

1 Curves and line integrals of scalar functions

Exercise 1 Find the curve of intersection of the cylinder $(x-3)^2+(z+1)^2 = 9$ and the plane $2x + 3y - 2z = 5$

Hint: First find $x(t)$ and $z(t)$ so that no matter how you define $y(t)$, the curve you get lives in the cylinder and wraps around once.

Exercise 2 Find parametrizations of the curves that form the sides of the triangle Σ , where Σ is the intersection of the plane $-x - 2y + z = 7$ with the first octant.

Hint: For points p and q , the line segment from p to q can be parametrized as $\gamma(t) = p + t(q - p)$ with $0 \leq t \leq 1$.

Exercise 3 Find parametrizations of the curves that form the edges of the surface Σ , where Σ is the portion of the sphere $(x-3)^2 + (y-2)^2 + (z-5)^2 = 36$ with $x \geq 3$ and $z \geq 5$.

Hint: Σ is like the peel of an orange slice. The edges are two half-circles going from $(3, -4, 5)$ to $(3, 8, 5)$

Exercise 4 Determine whether the curve

$$\gamma(t) = (t \sin t, t^2 - 1, 3t + \cos t)$$

passes through the point $(\pi/2, \pi^2/4 - 1, 3\pi/2)$. If it does, determine

- the times at which it does
- the lines tangent to the curve at those times
- the speed of the curve at those times

Exercise 5 Determine whether the curve

$$\gamma(t) = (t^2 - 2, t^3 - 2, 5t + t^2)$$

passes through the point $(7, 25, 24)$. If it does, determine

- the times at which it does
- the lines tangent to the curve at those times
- the speed of the curve at those times

Exercise 6 Determine whether the curve

$$\gamma(t) = (5 + t, t^2 - 1, 3t^2 - 2t)$$

passes through the point $(7, 2, 8)$. If it does, determine

- the times at which it does
- the lines tangent to the curve at those times
- the speed of the curve at those times

Exercise 7 Determine whether the curve

$$\gamma(t) = (t^2 + 2t + 1, t^3 + 3t^2 - t + 2, t^3 - 7t + 3)$$

passes through the point $(4, 5, -3)$. If it does, determine

- the times at which it does
- the lines tangent to the curve at those times
- the speed of the curve at those times

Exercise 8 Find the length of the curve

$$\gamma(t) = (e^t \cos t, e^t \sin t)$$

with $0 \leq t \leq 8\pi$. Draw a sketch of the curve.

Hint: The length of a curve is the integral of the speed $|\gamma'(t)|$ from initial time to final time.

Exercise 9 Find the length of the curve

$$\gamma(t) = (t - \sin t, 1 - \cos t)$$

with $0 \leq t \leq 4\pi$. Draw a sketch of the curve.

Hint: The length of a curve is the integral of the speed $|\gamma'(t)|$ from initial time to final time.

Exercise 10 Find the length of the curve

$$\gamma(t) = (2t, e^t + e^{-t})$$

with $-\log 2 \leq t \leq \log 2$. Draw a sketch of the curve.

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Hint: The length of a curve is the integral of the speed $|\gamma'(t)|$ from initial time to final time.

Exercise 11 Compute the line integral

$$\int_C (x + 3y + z^2) ds$$

where C is the curve with parametrization $\gamma : [0, 2] \rightarrow \mathbb{R}^3$ given by

$$\gamma(t) = (t - 2, 2t + 3, 5 - t)$$

Hint: The parametrization of a curve can be used as a change of variables. In this case, $x = t - 2$, $y = 2t + 3$, $z = 5 - t$. This integral is computed using this change of variables with the Jacobian $|\gamma'(t)|$.

Exercise 12 Compute the line integral

$$\int_C (-8x + 3y - 5z) ds$$

where C is the curve with parametrization $\gamma : [-1, 3] \rightarrow \mathbb{R}^3$ given by

$$\gamma(t) = (t^2 - 4, t^2 - 2t - 5, 3 - t^2)$$

Hint: The parametrization of a curve can be used as a change of variables. In this case, $x = t^2 - 4$, $y = t^2 - 2t - 5$, $z = 3 - t^2$. This integral is computed using this change of variables with the Jacobian $|\gamma'(t)|$.

Exercise 13 Compute the line integral

$$\int_C (x + y + z) ds$$

where C is the curve with parametrization $\gamma : [0, 3\pi/2] \rightarrow \mathbb{R}^3$ given by

$$\gamma(t) = (\cos t, \sin t, t)$$

Hint: The parametrization of a curve can be used as a change of variables. In this case, $x = \cos t$, $y = \sin t$, $z = t$. This integral is computed using this change of variables with the Jacobian $|\gamma'(t)|$.

Exercise 14 Compute the line integral

$$\int_C x ds$$

where C is the portion of the parabola $y = x^2 + 1$ between $(0, 1)$ and $(2, 5)$.

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Hint: First find a parametrization of the curve. Then compute like the previous problems.

Exercise 15 Compute the line integral

$$\int_C x e^y ds$$

where C is the line segment from $(0, 2)$ to $(5, 0)$.

Hint: First find a parametrization of the curve. Then compute like the previous problems.

Exercise 16 Compute the line integral

$$\int_C \sqrt{2 + 4x + 8y} ds$$

where C is the portion of the curve of intersection of the cylinder $y = x^2$ and the plane $x + y + z = 0$ from $(0, 0, 0)$ to $(2, 4, -6)$

Hint: First find a parametrization of the curve. Then compute like the previous problems.