1 Stokes Theorem

Exercise 1 *Let $C \subset \mathbb{R}^3$ be the intersection of the cylinder $x^2 + y^2 = 16$ and plane $z = 2$, oriented counterclockwise when viewed from above. Compute*

$$\int_C \langle -y, x, 2z \rangle \cdot d\gamma$$

Exercise 2 *Let $C \subset \mathbb{R}^3$ be the trajectory that travels along straight lines through the points $(0,1,0)$, $(4,1,0)$, $(4,1,2)$, $(0,1,2)$, and back to $(0,1,0)$. Compute*

$$\int_C \langle y^2 + z, xy, x - z \rangle \cdot d\gamma$$

Exercise 3 *Let $C \subset \mathbb{R}^3$ be the intersection of the cylinder $y^2 + z^2 = 16$ and plane $x = 5$, oriented clockwise when seen from the tip of the x-axis. Compute*

$$\int_C \langle 5x - 3, 1 - z, 2y + 4 \rangle \cdot d\gamma$$

Exercise 4 *Let $C \subset \mathbb{R}^3$ be the curve that travels along straight lines first from $(3,0,0)$ to $(0,5,0)$, then from $(0,5,0)$ to $(0,0,15)$, and then from $(0,0,15)$ to $(3,0,0)$. Compute*

$$\int_C \langle 2y, -x + z, y \rangle \cdot d\gamma$$

Exercise 5 *Let $C \subset \mathbb{R}^3$ be the curve that goes from $(2,0)$ to $(-2,0)$ along the arc $x^2 + y^2 = 4$, $y \geq 0$, in the xy-plane, followed by the curve that goes back to $(2,0)$ along the parabola $z = x^2 - 4$ in the xz-plane. Compute*

$$\int_C \langle x + y, z + x, y + z \rangle \cdot d\gamma$$

Exercise 6 *Let $C_1 \subset \mathbb{R}^3$ be the circle $x^2 + y^2 = 1$ in the xy-plane, oriented counterclockwise, and $C_2 \subset \mathbb{R}^3$ the circle $x^2 + y^2 = 1$ in the plane $z = 3$, oriented counterclockwise. Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field with*

$$\mathit{Curl}(F)(x, y, z) = \langle x, y, 0 \rangle.$$

Which one is larger?

$$\int_{C_1} F \cdot d\gamma_1 \qquad \text{or} \qquad \int_{C_2} F \cdot d\gamma_2$$

Exercise 7 *Let $C_1 \subset \mathbb{R}^3$ be the circle $x^2 + y^2 = 1$ in the xy-plane, oriented counterclockwise, and $C_2 \subset \mathbb{R}^3$ the circle $x^2 + y^2 = 16$ in the xy-plane, oriented counterclockwise. Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field with*

$$Curl(F)(x, y, z) = \langle 0, 0, 1 \rangle.$$

Which one is larger?

$$\int_{C_1} F \cdot d\gamma_1 \qquad \text{or} \qquad \int_{C_2} F \cdot d\gamma_2$$

Exercise 8 *Let $C_1 \subset \mathbb{R}^3$ be the curve that travels along straight lines from $(4, 0, 0)$, to $(0, 4, 0)$, from $(0, 4, 0)$ to $(0, 0, 4)$, and from $(0, 0, 4)$ to $(4, 0, 0)$. Let $C_2 \subset \mathbb{R}^3$ be the curve that travels along straight lines from $(1, 0, 0)$, to $(0, 1, 0)$, from $(0, 1, 0)$ to $(0, 0, 1)$, and from $(0, 0, 1)$ to $(1, 0, 0)$. Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field with*

$$Curl(F)(x, y, z) = \langle x, y, z \rangle.$$

Which one is larger?

$$\int_{C_1} F \cdot d\gamma_1 \qquad \text{or} \qquad \int_{C_2} F \cdot d\gamma_2$$