1 Polar, cylindrical, and spherical coordinates

Exercise 1 *Using polar coordinates, compute*

$$\int_{-4}^{0}\int_{-\sqrt{16-x^2}}^{0} y\,dydx$$

Hint: *This is a quarter of the disk of radius 4*

Exercise 2 *Using polar coordinates, compute*

$$\int_{0}^{3/\sqrt{2}}\int_{y}^{\sqrt{9-y^2}} x^2y\,dxdy$$

Hint: *This is a sector with angle $\pi/4$ of the circle of radius 3*

Exercise 3 *Using polar coordinates, compute*

$$\iint_B x\,dxdy$$

where $B \subset \mathbb{R}^2$ is the region in the first quadrant inside the circle $x^2+(y-3)^2 = 9$

Hint: *This is a quarter of the unit disk*

Exercise 4 *Using polar coordinates, show that*

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx = \sqrt{2\pi}$$

Hint: *The square of the integral on the left is $\iint_{\mathbb{R}^2} e^{-\frac{x^2+y^2}{2}}\,dA$.*

Exercise 5 *Let $D \subset \mathbb{R}^2$ be the interior of the circle $(x-1)^2+(y-1)^2 = 2$. Using polar coordinates, find*

$$\iint_D \frac{x}{x^2+y^2}dA$$

Hint: *The notes have a section on how to describe a region like this in polar coordinates.*

Exercise 6 *Using cylindrical coordinates, compute*

$$\int_{0}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{y}^{4} x^2\,dzdxdy$$

Hint: *Deal separately with the x, y variables and the z variable. For cylindrical coordinates, most of the time you don't need to do anything to the z variable*

Exercise 7 *Using cylindrical coordinates, compute*

$$\int_0^1 \int_{-1}^1 \int_z^{\sqrt{1-y^2}} y^2 \, dxdydz$$

Hint: *This is the region inside the cylinder $x^2 + y^2 = 1$, above the xy-plane, and below the plane $z = x$. Verify this!!!*

Exercise 8 *Using spherical coordinates, compute*

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} y^2 \, dzdydx$$

Hint: *This is a portion of the ball of radius 2*

Exercise 9 *Using spherical coordinates, compute*

$$\iiint_E zy^2 \, dV$$

where E is the region above the cone $z = -\sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 25$

Hint: *The equation of the cone is $\phi = 3\pi/4$ and the equation of the sphere is $\rho = 5$*

Exercise 10 *Using spherical coordinates, compute*

$$\iiint_E \frac{zx^2}{x^2 + y^2} \, dV$$

where E is the region below the plane $z = 3$ and above the cone $z = \sqrt{x^2 + y^2}/\sqrt{3}$

Hint: *The equation of the plane is $\rho = 3/\cos\phi$*

Exercise 11 *Using spherical coordinates, compute*

$$\iiint_E z \, dV$$

where E is the region inside the sphere $x^2 + y^2 + (z-3)^2 = 9$

Hint: *The notes have a section on how to describe a region like this in polar coordinates.*