

1 General changes of coordinates

Exercise 1 Using an appropriate change of variables, compute

$$\iint_D x^2 y \, dA$$

where $D \subset \mathbb{R}^2$ is the region between the hyperbolas $y = 2/x$ and $y = 4/x$ and the lines $y = 1$ and $y = 5$

Hint: The region can be described as

$$\begin{aligned} 2 &\leq xy \leq 4 \\ 1 &\leq y \leq 5 \end{aligned}$$

Exercise 2 Using an appropriate change of variables, compute

$$\iint_D x^2 \, dA$$

where $D \subset \mathbb{R}^2$ is the region between the hyperbolas $y = 2/x$ and $y = 3/x$ and the lines $y = x$ and $y = 4x$

Hint: The region can be described as

$$\begin{aligned} 2 &\leq xy \leq 3 \\ 1 &\leq y/x \leq 4 \end{aligned}$$

Exercise 3 Using an appropriate change of variables, compute

$$\iint_D 3x \, dA$$

where $D \subset \mathbb{R}^2$ is the parallelogram with vertices $(2, 0)$, $(4, 3)$, $(3, 4)$, $(1, 1)$.

Hint: The sides of the parallelogram have slopes $3/2$ and -1

Exercise 4 Using an appropriate change of variables, compute

$$\iint_D \frac{y}{3y+x} \, dA$$

where $D \subset \mathbb{R}^2$ is the parallelogram with vertices $(-1, 2)$, $(2, 1)$, $(3, 3)$, $(0, 4)$.

Hint: The sides of the parallelogram have slopes 2 and $-1/3$