1 Line integrals of vector fields

Exercise 1 *Let $C \subset \mathbb{R}^3$ be the curve with parametrization $\gamma : [0,1] \to \mathbb{R}^3$ given by*

$$\gamma(t) = (t + t^2, t^4, 2t + t^2 - 3).$$

Compute

$$\int_C \langle -y, x + 2, z + 1 \rangle \cdot d\gamma$$

Exercise 2 *Let $C \subset \mathbb{R}^3$ be the curve with parametrization $\gamma : [0,\pi] \to \mathbb{R}^3$ given by*

$$\gamma(t) = (\sin t, e^t, \cos t).$$

Compute

$$\int_C \langle x + y, y + z, z + x \rangle \cdot d\gamma$$

Exercise 3 *Let $C \subset \mathbb{R}^3$ be the curve with parametrization $\gamma : [0,1] \to \mathbb{R}^3$ given by*

$$\gamma(t) = (t^2 \cos(4\pi t), t^2 + t^3, t^3 + t^4 + t^5).$$

Compute

$$\int_C \left[(yz + 2x)\, dx + (xz + z)\, dy + (xy + y + 3z^2)\, dz \right]$$

Exercise 4 *Let $C \subset \mathbb{R}^3$ be the piece of the parabola $x = y^2$ from $(0,0)$ to $(4,2)$. Compute*

$$\int_C \left[e^{y^2}\, dx + x\, dy \right]$$

Exercise 5 *Let $C \subset \mathbb{R}^3$ be the unit circle in the xy-plane travelled counter-clockwise when viewed from above. Compute*

$$\int_C \langle e^{x^2}, \sin(y^2), \cos(z^3) \rangle \cdot d\gamma$$

Exercise 6 *Let $C \subset \mathbb{R}^3$ be portion from $(-2,0,2)$ to $(2,0,-2)$ of the curve of intersection of the plane $x + y + z = 0$ and the cylinder $x^2 + z^2 = 8$. Compute*

$$\int_C \langle x + z, \cos y, e^z + x \rangle \cdot d\gamma$$

Exercise 7 *Let $C \subset \mathbb{R}^3$ be the curve with parametrization $\gamma : [0,1] \to \mathbb{R}^3$ given by*

$$\gamma(t) = (t^2 + 2t - 1, (3 + t)\sqrt{t^2 + 1}, e^t + 2t^2).$$

Compute

$$\int_C \langle e^x(z-1), z\cos y, e^x + \sin y\rangle \cdot d\gamma$$

Exercise 8 *Let $C \subset \mathbb{R}^3$ be the curve with parametrization $\gamma : [0, \pi/2] \to \mathbb{R}^3$ given by*

$$\gamma(t) = (2\sin t, \sin t, 3\cos t).$$

Compute

$$\int_C [yz\,dx + xy\,dy + xz\,dz]$$