7}{z + 6} + \frac{8}{(z + 6)^2} - \frac{4}{(z + 6)^3}$$

Problem 2.10 $\frac{-10z^4 + z^3 - 19z^2 + z - 10}{z^5 + 2z^3 + z}$

$$\frac{-10z^4 + z^3 - 19z^2 + z - 10}{z^5 + 2z^3 + z} = -\frac{10}{z} + \frac{1}{z^2 + 1} + \frac{z}{(z^2 + 1)^2}$$

Problem 2.11 $\frac{4s^3 - 9s^2 + 12s + 12}{s^4 - 4s^3 + 8s^2 - 16s + 16} = \frac{1}{s - 2} + \frac{4}{(s - 2)^2} + \frac{3s + 1}{s^2 + 4}$

Problem 2.12 $\frac{2s^2 + 3s + 14}{(s^2 + 2s + 9)(s^2 + s + 5)}$

$$\frac{2s^2 + 3s + 14}{(s^2 + 2s + 9)(s^2 + s + 5)} = \frac{1}{s^2 + 2s + 9} + \frac{1}{s^2 + s + 5}$$

Problem 3 Find a partial fraction decomposition of $R(z) = \frac{4}{z^4 - 1}$ over the complex numbers.

$$R(z) = \frac{4}{z^4 - 1} = \frac{1}{z - 1} - \frac{1}{z + 1} + \frac{i}{z - i} - \frac{i}{z + i}$$

Problem 4 In light of Theorem ??, we know all polynomial functions can be reduced to a product of linear factors - if we use complex numbers. It turns out that Theorem ?? holds true with complex numbers as well (though when using complex numbers, there are no irreducible quadratics.) Discuss with your classmates how this ultimately means every rational function is a sum of shifted Laurent Monomials.¹

¹See Section ??.

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Problem 5 *One of the most common algebraic error the authors encounter when teaching Calculus is along the lines of:*

$$\frac{8}{x^2 - 9} \neq \frac{8}{x^2} - \frac{8}{9}$$

Think about why if the above were true, this section would have no need to exist.