1 Surfaces and surface integrals of scalar functions

Exercise 1 *Find a parametrization of the portion of the sphere $x^2+y^2+z^2 = 9$ above the plane $z = 3/2$. Specify the domain and the expression of the parametrization.*

Exercise 2 *Find a parametrization of the portion of the cylinder $x^2+y^2 = 16$ between the planes $x + y + z = 0$ and $2x + y - z = -10$. Specify the domain and the expression of the parametrization.*

Exercise 3 *Find a parametrization of the portion of the cone $z = \sqrt{3x^2 + 3y^2}$ outside the sphere $x^2+y^2+(z-2)^2 = 4$ and inside the sphere $x^2+y^2+(z-5)^2 = 25$. Specify the domain and the expression of the parametrization.*

Exercise 4 *Find a parametrization of the portion of the graph $z = \sin x+\sin y$ between the lines $x = 0$, $x = \pi$, $y = x$, and $y = \pi$. Specify the domain and the expression of the parametrization.*

Exercise 5 *For each of the previous problems, take your parametrization*

$$\varphi(u, v) = (x(u, v), y(u, v), z(u, v))$$

and consider the vector $\dfrac{\partial\varphi}{\partial u} \times \dfrac{\partial\varphi}{\partial v}$. Identify which side of the surface it comes out of. Construct a second parametrization of the same surface where the corresponding vector points in the opposite direction.

Exercise 6 *Using a parametrization and its Jacobian, find the area of the portion of the plane $5x+3y+4z = 60$ in the first octant ($x, y, z \geq 0$). Assumme a metal lamina has this shape and its density is given by $\rho(x, y, z) = \sin x + 3$. Find its mass.*

Exercise 7 *Find the area of the portion of the paraboloid $x = 9 - y^2 - z^2$ with $x \geq -16$. Assume a metal lamina has this shape and its density is given by $\rho(x, y, z) = x - y + 21$. Find its mass.*

Exercise 8 *Find*

$$\iint_\Sigma (z^2 + x + 3)\, dS$$

where Σ is the portion of the sphere $x^2 + y^2 + z^2 = 16$ above the plane $z = -2$.

Exercise 9 *Find*

$$\iint_\Sigma (x^2 + 3x + 2y^2 - 2y + z^2 + 3)\, dS$$

where Σ is the portion of the cone $z = \sqrt{3x^2 + 3y^2}$ outside the sphere $x^2 + y^2 + (z-2)^2 = 4$ and inside the sphere $x^2 + y^2 + (z-5)^2 = 25$.

Exercise 10 *Consider the surface $\Sigma \subset \mathbb{R}^3$ with parametrization*

$$\varphi : [0, 2\pi] \times [0, 2\pi] \to \mathbb{R}^3$$

given by

$$\varphi(u, v) = ((3 + \cos v)\cos u, (3 + \cos v)\sin u, \sin v)$$

Describe the surface Σ and find its area. Compute

$$\iint_\Sigma (x + 2y + z^2 + 3)\, dS$$

Exercise 11 *Compute*

$$\iint_\Sigma (xz + y)\, dS$$

where $\Sigma \subset \mathbb{R}^3$ is the helicoid with parametrization

$$\varphi(u, v) = (u\cos v, u\sin v, v)$$

with $1 \leq u \leq 3$, $0 \leq v \leq 4\pi$.