

1 Forms HW

Exercise 1 Consider the following 1-form in \mathbb{R}^2 :

$$\omega = x^2 e^y dy.$$

Compute and simplify $d\omega$.

Exercise 2 Consider the following 1-form in \mathbb{R}^2 :

$$\omega = 3xy dx + \cos x dy.$$

Compute and simplify $d\omega$.

Exercise 3 Consider the following 1-form in \mathbb{R}^3 :

$$\omega = xe^y dx + 5y^2 \sin z dy + 7xz dz.$$

Compute and simplify $d\omega$.

Exercise 4 Consider the following 1-form in \mathbb{R}^3 :

$$\omega = x^2 dy - 3y dz.$$

Compute and simplify $d\omega$.

Exercise 5 Consider the following 2-form in \mathbb{R}^3 :

$$\omega = xyz^2 dy \wedge dz + 2 \sin x \cos y dz \wedge dx + ye^z dx \wedge dy.$$

Compute and simplify $d\omega$.

Exercise 6 Consider the following 1-form in \mathbb{R}^2 :

$$\omega = 2x dx + \cos y dy.$$

Let $C \subset \mathbb{R}^2$ be the straight line from $(-1, 0)$ to $(3, \pi/2)$. Compute

$$\int_C \omega d\gamma.$$

Exercise 7 Consider the following 1-form in \mathbb{R}^3 :

$$\omega = e^x dx + \cos z dz.$$

Let $C \subset \mathbb{R}^3$ be the portion of the circle $z = \sqrt{1 - x^2}$ in the plane $y = 0$ from $(1, 0, 0)$ to $(0, 0, 1)$. Compute

$$\int_C \omega d\gamma.$$

Exercise 8 Consider the following 2-form in \mathbb{R}^3 :

$$\omega = e^x dy \wedge dz + (5 - y) dz \wedge dx + x^3 \cos y dx \wedge dy.$$

Let $\Sigma \subset \mathbb{R}^3$ be the square with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 0, 1)$, $(0, 0, 1)$, oriented towards the y -axis. Compute

$$\iint_{\Sigma} \omega dS.$$